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Departamento de Física.

Phenomenology of a dark sector with a vector-like fermion triplet

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Thesis work is presented in partial fulfillment for the requirements
to acquire the degree of Master of Science, Physics Mention, from
the Universidad Técnica Federico Santa María.

December, 2024

Abstract

We extend the Standard Model with a dark sector composed of isospin gauge group $SU(2)_D$ non-quiral, a new scalar in the fundamental representation and a fermion triplet in the adjoint representation, with its own mass term. Due to the different structures, we cannot introduce a Yukawa term for the fermions, so they do not notice the symmetry breaking and remain degenerate. The model presents three dark matter species, which can be identified with their quantum numbers under the discrete symmetries Z_2 of dark charge and dark fermion number.

What makes this model interesting, is the fact that the dark fermions are classically disconnected from the standard model. In that case, we will consider loop-induced processes, that can only happen due to quantum fluctuations, to compute the relevant quantities needed to test the model. In this thesis, we develop the different diagrams applying the Passarino-Veltamn reduction algorithm, which makes the programming a lot easier. In the end, we managed to put the first lower limits to the parameter space and compute the behavior of the cross section for dark fermion annihilation into SM final states.

Acknowledgments

This small section is going to be in Spanish, my mother tongue.

Quisiera empezar agradeciendo a mi tutor, el Dr. Alfonso Zerwekh. Hace 2 años ya, decidí cambiarme drásticamente de área de investigación llegando a la física de partículas, teniendo que aprender de manera autodidacta. Gracias a los apuntes y a la paciencia del profesor Alfonso, a la hora de resolver dudas, pude adentrarme en este bello e intrincado mundo de la física teórica. Ha sido un tremendo gusto trabajar con usted, gracias por su tiempo y sus ideas. También quisiera incluir aquí a Paulo Areyuna y Gonzalo Benitez, por su gran ayuda para resolver dudas en temas de programación y física de partículas.

En segundo lugar, a mi familia, mis papás Arturo y Macarena, mi hermana Amanda y Vitto. Estoy inmensamente agradecido de haber crecido junto a ustedes. Me han enseñado a tener criterios y a siempre hacer lo correcto. Fue un lindo año, tuvimos que atravesar algo muy difícil, pero logramos superarlo y salir adelante juntos. A pesar de que con mi hermana estamos más grandes y por ende, más independientes, hemos logrado encontrar los espacios donde compartir como familia. Ustedes son mi red de apoyo principal, mis confidentes y principales consejeros, y por eso, les estoy infinitamente agradecido.

Finalmente, los cálculos descritos en esta tesis, fueron realizados en un pequeño rincón del Cerro Placeres, en Valparaíso. En un rincón con dos escritorios, un excelente sillón para dormir, 5 sillas de cuestionable comodidad y las risas, locuras, amor y compañía de 10 personitas, que con justa razón me distraían. Aquel lugar se llama Cenade. Poca gente lo conoce, es chiquito, caluroso y acogedor. Llegué a ese espacio cuando por una casualidad de la vida y un poco de ayuda de la Limón, conocí al Ale, este pequeño actor y gran cocinero. Gracias por acogerme en tu círculo y aceptarme, con ansiedades y todo.

Gracias al Max, el tata del grupo, con un excelente gusto musical, y a la José, amiga y confidente, por introducirme al Visual Studio Code, y tenerme paciencia aquel único día (o quizás aquella semana) donde reaprendí a programar. A nuestra jefecita la Gabi y la Anto que a veces se aparecían. Al Luquitas por sus chistes tan peculiares y sus ruidosos bostezos. Al Joche por enseñarme a relajarme más. A la Maida, la guagua del grupo, que pase a todo, tiene más energía que nadie. Y a la Catu, gymsis, roller partner, apañe de chanches y una de mis mejores amigas. Gracias a todos y todas, son un grupo de personas increíble y bakan, y me encantó compartir este año con cada uno de ustedes, con risas, fajitas y gohans. Espero reír, llorar, gritar y cantar con ustedes este nuevo año que viene.

Por último, una mención especial a mi profesor de física del colegio Winterhill, Boris Padilla, ya que sin él no hubiese llegado a esta senda de aprendizaje. También entran aquí, Grego y Sean, Churiflais y Marga, amigos eónicos. Inclusive, todas aquellas personas de la enseñanza media que ya no están, y que para mal o para bien, me hicieron quién soy hoy.

Una última mención especial para la Senpai Ivette, mi segunda madre y compañera de instrucción, y a mi pequeña familia Samurai, repartida entre Viña del Mar e Iquique.

Este trabajo fue posible gracias al financiamiento del proyecto Fondecyt regular N° 1230110 y a la beca Magister UFTSM N° 046/2024.

‘If I have seen further, it is by standing on the shoulders of giants’
Sir Issac Newton in a letter to Robert Hooke, 1675.

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The Standard Model of Particle Physics: A brief review

In this chapter, we review the principal tools and aspects of the Standard Model of Particle Physics, the most recent and accurate theory describing three of the four fundamental interactions: electromagnetic, weak, and strong (gravity is not yet included). Currently, one of the most frequently mentioned issues in theoretical physics is the incompatibility between Quantum Mechanics (QM) and Relativity. However, the truth is that Quantum Field Theory (QFT), the framework in which the Standard Model (SM) operates, successfully reconciles them. The key detail here is that QFT unifies Special Relativity (which governs objects moving at speeds close to that of light) with QM. Gravity, on the other hand, remains the major unresolved challenge today¹.

Before proceeding, an important disclaimer: we will not delve into the mathematical foundations of QFT, as these are well explained in [1, 2, 3]. Nevertheless, the concepts presented here will serve as a reference point for comparison with our Dark Matter model in Chapter 3.

With that in mind, we begin with an overview of the particles and interactions that constitute the SM. Next, Noether's Theorem plays a central role, providing the fundamental tool for constructing physical theories (i.e., Lagrangians) based on symmetries and group theory. Quantum Electrodynamics (QED) will serve as an introductory example of gauge theories, leading us to more complex structures beyond $U(1)$ and ultimately introducing the symmetry groups of the SM. In particular, we will take a closer look at the Electroweak sector and, last but not least, the Higgs Mechanism. These last two topics will be crucial for the development of the model proposed in this thesis.

1.1 The known particles

Since the time of Ancient Greece, philosophers and later scientists wondered about the question: What are we made of? The idea emerged of a fundamental building block that could no longer be divided, which builds up everything in reality. Hence, the concept of the atom was born, and many models and experimental discoveries helped clarify the picture of this fundamental building block, from the atom to the proton and quarks. Dalton, Thomson, Rutherford, Bohr, Schrödinger, Chadwick, Pauli, Dirac, Feynman, Gell-Mann, and many more contributed to shaping the understanding we have today.

The SM describes twelve fermionic particles (and their antiparticles) and five gauge bosons. Fermions are the constituents of regular matter, with spin-1/2, while bosons are the carriers of interactions, more commonly known as forces, with spin 1 or 0.

1.1.1 Quarks and Gluons

The first group of fermions are the Quarks, which are the fundamental components of the proton and neutron, and they are capable of “feel” the three interactions of the SM. The most important of these forces (at least for the quarks) is the strong interaction, mediated by the massless bosons known as gluons. Quarks, among other

¹And clearly not the focus of this chapter.

particles, have an "extra" and exclusive charge known as color charge (red, green, and blue). It has nothing to do with the colors of visible light; rather, it is a charge that can take three values, so we had to come up with something to distinguish it from the regular electric charge.

