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Some issues about FL and LAB servo control

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Abstract: In this paper, the servo control design methodologies based on Feedback Linearization (FL) and Linear Algebra Based (LAB) are applied to nonlinear systems with and without zero dynamics, analysing the case of unstable zero dynamics. Their advantages and drawbacks are discussed. Some examples are developed to illustrate the results.

Keywords: Nonlinear plants, Linear Algebra Based control, feedback linearization.

1. INTRODUCTION

There is a pleiad of design methodologies to tackle the problem of controlling the trajectory of a liner/nonlinear plant (see, for instance, Gruyitch [2017], Jarzebowska [2012]), and there is a lot of literature dealing with the control design for nonlinear processes (see, e.g., Grimble and Majecki [2020], Khalil [2002], Slotine and Li [1991] or Isidori [1995]). Trajectory tracking of nonlinear systems is a fundamental and essential control problem in many applications (see, e.g., Yuan et al. [2020] and the references therein, Ren and Beard [2004]; Luo et al. [2005]; Aguiar and Hespanha [2007]; Li et al. [2019]). Other than the simplest solution of linearizing the process, leading to a local control, Lyapunov theory, Kubo et al. [2020], passivity, Wu et al. [2019], sliding mode control, Okwudire and Altintas [2009] or backstepping approaches provide tools to design nonlinear controllers. In the literature, some tracking controller design methods for particular classes of nonlinear systems have been proposed. Most of these methodologies are based on complex control theories leading to complicated control design procedures, also requiring a deep knowledge of the geometrical properties of the plant model.

As there are so many process models under the umbrella of nonlinear plants, there is no general better approach to design the control. Among the many options to deal with this problem, if the plant model is well known, model inversion is a way to get an appropriate control. In our opinion, there are two methodologies that, in some cases, can be easily applied producing comparable results. Both are based on some kind of model inversion and thus, a precise model is required. Feedback linearization (FL), Isidori [1995], is a well-known appealing control design methodology simplifying the treatment of nonlinear systems by reducing the setting to the linear case. Unfortunately, the approach is not always applicable and the baseline involves two steps, first the process model linearization and then the process control design. And some nonlinearities could be advantageous from the control viewpoint and they do not need to be cancelled.

On the other hand, the new methodology denoted as LAB control design Scaglia et al. [2020a], Scaglia et al. [2020b], is also simple to apply but, again, it is not always applicable and its simplicity is lost if some algebraic computations come out to be complicated. Both methodologies present some similarities leading to similar control laws if the control design parameters are properly tuned.

In this paper, these methodologies are tentatively applied to different plants, with and without zero dynamics, and the control solutions are compared. Three different scenarios are considered. First, a simple academic example based on the van der Pol equation is studied to show the similarities of both techniques. Then, a chemical reactor is considered to emphasize the different steps in reaching the control law. Finally, the trajectory control for a typical nonlinear model is developed and the advantages and drawbacks of each methodology are summarized.

2. SIMPLE NONLINEAR SYSTEM

Let us start with a simple academic nonlinear system, by considering a system described by the van der Pol equation, van der Pol [1920], where an additional linear term has been added.

2.1 Nonlinear system without zero dynamics

Let us assume that the output is associated with the first state variable.

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$$\dot{x}_1(t) = bx_1(t) + x_2(t) \tag{1}$$
$$\dot{x}_2(t) = -x_1(t) + \mu(1 - x_1^2(t))x_2(t) + u(t) \tag{2}$$
$$y(t) = x_1(t) \tag{3}$$

where b is a parameter to be defined later on and μ represents the strenght of damping. In the following, the time argument, if it is not relevant, is omitted to simplify the notation.

FL control In this approach, the plant model should be analyzed and expressed in a normal form, the zero dynamics being determined, Isidori [1995]. In this way, the output (3) derivatives are obtained until the input appears:

$$\ddot{y} = b\dot{x}_1 + \dot{x}_2 = (b^2 - 1)x_1 + bx_2 + \mu(1 - x_1^2)x_2 + u \tag{4}$$

Thence it results that the relative degree is $\rho = 2$, and then, as the process model is a second order model, the zero dynamics is null. To apply the feedback linearization, the model is converted into the Byrnes-Isidori normal form:

$$\xi_1 = x_1; \quad \xi_2 = bx_1 + x_2 \tag{5}$$

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$$\dot{\xi}_1 = \xi_2 \tag{6}$$
$$\dot{\xi}_2 = -\xi_1 + b\xi_2 + \mu(1 - \xi_1^2)(\xi_2 - b\xi_1) + u \tag{7}$$
$$y = \xi_1 \tag{8}$$

