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Technical note

# Estimating effective volumes in industrial forced-air flotation cells

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ABSTRACT

Mean residence time (MRT) in industrial flotation cells is one of the key parameters for kinetic modelling and identification of effective volumes. However, not all plants have access to robust tracer techniques to reliably measure this parameter with reasonable accuracy. For this reason, flotation practitioners estimate the mean residence time from the volumetric pulp flowrate and the effective cell volume. The latter requires assumptions on the air and froth volumes inside the machines, which has led to inaccuracies in MRT estimations. To overcome this challenge, the present communication correlated the measured ($\tau_m$) and calculated MRTs from the rougher circuits of four copper flotation plants (twenty-eight surveys). The rougher stages of these plants consisted of forced-air mechanical cells of 100, 160 and 200 $m^3$. The correlation between the measured and calculated MRTs showed that the following equation can be used as an approach to predict the MRT in industrial forced-air flotation cells: $\tau_m = \alpha V_f/Q$, with $\alpha = 0.872$ representing the relative effective volume (95 % confidence interval of 0.839–0.905), $V_f$ the total cell volume, and $Q$ the volumetric feed flowrate of pulp. This interval for the relative effective volume is proposed as a reference range to consolidate current assumptions for cell sizing, or to revisit these assumptions in case of significant deviations regarding the observed interval.

## 1. Introduction

The mean residence time (MRT) is one of the crucial parameters for flotation modelling, estimation of effective volumes, and circuit sizing (Yianatos et al., 2015). This residence time must be defined such that it not only maximizes the recovery of targeted minerals but also minimizes the likelihood of transferring gangue minerals to the froth zone. This term is linked to flotation kinetics, and consequently, to flotation recovery. For reactor cell systems, Equation (1) has been used to determine the flotation cell volumes ($V_f$) as a function of an estimated residence time (Metso, 2006):

$$\alpha V_f = Q\tau S \tag{1}$$

where $Q$ is the volumetric feed flowrate of pulp, $\tau$ represents the residence time of one machine, $S$ denotes a scale-up factor (which depends on the source of the $\tau$ estimation), and $\alpha$ is an aeration factor accounting for the relative air content in the cell. The latter has been typically assumed as 0.85; however, this value has not been justified from industrial MRT measurements. The MRTs can be specified by conducting laboratory or continuous pilot tests, or selected from information as that shown in Table A1 (Appendix A), based on mineralogy-oriented criteria. According to Metso (2006), $S$ in Equation (1) is set as 1.0 if $\tau$ is obtained from industrial assessments, specified by the customer, or estimated from a continuous pilot tests. If $\tau$ is estimated from laboratory batch tests, $S$ is set in the range 1.6–2.6 as a scale-up factor.

Mineral processing plants are currently facing expansions in capacity due to the reduction in cut-off grades that has led to increases in production rates. Estimations of MRTs are thus critical to determine the volume and number of flotation cells required to achieve targeted grades and recoveries. For mass balancing and modelling purposes, flotation recoveries in continuous systems are typically expressed according to Equation (2) [HSC Chemistry's database (Outotec, 2020; Aquino et al., 2019)]:

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$$R(\%) = R_\infty\left(1 - \frac{1}{(1+k\tau)^n}\right) \tag{2}$$

where $R$ is the flotation recovery in continuous operation, $k$ is the flotation rate constant (1/min), $R_\infty$ is the maximum achievable recovery, and $n$ is the number of cells in series.

The mean residence time in a flotation machine can be calculated either from the pulp flowrate and effective volume (Levenspiel, 1999), as $\tau_c$ in Equation (3), or experimentally measured, as $\tau_m$ in Equation (4).

$$\tau_c = \frac{V_{eff}}{Q} = \frac{\alpha V_f}{Q} \tag{3}$$

Within Equation (3), $V_{eff}$ ($m^3$) denotes the effective volume of a flotation cell. Although measuring the volumetric flowrate is relatively straightforward, the effective volume ($V_{eff} = V_f - V_{Air} - V_{Froth}$) requires assumptions on the air and froth volumes ($V_{Air}$ and $V_{Froth}$, respectively). Equation (3) is a special case of Equation (1), in which $\alpha$ is a volumetric factor accounting for $V_{Air}$ and $V_{Froth}$.