There are six flavors (types) of quarks in nature, each one capable of taking any of the three colors. These are the up, down, charm, strange, top, and bottom quarks. They come in families or generations of two quarks, with the u-type having an electric charge of $2/3$ and the d-type having $-1/3$. Each generation is a copy of the former, but with higher mass². Quarks are bound in groups of two quarks, known as mesons, and three quarks, known as baryons (protons and neutrons, for example). Any particle made up of quarks is color neutral, which is known as color confinement.

On the other hand, there are eight types of gluons, which are charged in combinations of color-anticolor, but electrically neutral. Since they can have color charge, they can interact among themselves. This is due to the structure of the group $SU(3)_C$ in which Quantum Chromodynamics (QCD), the theory that models the strong interaction, is based. We will get to that a bit later in this chapter.

1.1.2 Leptons

Every fermion that cannot feel the strong interaction is called a lepton. They can only interact through the electromagnetic and weak interactions, so they must be electrically charged. Here we have the electron, the muon, and the tau, each carrying a charge of -1 and being heavier than its predecessor.

Additionally, every lepton has a small, neutral, and almost massless counterpart, called a neutrino³. Because they are neutral, they can only interact weakly. Moreover, they are the lightest particles in the universe (as far as we know), so they move extremely fast (they are ultra-relativistic) and do not interact with almost anything, making them very hard to detect. In Figure 1.1, we summarize the leptons and quarks grouped into their corresponding generations.

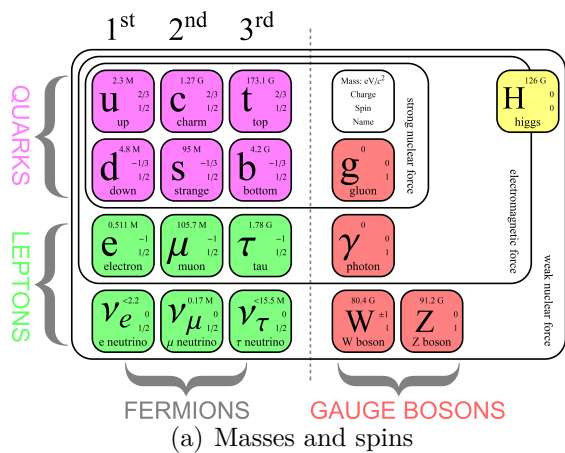


TABLE OF ELEMENTARY PARTICLES		
QUANTA OF RADIATION		
Strong Interactions	Eight gluons	
Electromagnetic Interactions	Photon (γ)	
Weak Interactions	Bosons $W^{\pm}, Z$	
Gravitational Interactions	Graviton (?)	
MATTER PARTICLES		
	Leptons	Quarks
1st Family	(ν_e, e^{-})	(u, d)
2nd Family	(ν_{μ}, μ^{-})	(c, s)
3rd Family	(ν_{τ}, τ^{-})	(t, b)
HIGGS BOSON		

(b) Particles summarized. Table taken from [1]

FIGURE 1.1: *Particles of the SM*

1.1.3 Electroweak bosons

As we will see later, the electromagnetic and weak interactions are intimately related. The electromagnetic force is mediated by the well-known photon, the quantum of light. It is massless and electrically neutral, so it

²For example, the top quark has a mass of $m_t \approx 173$ GeV, the heaviest one. Meanwhile, the up quark has $m_u \approx 2.2$ MeV
³The mass and the general study of neutrinos is an active field of research in high-energy physics today.

doesn't interact with itself. The weak interaction is a very short-range interaction ($\sim 10^{-18}$ m), so its mediators must be massive and are mainly responsible for particle decays. There are three of them: the charged $W^\pm$ bosons and the neutral Z boson. For example, the W^- can couple with an electron and an anti-neutrino in a β -decay (decay of a neutron into a proton and an electron). The particularity of the weak force is that it is a chiral interaction.

Quirality is a property of fermions that tells us about how the spinors transform under the Lorentz group. In the massless or ultra-relativistic limit, chirality is the same as helicity, i.e., the projection of the spin onto the momentum direction. If the projection of the spin points parallel to the momentum, i.e., positive helicity, then the particle is right-handed, and if it is anti-parallel, i.e., negative helicity, it is left-handed. The point is that the weak bosons only interact with left-handed fermions (whether a quark or a lepton) and right-handed anti-fermions. As we will see later, a left-handed lepton and its corresponding neutrino can form a vector-like structure, called a doublet of $SU(2)_L \otimes U(1)$ (the groups of the electroweak sector), in a way that the W boson can change particles within the same family (lepton to neutrino and vice versa, and u-type to d-type quarks and vice versa).

1.1.4 The Higgs Boson

The last and most important piece of the SM to be found is the Higgs boson. In this section, we will talk briefly about it. It was theorized in the 1960s and discovered at the LHC (Large Hadron Collider) in 2012⁴. This particle (actually, its field) is responsible for giving masses to every massive particle in the SM through the Higgs mechanism, which breaks the electroweak symmetry, a topic that we will explore later in this chapter. It is also responsible for making the SM consistent with unitarity. Some technical facts: it is electrically neutral, has spin-0, and can interact with any particle that has mass⁵.

1.2 Noether's Theorem and symmetries

Let's suppose we have a Lagrangian density $\mathcal{L}$ (simply "Lagrangian," for convenience), which is a function of the quantum fields φ_i and their derivatives $\partial_\mu \varphi_i$, where φ_i can be any scalar, vector, or spinor field. Then we can have an action:

$$S = \int d^4x \mathcal{L}(\varphi_i, \partial_\mu \varphi_i) \quad (1.1)$$

If we specify that this Lagrangian satisfies the principle of Least Action, then we can deduce the Euler-Lagrange equations for these quantum fields (see [3] for more details):

$$\frac{\partial \mathcal{L}}{\partial \varphi_i} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \varphi_i)} \right) = 0 \quad (1.2)$$

which gives us the equations of motion for the system. Now, let's suppose that our Lagrangian transforms under some transformation, which depends on the parameter α . Then, a variation in the Lagrangian can be written as:

$$\delta \mathcal{L} = \frac{\partial \mathcal{L}}{\partial \varphi_i} \delta \varphi_i + \frac{\partial \mathcal{L}}{\partial (\partial_\mu \varphi_i)} \delta (\partial_\mu \varphi_i) = \frac{\partial \mathcal{L}}{\partial \varphi_i} \delta \varphi_i + \frac{\partial \mathcal{L}}{\partial (\partial_\mu \varphi_i)} \partial_\mu (\delta \varphi_i) \quad (1.3)$$

Using the product rule for derivatives:

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \varphi_i)} \delta \varphi_i \right) = \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \varphi_i)} \right) \delta \varphi_i + \frac{\partial \mathcal{L}}{\partial (\partial_\mu \varphi_i)} \partial_\mu (\delta \varphi_i)$$

⁴Funny fact: Some of the papers cited in this thesis talk about the yet undiscovered Higgs boson, at least undiscovered at the time of publication.