Thus, the FL control input will be the sum of a state feedback linearizing control, u_l, obtained by the inversion of (7), a state feedback stabilization control, u_c, computed by using any control design methodology for linear systems, and a feedforward control, u_r, from the reference and its derivatives. That is

$$u = u_l + u_c + u_r \tag{9}$$
$$u_l = \xi_1 - b\xi_2 - \mu(1 - \xi_1^2)(\xi_2 - b\xi_1) \tag{10}$$
$$u_c = -k_1\xi_1 - k_2\xi_2 \tag{11}$$

where k_1, k_2 are the control parameters to, for instance, assign the closed-loop poles, leading to

$$\frac{\xi_1}{u_r} = \frac{1}{s^2 + k_2 s + k_1} \tag{12}$$

Thus, if the feedforward control u_r is obtained from the reference y_r and its derivatives, being

$$u_r = \ddot{y}_r + k_2\dot{y}_r + k_1 y_r \tag{13}$$

then

$$y = y_r$$

if the initial conditions are the same.

By using the initial state variables, (1)-(2), the feedback control (10)- (11) will be

$$u_l = x_1 - b^2 x_1 - bx_2 - \mu(1 - x_1^2)x_2 \tag{14}$$
$$u_c = -(k_1 + bk_2)x_1 - k_2 x_2 \tag{15}$$
$$u_b = u_l + u_c \tag{16}$$

The control decision is the selection of the coefficients in (11), to define the position of the closed-loop poles.

LAB control Let us now design the control based on the LAB approach, Scaglia et al. [2020a]. The process model is written as

$$\dot{y} = by + z \tag{17}$$
$$\dot{z} = -y + \mu(1 - y^2)z + u \tag{18}$$

being: $[x_1\, x_2]^T = [y\, z]^T$, where y is the process variable to track the reference and z is the so-called sacrificed variable.

The reference for the output as well as its derivatives are assumed to be accessible. Following the LAB methodology, the trajectory approaching is defined by

$$\Delta y = \dot{y}_r + k_y(y_r - y) = by + z \tag{19}$$
$$\Delta z = \dot{z}_r + k_z(z_r - z) = -y + \mu(1 - y^2)z + u \tag{20}$$

In this methodology, the design parameters are the trajectory approaching coefficients, k_y, k_z. The reference for the sacrificed variable is determined by inversion of (19), being

$$z_r = \Delta_y - by; \quad \dot{z}_r = \dot{\Delta}_y - b\dot{y} \tag{21}$$

Thus, taking into account (19), (21), the control input is obtained from (20), being composed by the feedforward term, u_r, and the feedback one, u_c,

$$u = u_r + u_b; \tag{22}$$
$$u_r = \ddot{y}_r + (k_y + k_z)\dot{y}_r + k_y k_z y_r \tag{23}$$
$$u_b = -(k_y b + b^2 + k_z k_y + bk_z - 1)y \\ -(k_y + k_z + b + \mu(1 - y^2))z \tag{24}$$

Comparison It is easy to verify that the control signal is the same in both cases: that is (16) and (24) are similar, assuming the equivalence between the controller parameters defined in both approaches

$$k_1 = k_y k_z; \quad k_2 = k_y + k_z \tag{25}$$

In the LAB approach, the controlled plant model, combining (24) and (18), will be

$$\dot{y} = by + z \tag{26}$$
$$\dot{z} = -(k_y b + b^2 + k_z k_y + bk_z)y - (k_y + k_z + b)z + u_r \tag{27}$$

Thus, the controlled plant transfer function is similar to (12) and the feedforward control is like (13). The control solution is the same although the design parameters are different but equivalent (25). In the FL approach the design parameters refer to the closed loop poles whereas in the LAB approach they are defined based on the convergence speed of the state variables to their references.

2.2 Nonlinear system with zero dynamics

Let us now consider the same system (1) - (2) but a new output, $y = x_2$. Assuming

$$z = x_1; \qquad \xi = y = x_2 \tag{28}$$

the Byrnes-Isidori normal form is, simply:

$$\dot{z} = bz + y \tag{29}$$

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$$\dot{\xi} = -z + \mu(1 - z^2)y + u \tag{30}$$

Thence, the relative degree is $\rho = 1$ and the zero dynamics, (29), is order one.