Equation (4) implies concentration measurements of a tracer as a function of time, $C(t)$, in the outlet stream of a flotation cell. An unbiased estimation of $C(t)$ requires inlet and outlet concentration measurements, and the respective deconvolution procedures to obtain the residence time distribution (Yianatos et al., 2015).

$$\tau_m = \frac{\int_0^\infty t \times C(t)dt}{\int_0^\infty C(t)dt} \tag{4}$$

Conditions with $\tau_c > \tau_m$ are justified by possible dead zones related to solid settling or short-circuits within a flotation cell (Yianatos and Díaz, 2011; Yianatos et al., 2012). Lelinski et al., (2002) reported $\tau_c > \tau_m$ for Dorr-Oliver (148 $m^3$), Outokumpu (160 $m^3$), and WEMCO (160 $m^3$) cells installed in the rougher stage at the Chuquicamata Division of CODELCO, Chile. Significant differences between $\tau_c$ and $\tau_m$ were reported (up to 50 %); however, the comparisons presented some potential bias due to: (a) an overestimation in the gas holdup when evaluating Equation (3) ($\varepsilon_G = 25$ %); and (b) there were uncertainties in the MRT estimated from the RTD data, Equation (4), caused by insufficient overall sampling time to characterize the RTD tails, which typically underestimates $\tau_m$.

Yianatos et al., (2015) used the N tanks-in-series model to represent the mixing regime along flotation banks, which were operated with MRTs in the range of 18–53 min. The number of equivalent perfectly mixed tanks-in-series was typically around 75 %–100 % of the number of installed cells in series. This result was related to a bypass flow throughout the flotation banks. More recently, Guner et al., (2023) measured MRTs in a 16 L Reflux™ flotation cell (RFC 100), showing an overestimation of 1–10 % in the calculated MRT values (assuming $\alpha = 1$) compared to the measured MRTs. This difference proved to be dependent upon the operating conditions in the flotation machine (water, air and bias flow rates).

Since most of the beneficiation plants do not have access to robust tracer technologies to measure residence time in their flotation circuits, a reliable estimation of $\alpha$ in Equation (3) is beneficial for metallurgical assessments and modelling purposes. This short communication summarizes industrial MRT data that allows the $\alpha$ values to be bounded, which is critical for machine sizing and kinetic characterizations.

## 2. Industrial measurement of MRTs

RTD measurements were conducted in four industrial plants, using radioactive tracers. Twenty-eight tests were carried out in the first mechanical cell of different rougher banks. Forced-air machines of 100, 160, and 200 $m^3$ were evaluated. Bromine 82 in water solution was used to characterize the mean residence time of the liquid phase. Given that segregation in the −45 μm size fraction has proven to be negligible compared to the liquid phase, irradiated solid tracer (<45 μm) was also incorporated in the analysis. Thus, the tracers had similar physical and chemical properties to those of the evaluated components. The mean lifetimes for the liquid and solid tracers were 36 h and 15 h, respectively. The radioactive tracers were injected into the feed of each cell, and concentration data (activity, in counts per second) were recorded from the inlet and outlet streams by means of non-invasive detectors. Although the mean lifetimes of the assessed tracers were significantly longer than an RTD measurement in a single cell, all datasets presented in this paper were corrected by radioactive decay. Fig. 1a illustrates measured inlet and outlet concentrations, which were normalized to obtain a unit area under the curve.