⁵Yes, mass can be seen as another type of charge.

we then get:

$$\delta\mathcal{L} = \left[\frac{\partial\mathcal{L}}{\partial\varphi_i} - \partial_\mu \left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi_i)} \right) \right] \delta\varphi_i + \partial_\mu \left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi_i)} \delta\varphi_i \right)$$

So, if the equations of motion are satisfied and we also demand that our Lagrangian is invariant under the transformation, then:

$$\delta\mathcal{L} = \partial_\mu \left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi_i)} \delta\varphi_i \right) = 0 \quad (1.4)$$

Thus, the quantity

$$J^\mu = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi_i)} \delta\varphi_i \quad (1.5)$$

is conserved. This quantity is called Noether's current, and the expression above is known as Noether's theorem. If the Lagrangian is invariant under some transformation, it's symmetric under it, and there is a conserved current associated with it, then, in particular, we can have a conserved "charge" by integrating over the zero component of the current:

$$Q = \int d^3x J^0$$

Some examples that are useful to understand this theorem come from classical mechanics. Symmetry under time translation leads to the conservation of energy, while invariance under translations in spatial directions or rotations with respect to some axis leads to the conservation of the corresponding components of linear and angular momentum, respectively. But what about symmetries under more abstract transformations?

1.2.1 Quantum Electrodynamics

Trying to take the "square root" of the Klein-Gordon equation, Paul Dirac obtained its famous equation (this is a story for another time), which describes a free relativistic electron, or a free spin-1/2 fermion:

$$(i\gamma^\mu\partial_\mu - m)\psi = 0 \quad (1.6)$$

And its corresponding Lagrangian:

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi \quad (1.7)$$

where γ^μ are the Dirac matrices, m is the mass of the fermion, and ψ is a Dirac spinor. It's easy to see that this Lagrangian is invariant under the transformation:

$$\psi \longrightarrow e^{-i\alpha}\psi$$

where α is the parameter of the transformation, which is continuous and constant. This is known as a global phase symmetry (actually, it's a global $U(1)$ symmetry, but we will get to that later). What if we make the transformation local? That is, $\alpha = \alpha(x)$. The Lagrangian transforms under this as follows:

$$\begin{aligned} \mathcal{L}' &= i\bar{\psi}e^{i\alpha(x)}\gamma^\mu\partial_\mu(e^{-i\alpha(x)}\psi) - m\bar{\psi}e^{i\alpha(x)}e^{-i\alpha(x)}\psi \\ &= i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi + \bar{\psi}\gamma^\mu\psi\partial_\mu\alpha(x) \\ &= \mathcal{L} + \bar{\psi}\gamma^\mu\psi\partial_\mu\alpha(x) \end{aligned}$$

As we can see, because $\partial_\mu\alpha(x)$ is not zero, the Lagrangian is not invariant under this local transformation. We want to find a Lagrangian that is symmetric not only globally but also locally. We then introduce the covariant derivative following the minimal coupling⁶:

$$D_\mu = \partial_\mu - ieA_\mu \quad (1.8)$$

⁶There's an elegant way (in my opinion) to introduce this, which involves Wilson lines, in [3].

and we demand that it transforms in the same way as ψ . That is:

$$(D_\mu \psi)' = e^{-i\alpha(x)} D_\mu \psi$$

Notice that, to do this, we were forced to introduce a new field, one that is vector-like. If we demand the above condition, we obtain the following transformation law:

$$\begin{aligned} (D_\mu \psi)' &= D_\mu (e^{-i\alpha(x)} \psi) = (\partial_\mu - ieA_\mu) (e^{-i\alpha(x)} \psi) \\ &= e^{-i\alpha(x)} \left[\partial_\mu - ie \left(A_\mu + \frac{1}{e} \partial_\mu \alpha(x) \right) \right] \psi \end{aligned}$$

We can redefine A_μ as follows:

$$A_\mu \longrightarrow A_\mu + \frac{1}{e} \partial_\mu \alpha(x) \quad (1.9)$$

Having that transformation law, the covariant derivative transforms as we want, and the Dirac Lagrangian becomes:

$$\mathcal{L} = i\bar{\psi}\gamma^\mu D_\mu \psi - m\bar{\psi}\psi = i\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi + e\bar{\psi}\gamma^\mu A_\mu \psi \quad (1.10)$$

Notice that this new field couples to the electron, and that this coupling is proportional to the electric charge. Very peculiar, huh? It turns out that this field corresponds to the photon field, and like any dynamical field, it must have a kinetic term⁷:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad (1.11)$$

Then we put it into the modified Dirac Lagrangian in a way that preserves both the local and Lorentz symmetries. I present to you the Lagrangian of Quantum Electrodynamics:

$$\mathcal{L}_{QED} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi}\gamma^\mu D_\mu \psi \quad (1.12)$$

In summary, by demanding that our Lagrangian (and our theory) be invariant under this local symmetry, commonly known as gauge symmetry, we must introduce a massless spin-1 field (and particle) A_μ , with its own gauge transformation, which turns out to be the photon field. So, starting from a free theory with a global symmetry, by turning the symmetry into a local one, we end up with a theory that includes a gauge interaction, which allows us to describe electromagnetism, with the photon as the carrier of this force. The interaction is hidden in the covariant derivative, and its strength, the coupling constant, is given by the electric charge, as expected. The conserved current is:

$$J^\mu = \bar{\psi}\gamma^\mu \psi \longrightarrow J^0 = \bar{\psi}\gamma^0 \psi = \psi^\dagger \psi$$

This is commonly interpreted as the electric charge density. In conclusion, gauge symmetry leads to the conservation of electric charge.

Mass terms of bosons: The Proca Lagrangian

In this section, we introduced a gauge theory for the electromagnetic interaction, mediated by a massless boson: the photon. Can we do the same with the weak interaction? Remember that the $W^\pm$ and Z bosons are massive ($M_W \approx 80$ GeV, $M_Z \approx 91$ GeV). The answer is yes, but not exactly. The kinetic and mass terms for such a massive gauge boson are known as the Proca Lagrangian:

$$\mathcal{L}_{Proca} = -\frac{1}{4} W_{\mu\nu} W^{\mu\nu} + \frac{1}{2} m^2 W_\mu W^\mu \quad (1.13)$$

⁷This comes out naturally when we use gauge invariance to embed a spin-1 particle into a 4-dimensional field A_μ [3].

This is just a generic massive gauge boson, and it's all we need for now. Let's see how a gauge transformation of a gauge field works here. The kinetic term is invariant by construction, but the mass term:

$$\begin{aligned} \left(\frac{1}{2}m^2 W_\mu W^\mu\right)' &= \frac{1}{2}m^2 \left(W_\mu + \frac{1}{g}\partial_\mu \alpha\right) \left(W^\mu + \frac{1}{g}\partial^\mu \alpha\right) \\ &= \frac{1}{2}m^2 W_\mu W^\mu + \frac{m^2}{2g}(W_\mu \partial^\mu \alpha + W^\mu \partial_\mu \alpha) + \frac{m^2}{2g^2} \partial_\mu \alpha \partial^\mu \alpha \end{aligned}$$

We see that this Lagrangian is not gauge invariant. Such a mass term for a gauge boson explicitly breaks the symmetry. That's something we do not want when building our theories. But then, how can we get a theory with massive spin-1 particles? The answer to this question will reveal some deep truths about the nature of interactions in the early universe.

1.3 The groups of the standard model

Until now, we have seen that QED was built based on a gauge symmetry, which led to the inclusion of the photon field and charge conservation. This symmetry corresponds to the group $U(1)$. This is the starting point for building other theories, which can include one neutral boson and two charged ones, like the weak interaction, or eight bosons like the gluons of QCD.