FL control The input/output FL for this system is (30)

$$u = x_1 - \mu(1 - x_1^2)x_2 + u_c + u_r \tag{31}$$

where u_c is the control feedback to define the controlled plant dynamics and u_r is the feedforward control to follow the reference. The linearized plant model is given by

$$\begin{aligned} \dot{x}_1 &= bx_1 + x_2 \\ \dot{x}_2 &= u_c + u_r \end{aligned} \tag{32}$$

Thus, the linear control

$$u_c = -c_2 x_2 \tag{33}$$

will stabilize the controlled plant if $c_2 > 0$ and $b < 0$. The controlled plant transfer function is given by

$$\frac{y}{u_r} = \frac{1}{s + c_2} \tag{34}$$

and the feedforward

$$u_r = \dot{y}_r + c_2 y_r \tag{35}$$

will allow for a perfect trajectory tracking.

It should be noticed that the pole at $-b$ is unaffected by the control. Nevertheless, as the system (32) is controllable, to modify this pole linked to the first variable the feedback control should be

$$u_c = -c_1 x_1 - c_2 x_2 \tag{36}$$

and c_1, c_2 are the controller parameters in this setting. Thus, the closed-loop system is

$$\dot{x}_1 = bx_1 + x_2 \tag{37}$$

$$\dot{x}_2 = -c_1 x_1 - c_2 x_2 + u_r \tag{38}$$

$$y = x_2 \tag{39}$$

and the closed-loop transfer function is given by

$$\frac{y}{u_r} = \frac{s - b}{s^2 + (c_2 - b)s + c_1 - bc_2} \tag{40}$$

To ensure the closed-loop stability it should be

$$c_2 > b; \quad c_1 > bc_2 \tag{41}$$

and the feedforward control, in order to achieve perfect reference tracking, should be the composed signal of the reference and its derivatives,

$$u_r = \ddot{y}_r + (c_2 - b)\dot{y}_r + (c_1 - bc_2) y_r \tag{42}$$

filtered by

$$F(s) = \frac{1}{s - b} \tag{43}$$

Thence, to achieve internal stability, it should be $b < 0$.

LAB control The new plant model will be

$$\dot{z} = bz + y \tag{44}$$

$$\dot{y} = -z + \mu(1 - z^2)y + u \tag{45}$$

and the desired behavior is defined by the control coefficient c_y as

$$\dot{y} = \dot{y}_r + c_y(y_r - y) \tag{46}$$

Combining (45) and (46) it yields

$$u = \dot{y}_r + c_y(y_r - y) + z - \mu(1 - z^2)y \tag{47}$$

In this case, the control feedback by using the original state variables is given by

$$u_b = z - c_y y - \mu(1 - z^2)y \tag{48}$$

whereas the feedforward component will be

$$u_r = \dot{y}_r + c_y y_r \tag{49}$$

similar to (31). But, what about the stability of the controlled plant? Internally there is a pole at b, which would be unstable for $b > 0$. Thus, the controlled plant would be unstable as the control signal is not affecting the b pole, and a complementary state feeddback, as in (36), should be added.

2.3 Non minimum phase (NMP) plants

If $b > 0$, the zero dynamics is unstable. To avoid the filter (43) unstability, the original system (1) - (2) can be approximated by

$$\begin{aligned} \dot{x}_1 &= -bx_1 + x_2 \\ \dot{x}_2 &= -x_1 + \mu(1 - x_1^2)x_2 + \bar{u} \\ \dot{\bar{u}} + b\bar{u} &= \dot{u} - bu \end{aligned} \tag{50}$$

and, taking into account the time delay Padé approximation for $T = 2b$

$$e^{-Ts} \approxeq \frac{1 - T/2s}{1 + T/2s},$$

the controlled output will be

$$y(t) \approxeq y(t - 2b) \tag{51}$$

Thus, the control $\bar{u}$ for the system (50) is designed as before, by using the stable low pass filter

$$F(s) = \frac{1}{s + b} \tag{52}$$

3. CONCENTRATION TRACKING IN CSTR

Let us now consider a more involved process model to make more explicit the difficulties in deriving the control signals. A continuous stirred tank reactor (CSTR) as depicted in Figure 1, is used.