The RTD [$\xi(t)$] was estimated by parametric deconvolution. This approach consists of convoluting the inlet concentration with an RTD model in order to fit the outlet concentration. As the injection points do not always coincide with the inlet stream to a flotation machine, some dispersion is expected in the inlet signal, $x(t)$, and thus direct RTD estimations tend to be unreliable due to the $x(t)$ autocorrelation. Ill-posed RTD estimations are consequently common in flotation machines, making parametric deconvolution a robust approach for $\xi(t)$ characterizations. Fig. 1a presents a model fitting for the outlet signal. Different RTD models were evaluated for each experimental test as described by Yianatos et al. (2015), choosing that with the best goodness-of-fit. In all RTD models, a transport delay $\tau_D$ was incorporated. From the methodology reported by Yianatos et al. (2015), the minimum adjusted coefficient of determination was $R^2_{adj} = 0.971$, with an inter-decile range of $R^2_{adj} = 0.976$–$0.995$. For the inlet and outlet signals illustrated in Fig. 1a, the LSTS model of Fig. 1b proved to be an adequate model structure for $\xi(t)$, leading to $R^2_{adj} = 0.986$ in the outlet reconstruction. From the $\xi(t)$ model and its parameters, $\tau_m$ was obtained from Equation (4), considering that $\xi(t)$ is a normalized representation for $C(t)$. To evaluate $\tau_c$ from Equation (3), the volumetric feed flowrate was calculated from the solid tonnage and the pulp density. The latter was obtained from feed samples or from the plant instrumentation, according to its availability. Equation (3) was evaluated assuming that the pulp occupied the entire cell volume (*i.e.*, no air and no froth, and $\alpha = 1$). Thus, the $\tau_m/\tau_c$ ratio indicated the relative effective volume occupied by the pulp. Equations (3) and (4) were then compared to estimate a typical value for $\alpha$ in forced-air flotation cells. For further details on the experimental procedure and RTD estimation, please refer to Yianatos et al. (2015).

## 3. Results and discussions

Fig. 2a shows the correlation between the MRTs obtained from Equation (3) with $\alpha = 1$, and Equation (4). In the former, the design volume (*i.e.*, no air and no froth) was used such that the linear model $\tau_m = \alpha V_f/Q$ was proposed, with $\alpha$ representing the relative effective volume. An $\alpha$ value of 0.872 was obtained, with a 95 % confidence interval of 0.839–0.905. The results shown in Fig. 2a are in good agreement with those presented by Guner et al., (2023) for Reflux™ flotation cells.

Given the variability shown in Fig. 2a, the sensitivity of the regression results to influential observations was assessed. The relative effective volume was then estimated after removing one data point at a time. The procedure was repeated for each data point, recalculating the relative effective volume 28 times (using 27 data points). Fig. 2b shows the cumulative distribution function of α, which was bounded by a limited range between 0.86 and 0.88. This observation highlighted the reliability of α estimations, despite the experimental variability.

Following the correlation between the measured and calculated residence times assuming $\alpha = 1$, a reliable range for the relative effective volume of $\alpha = 0.84 - 0.91$ was observed. This result is applicable to forced-air flotation cells installed in rougher banks. The outcome allows process engineers to consolidate assumed effective volumes or revisit cell sizing criteria. The database presented here must be expanded to reduce uncertainties. Thus, the design of flotation circuits will be subject

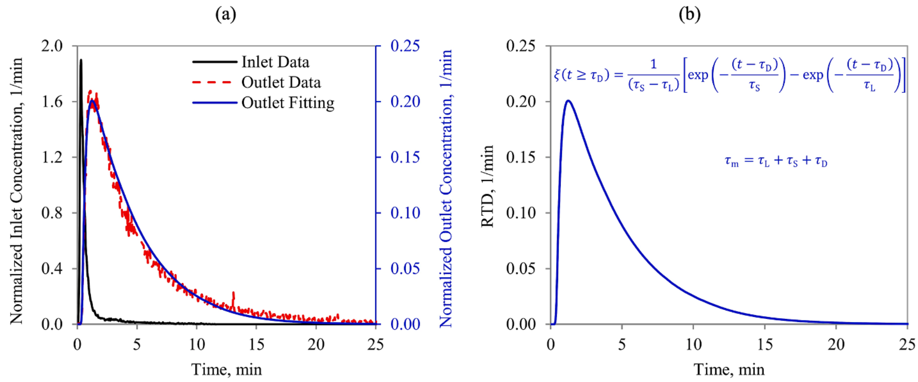


**Fig. 1.** (a) Example of normalized inlet and outlet concentrations, along with the outlet signal reconstruction, and (b) residence time distribution [Large and Small Tanks in Series].