Let's generalize our $U(1)$ theory to another group composed of unitary N -dimensional square matrices, with determinant equal to 1, which in principle, do not commute⁸. This group is known as $SU(N)$. In the case of the $SU(N)$ gauge group, the QED transformation rules are promoted to:

$$\begin{aligned} \psi &\longrightarrow e^{-iT^a \alpha^a(x)} \psi \\ W_\mu^a &\longrightarrow W_\mu^a - \frac{1}{g} \partial_\mu \alpha^a(x) - f^{abc} \alpha^b(x) W_\mu^c \end{aligned} \tag{1.14}$$

where g is the coupling constant of the theory. Alright, in these two seemingly simple expressions, we have just introduced many definitions and concepts. First of all, T^a are known as the generators of the group. For a group like $SU(N)$, there are $N^2 - 1$ generators, so $a = 1, \dots, N^2 - 1$. For example, in the case of $SU(2)$, there are $2^2 - 1 = 3$ generators:

$$T^a = \frac{\sigma^a}{2}$$

where σ^a are the Pauli matrices. In $SU(3)$, there are 8 generators (the Gell-Mann matrices), which explains the eight gluons of QCD. Then, the generators of an $SU(N)$ group are $N \times N$ matrices, and they are in the fundamental representation of the group⁹. A general transformation matrix of this group can be written as:

$$U(x) = e^{-iT^a \alpha^a(x)}$$

We can see that the exponential is no longer just a number; now it is a matrix, and there are $N^2 - 1$ scalar functions $\alpha^a(x)$. More importantly, ψ is no longer just a field, but an N -plet composed of N different fields, which transform in the fundamental representation of $SU(N)$, as we established above. On the other hand, for each generator, there exists a gauge field that transforms in the adjoint¹⁰ representation of $SU(N)$. The f^{abc} term corresponds to the structure factor of the group. In the case of $SU(2)$, it is the Levi-Civita tensor ϵ^{abc} .

⁸Known as Non-Abelian group

⁹The smallest non-trivial representation [3].

¹⁰Matrices that act on the vector space spanned by the $N^2 - 1$ generators.

If we want to build a Lagrangian based on an $SU(N)$ gauge symmetry, we will have to promote our covariant derivative and field strength tensor to:

$$\begin{aligned} D_\mu &= \partial_\mu - igT^a W_\mu^a \\ W_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc} A_\mu^b A_\nu^c \end{aligned} \quad (1.15)$$

The field strength tensor looks very similar to its $U(1)$ analog, but it includes an extra term that allows interactions among the gauge bosons themselves, due to been constructed in a non-Abelian group. Finally, the Lagrangian is:

$$\mathcal{L} = -\frac{1}{4}W_{\mu\nu}^a W^{\mu\nu a} + \bar{\psi}\gamma^\mu D_\mu\psi - m\bar{\psi}\psi \quad (1.16)$$

A theory with a Lagrangian like this, with all the considerations and definitions made above, is known as a Yang-Mills gauge theory. For example, QCD is a Yang-Mills theory based on the gauge group $SU(3)_C$, which conserves color charge. A quark field q is a triplet in the fundamental representation (each component is the same flavor quark charged in a different color). Since $SU(3)_C$ has eight generators, there are eight different gluons which transform in the adjoint representation and can interact among themselves. The rest of the particles in the SM are color singlets of $SU(3)_C$, because they are not color-charged.

The groups of the SM are a mixture of $SU(N)$ groups, including different gauge symmetries, like left-handed isospin $SU(2)_L$ and hypercharge $U(1)_Y$:

$$G_{SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

The last two groups will be introduced in major detail as follows.

1.3.1 Glashow-Weinberg-Salam Theory of Electroweak interactions

When we introduced the weak bosons, we saw that this interaction treats left-handed spinors differently from right-handed ones. Therefore, this interaction is chiral. A spinor can be separated into its left- and right-handed components as follows:

$$\psi = \psi_L + \psi_R \quad (1.17)$$

where each component is a Dirac 4-dimensional spinor, with two of its components set to zero (to match the original one). Each chiral component can be written in terms of ψ using the chirality projectors:

$$P_L = \frac{1 - \gamma^5}{2} ; P_R = \frac{1 + \gamma^5}{2} \longrightarrow \psi_L = \frac{1 - \gamma^5}{2}\psi ; \psi_R = \frac{1 + \gamma^5}{2}\psi \quad (1.18)$$

where $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. On the other hand, we mentioned that there are three bosons for the weak interaction: two electrically charged and one neutral. Additionally, this interaction can convert an electron into a neutrino, and vice versa. Given this situation, it's tempting to construct a Yang-Mills theory based on $SU(2)$, which has three generators. In this way, we could arrange a lepton and its neutrino partner in a left-handed doublet:

$$L = \begin{pmatrix} \nu_l \\ l \end{pmatrix}_L \quad (1.19)$$

where l is some charged lepton. Meanwhile, the right-handed spinors will be singlets. However, we encounter a problem. First, unlike the gluons, the weak bosons are electrically charged, so they can interact with the photon. This means we must include electromagnetism in the picture. This brings us to the second problem: the electromagnetic (EM) interaction is not chiral:

$$\bar{e}\gamma^\mu e A_\mu = (\bar{e}_R\gamma^\mu e_R + \bar{e}_L\gamma^\mu e_L)A_\mu$$

and the photon does not interact with the neutrino because it is neutral. Therefore, if we unify electromagnetism and the weak interaction in a theory based solely on $SU(2)$, we would be treating the components of the doublet in two different ways, which would break the symmetry. It would be better if we could assign another label to the doublet as a whole, i.e., another quantum number. Here, we define the electroweak hypercharge Y based on the electric charge of each particle and the isospin eigenvalues of the generator T^3 of $SU(2)_L$:

$$q = t_3 + \frac{Y}{2} \longrightarrow Y = 2(q - t_3) \quad (1.20)$$

For example, the neutrino has $t_3 = \frac{1}{2}$ and is electrically neutral, so:

$$Y = 2\left(0 - \frac{1}{2}\right) = -1$$

and for the electron (or its cousins in the other generations), $t_3 = -\frac{1}{2}$ and $q = -1$, so:

$$Y = 2\left(-1 + \frac{1}{2}\right) = -1$$

Thus, we see that the electroweak doublet L has hypercharge $Y = -1$. We can apply the same reasoning to the other generations of leptons and even extend it to the quark families, defining the doublet:

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \quad (1.21)$$

Meanwhile, the right-handed quarks are singlets of $SU(2)_L$. In Figure 1.2, we show the principal quantum numbers of the particles¹¹ of the Standard Model. Finally, we can promote the hypercharge to a $U(1)$ gauge group with its corresponding B_μ boson. Thus, we obtain the group of this electroweak theory:

$$G_{EW} = SU(2)_L \otimes U(1)_Y$$

and its Lagrangian:

$$\mathcal{L}_{EW} = -\frac{1}{4}W_{\mu\nu}^a W^{\mu\nu a} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + i\bar{L}^j \gamma^\mu D_\mu L^j + i\bar{\psi}_R^j \gamma^\mu D_\mu \psi_R^j \quad (1.22)$$

where j runs over every lepton and quark in the Standard Model. Notice that the covariant derivative must act differently on L than on ψ_R . Thus, we have:

$$\begin{aligned} D_\mu L &= \partial_\mu L - igT^a W_\mu^a L - ig' \frac{Y_L}{2} B_\mu L \\ D_\mu \psi_R &= \partial_\mu \psi_R - ig' \frac{Y_{\psi_R}}{2} B_\mu \psi_R \end{aligned} \quad (1.23)$$

where g and g' are the coupling constants associated with $SU(2)_L$ and $U(1)_Y$, respectively.