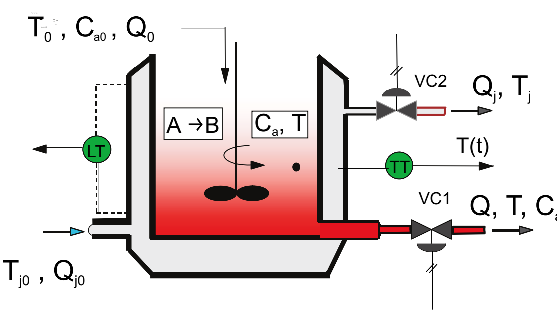

Fig. 1. CSTR general schema

A first order, exothermic, irreversible reaction: A → B, as shown in figure 1, is considered. The reaction heat is removed by a cooling jacket surrounding the reactor. Negligible heat losses and perfectly mixing are assumed, Coughanowr [1991], Luyben [1990]. The jacket water is assumed to be perfectly mixed and the mass of the metal walls is considered negligible, so the thermal inertia of the metal is not considered.

A model based on material and energy balances can be described by, Perez and Albertos [2004]:

$$V \frac{dC_a(t)}{dt} = q(t)(C_{a,0} - C_a(t)) - \alpha(t)C_a^2(t); \; \alpha(t) = \alpha_o e^{-\frac{E}{RT(t)}} \tag{53}$$

$$V \frac{dT(t)}{dt} = q(t)(T_0 - T(t)) + \frac{H\alpha(t)}{\rho c_p} C_a^2(t) - \frac{UA}{\rho c_p}[T(t) - T_j(t)] \tag{54}$$

$$V_j \frac{dT_j(t)}{dt} = q_j(t)[T_{j,o} - T_j(t)] + \frac{UA}{\rho c_p}[T(t) - T_j(t)] \tag{55}$$

Assuming the state vector being composed by $x_1 = C_a$, the product concentration at the tank outlet flow, $x_2 = T$, the reactor inner temperature, assumed to be homogeneous, and $x_3 = T_j$, the jacket outlet flow temperature, with $y = x_1$ being the output variable and the refrigerator flow in the reactor jacket as the control variable $u = q_j$, the internal representation will be

$$\dot{x}_1 = \frac{q}{V}(x_{1,o} - x_1) - \alpha(t)x_1^2; \quad \alpha(t) = \alpha_o e^{-\frac{E}{Rx_2}} \tag{56}$$

$$\dot{x}_2 = \frac{q}{V}(x_{2,o} - x_2(t)) + \frac{H\alpha}{\rho c_p V}x_1^2 - \frac{UA}{\rho c_p V}[x_2 - x_3] \tag{57}$$

$$\dot{x}_3 = \frac{UA}{\rho c_p V_j}[x_2 - x_3] + \frac{x_{3,o} - x_3}{V_j}u \tag{58}$$

3.1 FL control

To apply the FL control methodology, the process zero dynamics should be determined. Deriving the output equation, the following sequence is obtained:

$$\dot{y} = \frac{q}{V}(y_o - y) - \alpha(t)y^2 \tag{59}$$

$$\ddot{y} = -\frac{q}{V}\dot{y} - 2\alpha(t)y\dot{y} - \dot{\alpha}(t)y^2 \tag{60}$$

$$\dddot{y} = -\frac{q}{V}\ddot{y} - 2\dot{\alpha}(t)y\dot{y} - 2\alpha(t)(y\ddot{y} + \dot{y}^2) - \ddot{\alpha}(t)y^2 - 2\dot{\alpha}(t)y\dot{y} \tag{61}$$

where, from (56), it is

$$\dot{\alpha} = \alpha\frac{E}{Rx_2^2}[\dot{x}_2] \tag{62}$$

$$\ddot{\alpha} = \alpha\frac{E}{Rx_2^2}[\frac{E\dot{x}_2^2}{Rx_2^2} - \frac{2\dot{x}_2^2}{x_2} + \ddot{x}_2] \tag{63}$$

being

$$\ddot{x}_2 = -\frac{q}{V}\dot{x}_2 - \frac{H}{\rho c_p}(\dot{\alpha}y^2 + 2\alpha y\dot{y}) - \frac{UA}{\rho c_p V}\dot{x}_2 + \frac{U^2A^2}{\rho c_p V \rho c_p V_j}(x_2 - x_3)) + \frac{UA}{\rho c_p V V_j}(x_{3,o} - x_3)u \tag{64}$$