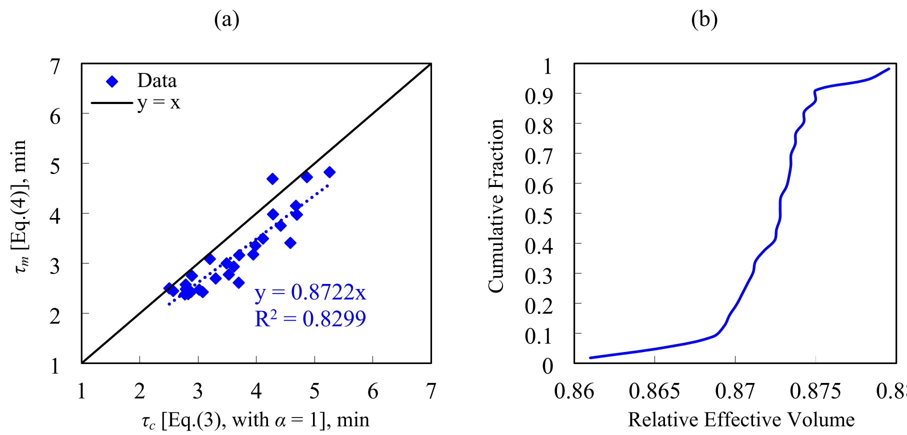


**Fig. 2.** (a) Correlation between the estimated MRTs from Equation (3) with $\alpha = 1$ and Equation (4), and (b) cumulative fraction of the estimated α values after removing one datapoint at a time.

to less prediction errors associated with large-scale flotation times.

## 4. Conclusions

This short communication presented MRT estimations in forced-air mechanical cells from industrial measurements conducted in four different rougher flotation circuits. The results showed that the mean residence time can be estimated from $\tau_m = \alpha V_f/Q$, where $\alpha = 0.872$ represents the relative effective volume with a 95 % confidence interval of 0.839–0.905, and $V_f/Q$ is an estimate for the residence time assuming that the pulp occupies the entire cell volume (*i.e.*, no air and no froth). Design criteria with $\alpha$ values in or close to this range are supported by these. On the other hand, $\alpha$ values far from the range 0.839–0.905 require further analysis to justify the respective assumption.

The database presented here must be expanded to decrease uncertainties in the estimated $\alpha$ values and to generalize the results to different flotation stages and machines.

### CRediT authorship contribution statement

**Luis Vinnett:** Conceptualization, Data curation, Formal analysis, Funding acquisition, Investigation, Project administration, Software, Validation, Visualization, Writing – original draft, Writing – review & editing, Resources. **Juan Yianatos:** Conceptualization, Formal analysis, Funding acquisition, Project administration, Resources, Supervision, Writing – review & editing. **Francisco Díaz:** Data curation, Funding acquisition, Methodology, Writing – review & editing. **Ahmad Hassanzadeh:** Validation, Visualization, Writing – original draft, Writing – review & editing, Conceptualization, Formal analysis, Investigation, Methodology, Resources, Supervision.

### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

### Data availability

Data will be made available on request.

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## Appendix A

**Table A1**
Reactor cell system flotation sizing for rougher flotation cell duty* (Metso, 2006).

| Mineral type | Solid content (%w/w) | τ (min) | No. of cells/bank |
|---|---|---|---|
| Barite | 30–40 | 8–10 | 6–8 |
| Copper | 32–42 | 13–16 | 8–12 |
| Fluorspar | 25–32 | 8–10 | 6–8 |
| Feldspar | 25–35 | 8–10 | 6–8 |
| Lead | 25–35 | 6–8 | 6–8 |
| Molybdenite | 35–45 | 14–20 | 10–14 |
| Nickle | 28–32 | 10–14 | 8–14 |
| Phosphate | 30–35 | 4–6 | 4–5 |
| Potash | 25–35 | 4–6 | 4–6 |
| Tungsten | 25–32 | 8–12 | 7–10 |
| Zinc | 25–32 | 8–12 | 6–8 |
| Silica (iron) | 40–50 | 8–10 | 8–10 |
| Silica (phosphate) | 30–35 | 4–6 | 4–6 |
| Sand (impurity) | 30–40 | 7–9 | 6–8 |
| Coal | 4–12 | 4–6 | 4–5 |
| Effluent | As received | 6–12 | 4–6 |

*For cleaning duty, the required retention time is suggested to be approximately 65 % of the rougher flotation time using 60 % of the rougher solid percentage.

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