¹¹Note that the right-handed neutrinos have all quantum numbers equal to zero.

Particle	Q	t_3	Y	Color
u_L, c_L, t_L	$2/3$	$1/2$	$1/3$	triplet
d_L, s_L, b_L	$-1/3$	$-1/2$	$1/3$	triplet
$(\nu_e)_L, (\nu_\mu)_L, (\nu_\tau)_L$	0	$1/2$	-1	singlet
e_L^-, μ_L^-, τ_L^-	-1	$-1/2$	-1	singlet
u_R, c_R, t_R	$2/3$	0	$4/3$	triplet
d_R, s_R, b_R	$-1/3$	0	$-2/3$	triplet
$(\nu_e)_R, (\nu_\mu)_R, (\nu_\tau)_R$	0	0	0	singlet
e_R^-, μ_R^-, τ_R^-	-1	0	-2	singlet
8 gluons	0	0	0	octet
$W^\pm$	± 1	± 1	0	singlet
W^0	0	0	0	singlet
B^0	0	0	0	singlet
Φ	$1, 0$	$\pm 1/2$	1	singlet

FIGURE 1.2: Charges, isospin, and hypercharges of the Standard Model particles. Table taken from [2].

One important observation is that, apart from the fact that we still do not have mass terms for the gauge bosons, we now cannot include mass terms for the fermions either. Here's why. Let's consider a mass term and expand it in its chiral components:

$$m\bar{\psi}\psi = m\begin{pmatrix}\bar{\psi}_R & \bar{\psi}_L\end{pmatrix}\begin{pmatrix}\psi_L \\ \psi_R\end{pmatrix} = m(\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R) \quad (1.24)$$

This term is not invariant under $SU(2)_L$ due to the mixing of the chiral components. As a result, we end up with a unified theory known as the electroweak theory, but at this point, both the bosons and fermions are massless. Additionally, we observe that electromagnetism and the weak force currently act as separate interactions. So, how do we go from this point to the current situation? How do we give mass to the particles in the Standard Model?

1.4 The Higgs Mechanism

In the 1960s, Peter Higgs proposed the idea of the Higgs field and predicted, with it, the existence of the Higgs boson—the missing piece of the Standard Model. This field is represented by a complex scalar field in the fundamental representation of $SU(2)_L$, so it forms a doublet with hypercharge $Y = 1$:

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} = \begin{pmatrix} h_1 + ih_2 \\ h_3 + ih_4 \end{pmatrix} \quad (1.25)$$

Here, h_j are real scalar fields. By convention, one component is electrically charged, and the other is neutral. We will add this field to the Lagrangian of the electroweak theory as follows:

$$\mathcal{L}_{Higgs} = (D_\mu H)^\dagger (D^\mu H) - \mu^2 H^\dagger H - \frac{\lambda}{2} (H^\dagger H)^2 \quad (1.26)$$

where

$$D_\mu H = \partial_\mu H - igT^a W_\mu^a H - ig' \frac{Y_H}{2} B_\mu H \quad (1.27)$$

with $Y_H = 1$.

1.4.1 Spontaneous symmetry breaking

The potential corresponding to the above Lagrangian is:

$$V(|H|) = \mu^2 |H|^2 + \frac{\lambda}{2} |H|^4 \quad (1.28)$$

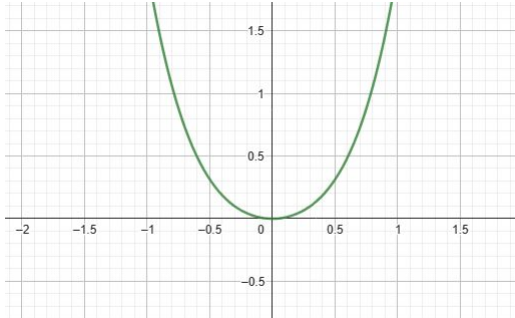
This potential has a minimum at $|H| = 0$ (see Figure 1.3(a)) when μ^2 is positive. Now, what happens if we set $\mu^2 < 0$? In other words, if we replace μ^2 with $-\mu^2$, the potential becomes:

$$V(|H|) = -\mu^2 |H|^2 + \frac{\lambda}{2} |H|^4 = -\mu^2 v + \frac{\lambda}{2} v^2 \quad (1.29)$$

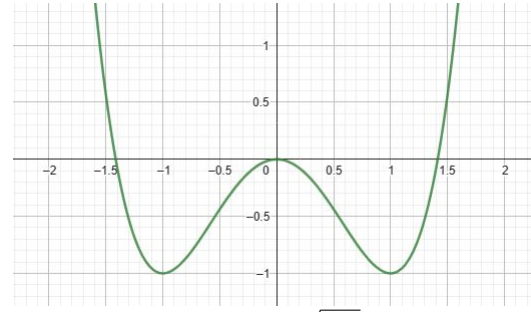
where now zero is a maximum, making it unstable. We have set $v^2 = |H|^2$. The new minima are given by:

$$\frac{dV}{dv} = (-2\mu^2 + \lambda v^2)v = 0 \longrightarrow v = \pm \sqrt{\frac{2\mu^2}{\lambda}}$$

As we can see in Figure 1.3(a), the potential is symmetric under a Z_2 symmetry. However, with this new minimum, and as shown in Figure 1.3(b), the symmetry is spontaneously broken. We do not refer to it as explicitly broken because the Lagrangian itself remains symmetric.



(a) Minimum at $|H| = 0$. $\mu^2 = 1$ and $\lambda = 2$.



(b) Minima at $|H| = \pm \sqrt{\frac{2\mu^2}{\lambda}}$. $\mu^2 = -2$ and $\lambda = 2$.

FIGURE 1.3: *Higgs potential for different signs of μ^2 .*

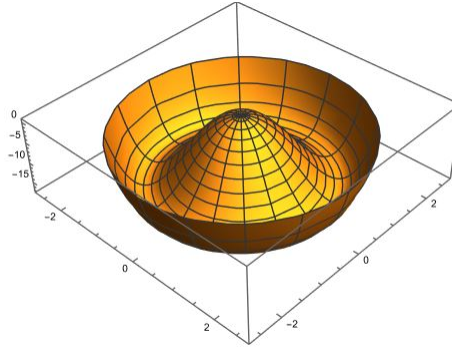


FIGURE 1.4: *Plot of the "Mexican hat" potential.*

Now, suppose we want to expand the Higgs doublet around one of these minima:

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \quad (1.30)$$

In fact, since H is a complex field, the potential takes the shape shown in Figure 1.4, known as the Mexican hat. We can observe that there is an infinite number of minima for the same field:

$$\langle H \rangle = ve^{i\theta} ; \theta \in [0, 2\pi]$$

Thus, we can expand the Higgs field around its minimum as:

$$H = \begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{pmatrix} e^{i\frac{T^a \pi^a(x)}{\sqrt{2}v}} \quad (1.31)$$

where $\pi^a(x)$ are the Goldstone bosons. According to Goldstone's theorem, for every broken symmetry, there is a massless Goldstone boson [3]. However, notice that our scalar-sector Lagrangian is symmetric under phase shifts, so we can simply set all $\pi^a(x)$ to zero, and then:

$$H = \begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{pmatrix} \quad (1.32)$$

This is known as the unitary gauge, and it will be used to simplify the following calculations. Also, by comparing the current Higgs field with the one we initially introduced, we can observe that $h_1 = h_2 = h_4 = 0$ and $h_3 = h$. The field h is known as the Higgs boson.