Thus, the zero dynamics is null. The new state vector will be

$$[\xi] = [y\,\dot{y}\,\ddot{y}]^T \tag{65}$$

To get the linearizing control, let us simplify the notation by denoting (61) as

$$\dddot{y} = \Gamma_1 + \gamma_1\ddot{\alpha} \tag{66}$$

as well as (63) by

$$\ddot{\alpha} = \Gamma_2 + \gamma_2\ddot{x}_2 \tag{67}$$

and (64) by

$$\ddot{x}_2 = \Gamma_3 + \gamma_3 u \tag{68}$$

Thus, the linearizing control will be

$$u_l = \frac{1}{\gamma_1\gamma_2\gamma_3}[\nu - \Gamma_1 - \gamma_1\Gamma_2 - \gamma_1\gamma_2\Gamma_3] \tag{69}$$

showing the complexity in its implementation and requiring access to all the state variables.

The stabilizing control, ν, to track the reference y_r will be defined by

$$\nu = \dddot{y}_r - k_1(\ddot{y} - \ddot{y}_r) - k_2(\dot{y} - \dot{y}_r) - k_3(y - y_r) \tag{70}$$

where k_1, k_2, k_3 are positive constants. In this way, the error dynamics, $e(t) = y(t) - y_r(t)$, will be

$$\dddot{e} + k_1\ddot{e} + k_2\dot{e} + k_3 e = 0 \tag{71}$$

3.2 LAB control

To derive the LAB control for this plant let us consider the state space representation (56)-(58).

Assuming a proportional approaching to the required reference, the desired behavior is expressed by

$$\begin{aligned} \dot{x}_1 &= \dot{x}_{1,r} + k_c(x_{1,r} - x_1) \\ \dot{x}_2 &= \dot{x}_{2,r} + k_t(x_{2,r} - x_2) \\ \dot{x}_3 &= \dot{x}_{3,r} + k_j(x_{3,r} - x_3) \end{aligned} \tag{72}$$

where $x_{1,r}$ is the given reference for the component A concentration and $x_{2,r}, x_{3,r}$ are the references for the temperatures, to be computed. The control design parameters, k_c, k_t, k_j should be positive.

Combining the process model (56)-(58) with the desired behavior (72) the controlled plant behavior is defined.

From the first equation in this set, the required value of α is derived:

$$\alpha_r = -\frac{1}{x_1^2}(\dot{x}_{1,r} + k_c(x_{1,r} - x_1) - \frac{q}{V}(x_{1,o} - x_1)) \tag{73}$$

and thus for the reactor temperature,

$$x_{2,r} = \frac{-E/R}{ln(\alpha_r/\alpha_o)} \tag{74}$$

From the second equation in the controlled model, the required value of $T_j = x_3$, the jacket temperature, is derived:

$$x_{3,r} = \frac{\rho c_p V}{UA}(\dot{x}_{2,r} + k_t(x_{2,r} - x_2) - \frac{q}{V}(x_{2,o} - x_2) - \frac{H\alpha}{\rho c_p V}x_1^2 + \frac{UA}{\rho c_p V}x_2)$$

Finally, from the third equation, the control action is derived

$$u = \frac{V_j}{x_{3,o} - x_3}[\dot{x}_{3,r} + k_j(x_{3,r} - x_3) - \frac{UA}{\rho c_p V_j}(x_2 - x_3)] \tag{75}$$

This expression is difficult to implement as not only the temperature references but also their derivatives should be computed.

3.3 Discussion

In this process both methodologies are applicable but there are some clear differences in the control design procedure. The FL approach will require the computation of (69), whereas the LAB approach control is computed by (75) needing the computation of the sacrificed variables and their derivatives. Clearly, the complexity of the solution depends very much on the plant model, not being easy to, a priori, determine which one would be more convenient. As before, the design coefficients have different meaning.

There are some applications where one of these methodologies cannot be applied or the domain of application is restricted, as shown in the next example.

4. SERVO CONTROL EXAMPLE

It has been shown that when both methodologies are applicable there are some differences in the procedure to compute the solution as well as in the control parameters tuning. Also the control action, mainly in the transient period, could be different. In order to evaluate the differences, let us consider the servo control of a simple nonlinear model such as

$$\dot{x}_1 = a\sin x_2 \tag{76}$$

$$\dot{x}_2 = -x_1^2 + u \tag{77}$$

The problem is to compute the control action to track an achievable reference $y_r(t)$, whose derivatives are accessible.

If the output equation is $y = x_2$, the relative degree is $\rho = 1$. Both methodologies can be applied and the outcome is similar to that in Section 2. The input/output relationship in the controlled plant can be linearized and stabilized but the zero dynamics is unchanged, being nonlinear.

If the output equation is $y = x_1$, there is no zero dynamics

$$\dot{y} = a\sin x_2 \tag{78}$$

$$\dot{x}_2 = -y^2 + u \tag{79}$$

4.1 FL

To apply the FL, the previous model should be transformed by, for instance, the diffeomorphism

$$\xi_1 = x_1 \tag{80}$$

$$\xi_2 = a\sin x_2 \tag{81}$$

leading to the new state space model

$$\dot{\xi}_1 = \xi_2 \tag{82}$$

$$\dot{\xi}_2 = a\cos(\sin^{-1}(\frac{\xi_2}{a}))(-\xi_1^2 + u) \tag{83}$$

which is FL by the control action

$$u = \xi_1^2 + \frac{1}{a\cos(\sin^{-1}(\frac{\xi_2}{a}))}v \tag{84}$$

yielding the linearized model

$$\dot{\xi}_1 = \xi_2 \tag{85}$$

$$\dot{\xi}_2 = v \tag{86}$$

To track a given reference $y_r(t)$, the control input should be generated as

$$u = \xi_1^2 + \frac{-k_1\xi_1 - k_2\xi_2 + \ddot{y}_r + (k_1 + k_2)\dot{y}_r + (k_1 k_2)y_r}{a\cos(\sin^{-1}(\frac{\xi_2}{a}))} \tag{87}$$

where k_1, k_2 are the design parameters assigning the closed loop poles. This control that can be expressed as a function of the original state variables as

$$u = x_1^2 + \frac{-k_1 x_1 - k_2 a\sin x_2 + \ddot{y}_r + (k_1 + k_2)\dot{y}_r + (k_1 k_2)y_r}{a\cos x_2} \tag{88}$$

As pointed out in (Khalil [2002], page 507), the diffeomorphism (80) is only well defined for $-\pi/2 < x_2 < \pi/2$, and the control (88) is only valid if the system state does not violate these limits. This constraint is due to the change of variables but it is not existing in the original system (76).

Due to the pole/zero cancellation in the $y(s)/y_r(s)$ transfer function, the reference tracking will be also perfect.

4.2 LAB

By using the LAB approach, denoting by $y = x_1$, $z = x_2$, the control action is derived from

$$\dot{y}_r + k_y(y_r - y) = a\sin(z_r) \tag{89}$$

$$\dot{z}_r + k_z(z_r - z) = -y^2 + u \tag{90}$$

Thus, from (89), the reference for the sacrificed variable and its derivative will be

$$z_r = \arcsin\frac{1}{a}(\dot{y}_r + k_y(y_r - y)) \tag{91}$$

$$\dot{z}_r = \frac{\ddot{y}_r + k_y(\dot{y}_r - \dot{y})}{a\cos(z_r)} \tag{92}$$

From (90), the control action would be

$$u = \dot{z}_r + k_z(z_r - z) + y^2$$
$$= \frac{\ddot{y}_r + k_y(\dot{y}_r - \dot{y})}{a\cos(z_r)} + k_z(\arcsin\frac{\dot{y}_r + k_y(y_r - y)}{a} - z) + y^2 \tag{93}$$

In this case, the trigonometric function imposes a constraint in the selection of k_y such that

$$|\frac{\dot{y}_r + k_y(y_r - y)}{a}| < 1 \tag{94}$$

limiting, in an indirect way, the possible references to be tracked.

4.3 Discusion

In this example, and considering the case where the output variable is $y = x_1$, the control signals can be generated by the FL (88) as well as by the LAB (93) approaches. Their time evolution is plotted in the following figures, assuming three different scenarios: 1) the reference is constant and, due to the initial conditions, the constraint $(-\pi/2 < x_2 < \pi/2)$ is not violated; 2) as before but, due to the initial conditions, the constraint $(-\pi/2 < x_2 < \pi/2)$ is violated; 3) the reference is sinousoidal and this constraint is violated. In the model (76), $a = 4$ has been assumed, and the selected control parameters are $k_y = k_z = 0.9$.