1.4.2 Bosonic mass terms and mixing

With the Higgs doublet expanded around the minimum, as we did previously, let us examine how our Lagrangian behaves with small perturbations around this value. Recall that the full form of the electroweak Lagrangian is:

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}W_{\mu\nu}^a W^{\mu\nu a} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\ & + i\bar{\psi}_L \not{D} \psi_L + i\bar{\psi}_R \not{D} \psi_R \\ & + (D_\mu H)^\dagger (D^\mu H) - \mu^2 H^\dagger H - \frac{\lambda}{2} (H^\dagger H)^2 \end{aligned} \quad (1.33)$$

where the "dashed" notation denotes $\not{D} = \gamma^\mu D_\mu$. Now, we focus on the kinetic term of the scalar sector and expand it around the minimum. We get:

$$\begin{aligned} D_\mu H &= \begin{pmatrix} \partial_\mu - i\frac{g}{2}W_\mu^3 - i\frac{g'}{2}B_\mu & -i\frac{g}{2}(W_\mu^1 - iW_\mu^2) \\ -i\frac{g}{2}(W_\mu^1 + iW_\mu^2) & \partial_\mu + i\frac{g}{2}W_\mu^3 - i\frac{g'}{2}B_\mu \end{pmatrix} \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix} \\ &= \begin{pmatrix} -i\frac{g(v+h)}{2}W_\mu^+ \\ \frac{\partial_\mu h}{\sqrt{2}} + i\frac{g(v+h)}{2\sqrt{2}}W_\mu^3 - \frac{g'(v+h)}{2\sqrt{2}}B_\mu \end{pmatrix} \end{aligned}$$

where we have defined the linear combination of fields:

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp iW_\mu^2) \quad (1.34)$$

commonly known as the $W^\pm$ bosons. Taking the adjoint of this matrix, we obtain:

$$\begin{aligned} (D_\mu H)^\dagger (D^\mu H) &= \frac{g^2 v^2}{4} W_\mu^- W^{\mu+} \\ &+ \frac{g'^2 v^2}{8} B_\mu B^\mu + \frac{g^2 v^2}{8} W_\mu^3 W^{\mu 3} - \frac{gg' v^2}{4} W_\mu^3 B^\mu \\ &+ \frac{g'^2 v}{2} h B_\mu B^\mu - \frac{gg' v}{2} h W_\mu^3 B^\mu - \frac{g^2 v}{4} h W_\mu^3 W^{\mu 3} + \frac{g^2 v}{2} h W_\mu^- W^{\mu+} \\ &+ \frac{g'^2}{8} h^2 B_\mu B^\mu + \frac{g^2}{4} h^2 W_\mu^- W^{\mu+} - \frac{gg'}{4} h^2 W_\mu^3 B^\mu + \frac{g^2}{8} h^2 W_\mu^3 W^{\mu 3} \\ &+ \frac{1}{2} \partial_\mu h \partial^\mu h \end{aligned} \quad (1.35)$$

From this, we observe a kinetic term for the Higgs boson and a mass term for the $W^\pm$ bosons¹². Thus, we define their mass as:

$$M_W = \frac{gv}{2} \quad (1.36)$$

Additionally, we have several interaction terms between the electroweak bosons and the Higgs boson, as well as mass terms for B^μ and W_μ^3 . There is also a quadratic term involving both of them, which corresponds to a mixing term. The mass matrix is:

$$\mathbb{M} = \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix}$$

This matrix is not diagonal, indicating that the bosons (the gauge bosons) are not yet physical fields. We must rotate this matrix, in other words, change to a basis where $\mathbb{M}$ is diagonal. To achieve this, let us define the physical fields:

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \mathcal{O} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos(\theta_W) & -\sin(\theta_W) \\ \sin(\theta_W) & \cos(\theta_W) \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}$$

where θ_W is the Weinberg angle. The new fields¹³ can be written as:

$$\begin{aligned} Z_\mu &= W_\mu^3 \cos(\theta_W) - B_\mu \sin(\theta_W) \\ A_\mu &= W_\mu^3 \sin(\theta_W) + B_\mu \cos(\theta_W) \end{aligned} \quad (1.37)$$

In this new basis, the mass matrix is:

$$\begin{aligned} \mathbb{M}' &= \mathcal{O} \mathbb{M} \mathcal{O}^T \\ &= \begin{pmatrix} (g \cos(\theta_W) + g' \sin(\theta_W)) & (g \sin(\theta_W) - g' \cos(\theta_W)) \\ (g \sin(\theta_W) - g' \cos(\theta_W)) & (g \sin(\theta_W) - g' \cos(\theta_W)) \end{pmatrix} \end{aligned}$$

For this matrix to be diagonal, we must impose the condition:

$$g \sin(\theta_W) - g' \cos(\theta_W) = 0 \longrightarrow \tan(\theta_W) = \frac{g'}{g} \quad (1.38)$$

Then, the mass matrix becomes:

$$\mathbb{M}' = \begin{pmatrix} \frac{g^2}{\cos^2(\theta_W)} & 0 \\ 0 & 0 \end{pmatrix}$$

This modifies the scalar sector of the electroweak Lagrangian to:

$$\mathcal{L}_{AZ} = \frac{v^2}{8} \begin{pmatrix} Z_\mu & A_\mu \end{pmatrix} \begin{pmatrix} \frac{g^2}{\cos^2(\theta_W)} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \frac{v^2 g^2}{8 \cos^2(\theta_W)} Z_\mu Z^\mu \quad (1.39)$$

Notice that A_μ is massless, so we identify this boson as the photon, while Z_μ corresponds to the neutral and massive Z boson, with a mass given by:

$$M_Z = \frac{gv}{2 \cos(\theta_W)} \quad (1.40)$$

What have we learned? The fact that the vacuum expectation value (i.e., the minimum) of the Higgs field is non-zero leads to the spontaneous symmetry breaking of the electroweak symmetry $SU(2)_L \otimes U(1)_Y$, breaking it down to a remnant symmetry $U(1)_{\text{EM}}$ of electromagnetism¹⁴. As a result, the weak bosons acquire mass. This process is known as the Higgs Mechanism. Recall that we are in the unitary gauge and remember Goldstone's theorem: since we now have three massive gauge bosons, there must be three Goldstone bosons present. These

¹²Note that $W^\pm$ form a particle-antiparticle pair.

¹³The letters used to define them might give away a bit.

¹⁴Thus, after symmetry breaking, the Standard Model group becomes $SU(3)_C \otimes U(1)_{\text{EM}}$.

Goldstone bosons are said to be absorbed into the longitudinal polarization of the weak bosons, thereby giving them their masses [3].

As an additional note, if we expand the kinetic terms of the fermion sector in a similar manner, we find that the photon couples equally to both left- and right-handed fermions, unlike the weak bosons. The Z boson has different couplings for each chirality, and the $W^\pm$ boson does not interact with right-handed fermions at all.

1.4.3 Yukawa terms

So, we have mass terms for the weak bosons, as expected, due to spontaneous symmetry breaking. However, we still do not have mass terms for the fermions. What about them? It turns out that the same mechanism gives them mass, but we need to include the proper term in the Lagrangian. Let us recall the left-handed lepton doublet from equation (1.19). We want the neutrino to remain massless (as an approximation). The term we are looking for is known as the Yukawa term:

$$\mathcal{L}_y = -y\bar{L}H\psi_R + h.c. \quad (1.41)$$

where y is the Yukawa coupling, and its value depends on the fermion. Now, let's expand this term around the vacuum (we will use the electron in this case):

$$\mathcal{L}_y = -y\begin{pmatrix} \bar{\nu}_L & \bar{e}_L \end{pmatrix} \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix} e_R + h.c. = -\frac{yv}{\sqrt{2}}\bar{e}_L e_R - \frac{y}{\sqrt{2}}\bar{e}_L e_R h + h.c. \quad (1.42)$$

We can clearly see that the chiral symmetry is broken. Now, let's make the Hermitian conjugate part explicit:

$$\mathcal{L}_y = -\frac{yv}{\sqrt{2}}(\bar{e}_L e_R + \bar{e}_R e_L) - \frac{y}{\sqrt{2}}(\bar{e}_L e_R + \bar{e}_R e_L)h = -\frac{yv}{\sqrt{2}}\bar{e}e - \frac{y}{\sqrt{2}}\bar{e}eh.$$

The electron mass is given by:

$$m_e = \frac{yv}{\sqrt{2}}. \quad (1.43)$$

Additionally, we have an interaction term with the Higgs boson, which is proportional to the electron's mass. In general, the Higgs boson interacts with every particle that has mass (so the Higgs can interact with itself), and this interaction is proportional to the mass of the particle.

1.5 Some brief conclusions

In this chapter, we have seen that to build a theory that attempts to explain the fundamental particles and their interactions, such as the SM, we can rely on abstract symmetries that lead to conservation laws (such as the conservation of electric charge in QED), grounding our theories in gauge groups like $SU(N)$, known as Yang-Mills theories. Demanding our theories to be gauge invariant naturally leads to the appearance of gauge bosons. However, in some cases, we cannot simply add mass terms to the Lagrangian without explicitly breaking the symmetry. We must be more subtle and break the symmetry in an “indirect” way, as the Higgs mechanism does. It is worth mentioning that there is another approach using the Feynman gauge, which allows the propagator to take a simpler form, as we will discuss later, but introduces terms with Goldstone bosons. The Feynman gauge can be used by defining the expansion around the vacuum as:

$$H = \begin{pmatrix} -iW_f^+ \\ \frac{1}{\sqrt{2}}(v + h + iZ_f) \end{pmatrix}, \quad (1.44)$$

where the subscript f indicates that these are the Goldstone bosons of their respective vectors. There is also the presence of unphysical particles, known as Ghost particles, which appear in any Yang-Mills theory. However,

we have not included them in this review, as they do not play a relevant role in the model discussed in this thesis.

There are some issues with the Standard Model, such as the mass hierarchy among the generations of fermions and the fact that experimental data show neutrinos have mass, due to neutrino oscillations. Nevertheless, the purpose of this chapter was to demonstrate the mathematical framework of Yang-Mills theories and the Higgs mechanism, and how they can be used to construct gauge theories like the Standard Model or extensions of it, such as attempts to include Dark Matter in the framework.

Dark Matter: A bit of everything we know so far

2.1 How do we know that DM exists?

2.1.1 Multi-scale evidence

To answer the question above, we will take a brief historical approach. The scientific community has been aware of the existence of a non-luminous component of matter in the universe since at least the 1930s. Using techniques such as the Doppler shift of stars, astronomers like Oort in the Milky Way (MW) [4] and Zwicky in the Coma Cluster¹ [5] calculated the velocities of stars in the outer arms of the galaxy and the velocity dispersion of nebulae in the cluster, respectively.

Oort found that the stars were moving too fast relative to the gravitational pull at their radius. Zwicky determined that the mass-to-luminosity ratio, calculated using only the velocity dispersion, was significantly larger compared to the value obtained using conventional methods based on the brightness of stars. The latter value was about 2% of the one derived from velocity dispersions. These two discoveries led to the same question: Is luminosity a reliable indicator of the mass distribution in galaxies?

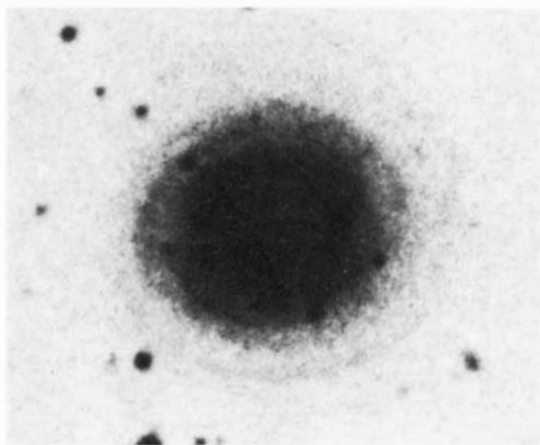
Assuming a spherically symmetric galactic disk and applying either the Virial Theorem or Newtonian mechanics, we arrive at the same result:

$$v(r) = \sqrt{\frac{GM(r)}{r}} \quad (2.1)$$

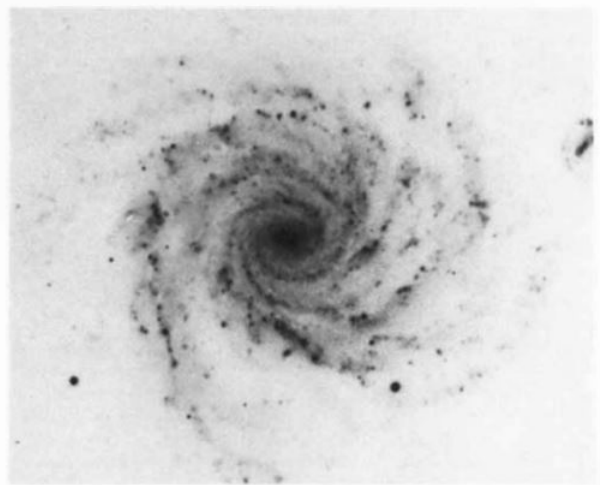
where G is the universal gravitational constant, and $M(r)$ represents the mass enclosed within a hypothetical Gaussian sphere of radius r . If we follow the traditional approach and use luminosity as our guide, most of the galaxy's mass should be concentrated at the center, where the majority of the brightness originates. Then, as the radius increases, less and less mass should be added—an analogy to our solar system, where the mass of the Sun vastly outweighs that of the planets. Consequently, for a radius such as the optical radius R_{opt} ², $M(r)$ should remain nearly constant, and the velocity dispersion should follow $v(r) \sim r^{-1/2}$. In this case, the velocity should decrease to zero, exhibiting Keplerian behavior. However, nature tells us otherwise.

¹They were observing it, not located there. Just a disclaimer.

²The radius that encloses approximately 83% of the galaxy's brightness [6].