1. No Constraints: Assume tnitial conditions $\{x_{10}, x_{20}\} = \{0, 0\}$, and $y_r = 5$. In this case, the evolution of both state

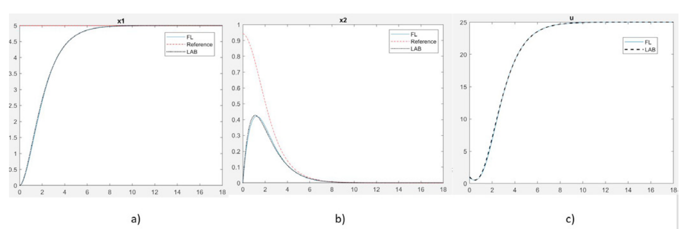

Fig. 2. Set-point tracking, $\{x_{10}, x_{20}\} = \{0, 0\}$

variables as well as the control input is the same for both control schemas, as shown in Figure 2.

2. State Constraints: Let us assume $y_r = 5$, with initial conditions $\{x_{10}, x_{20}\} = \{0, 2\}$. Note that $x_{20} > \pi/2$. The state variables and control signals are plotted in Figure 3. It can be seen that the FL control does not bring the second state variable to zero but to π, avoiding the constraint (Figure 3b). Moreover, due to the different initial reaction, the control signal is stronger in the LAB approach (Figure 3c).

3. Sinousoidal reference: $y_r = 5 + 5\sin(\frac{\pi}{5}t)$, with initial conditions $\{x_{10}, x_{20}\} = \{0, 2\}$.

The reference, state variables and control signals are plotted in Figure 4. It is worth to note that: i) The output

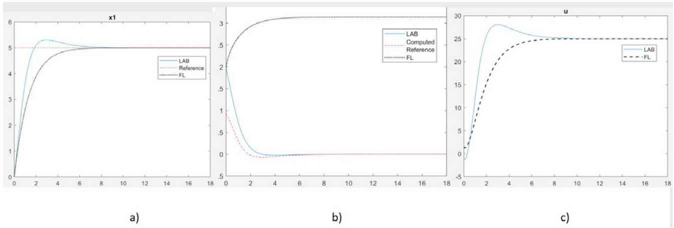

Fig. 3. Set-point tracking, $\{x_{10}, x_{20}\} = \{0, 2\}$

computed by both approaches follows the reference with minor differences; ii) the steady-state behavior is similar with both control signals (Figure 4a); iii) due to the constraint, the x_2 variable does not oscillates around zero, but around π, (Figure 4b); iv) the FL control transient is disturbed by the constraint (Figure 4c).

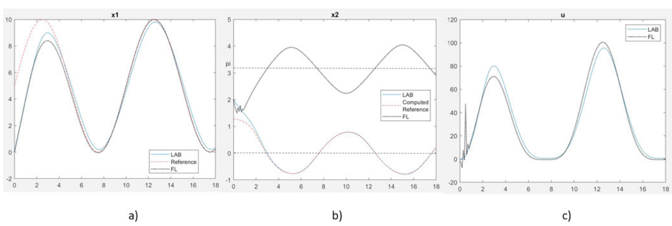

Fig. 4. Sinousoidal tracking, violating initial conditions

5. CONCLUSIONS

The possibility to apply FL or LAB to design the control for tracking a reference has been illustrated in several examples. Based on these cases, the following conclusions can be drawn:

(1) For simple process models, both approaches can be applied and they provide the same control signal.
(2) For perfect tracking, the nonzero dynamics should be cancelled. For NMP plants, the unstable pole/zero cancellation can be approximated by a time delay, if the zero is defined by a single parameter in the process model.
(3) The main difficulties are: a) to get the canonical form to apply FL, b) to solve the set of nonlinear algebraic equations, in the LAB approach.
(4) If both approaches are applicable, the steady-state dynamics are similar but the solutions may differ in the transient period (as illustrated in the simulations).
(5) The need of a diffeomorphism to get the canonical representation may introduce additional constraints in the computation of the FL control law (88).

Summing up it can be said that both approaches will lead to similar control actions in simple cases although the control parameters selection is done from a different perspective.

The control computation procedure is different and, as a consequence, its domain of application, complexity and solvability mainly depend on the process model.

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