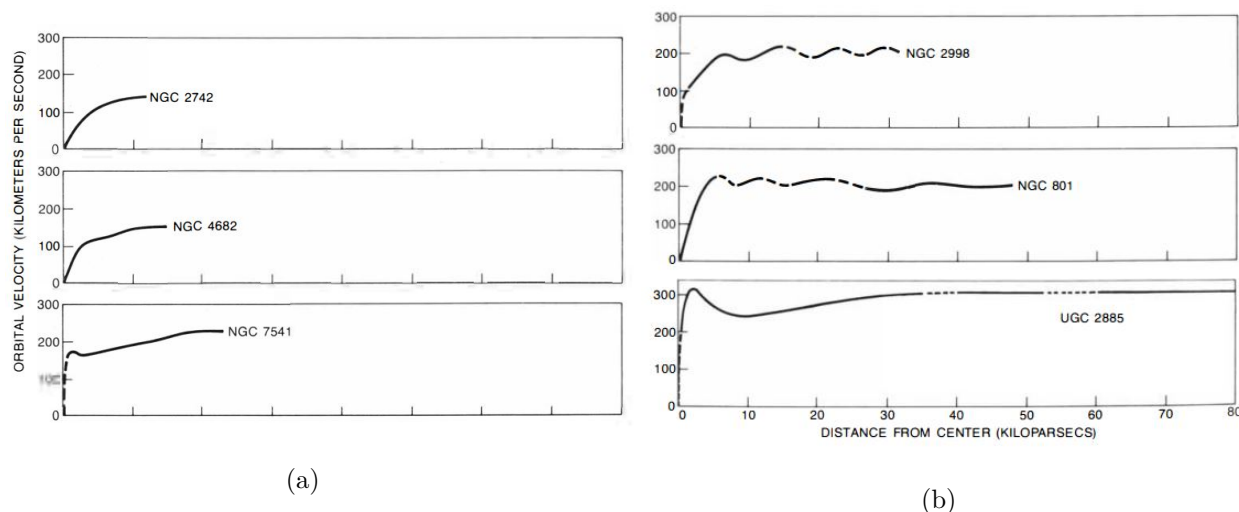
(a) Type Sa



(b) Type Sc

FIGURE 2.1: *Negative images of spiral galaxies studied by Rubin [7].*

Years later, scientists began to study other galaxies far from us, including those at distances of up to 60 Mpc. In particular, Rubin [7] in 1983 studied a group of 60 high-luminosity spiral galaxies with masses ranging from 10^9 to 10^{12} solar masses. These galaxies were classified into three groups: the first had a prominent and bright nucleus with tiny and almost indistinguishable arms, while others had larger arms in comparison to their center, similar to our own Milky Way (see Figure 2.1). Due to the great distances, the stars in the outer regions of these galaxies appear too faint. To address this, she studied the radiation emitted by the surrounding gas clouds instead, composed of hydrogen and helium, which encircle these stars and used it as a tracer.

FIGURE 2.2: *Velocity dispersion as a function of radius in Sc type galaxies. The luminosity and mass of the galaxy increases from top to bottom. Image taken from Rubin [7].*

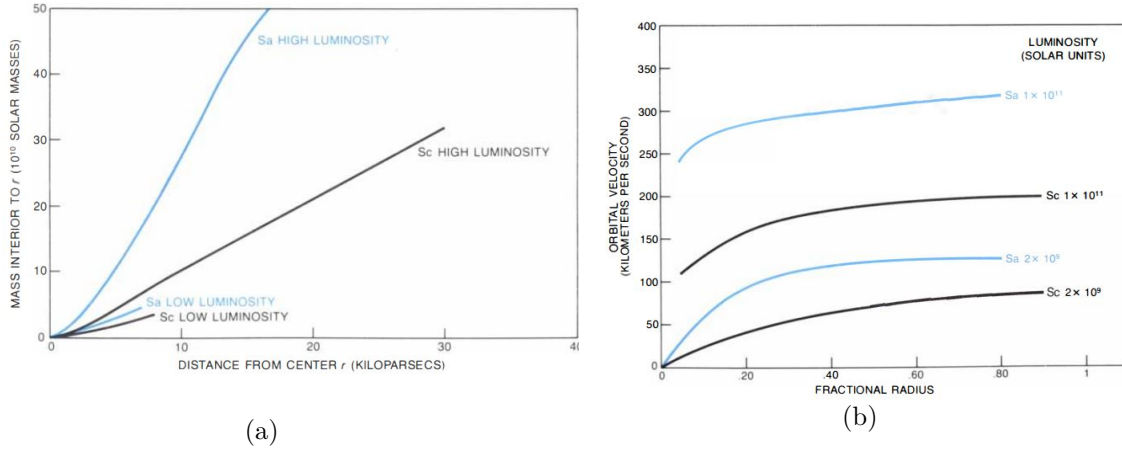


FIGURE 2.3: (a) Mass as a function of radius in Sa and Sc type galaxies with high and low luminosity. (b) Orbital velocity comparison between Sa and Sc type galaxies. Image taken from Rubin [7].

The results were astonishing. As seen in Figure 2.2, as luminosity increases, the velocity flattens abruptly and remains constant as the radius increases, contrary to what was predicted and previously believed. This implies that the mass distribution in Equation 2.1 must follow $M(r) \sim r$ to maintain a constant velocity dispersion. Furthermore, for brighter galaxies, this linear behavior becomes steeper, as shown in Figure 2.3(a). Where did all that mass come from? Why can't we see it? The term "Dark Matter" was coined by Rubin and other authors at the time, leading to one key conclusion: Unlike luminosity, mass is not concentrated at the center of galaxies, making luminosity alone an unreliable parameter for determining mass distribution. Moreover, it appears that more massive galaxies (such as Sa-type galaxies) contain a greater proportion of this non-luminous matter, a fact we will explore further.

Some recent researchers employ tools predicted by General Relativity, such as gravitational lensing. Gravitational lensing allows us to observe astronomical objects that are otherwise hidden, using potential wells to reveal their apparent positions. Using this method, gravitational lensing of background galaxies, with a massive object positioned in the foreground, can serve as a useful tracer for potential wells generated by possibly non-luminous matter. The challenge, however, is to distinguish the baryonic component of a system from the hypothetical dark massive component. A system such as a galaxy merger could provide an answer.

The Bullet Cluster is the result of a galaxy merger that occurred roughly 100 million years ago. As shown in Figure 2.4(b), the stars of the galaxies behave as collisionless particles, passing through each other without interaction, while the dust is dragged toward the center of the merger due to its interactions, forming plasma and emitting X-rays. These emissions can be observed as the yellow and orange colors around the center of Figure 2.4(a).

Clowe *et al.* [8] used weak gravitational lensing to create a two-dimensional map of the gravitational well of the system, represented by the green contours in Figure 2.4. Their hypothesis was that if the potential well traces the X-ray component, i.e., the baryonic matter, then there would be no non-luminous component. Their results, however, show that the peaks of each galaxy's potential wells are significantly displaced from their corresponding X-ray sources. Observing Figure 2.4(b), one might argue that the peaks coincide with the major concentrations of stars, but the stellar component accounts for only 1 to 2% of the mass, while the dust contributes 5 to 15%. If there were no dark matter, then the peaks should align with the centers of the X-ray sources. Instead, the dark component coincides with the spatial location of the stars in each galaxy.

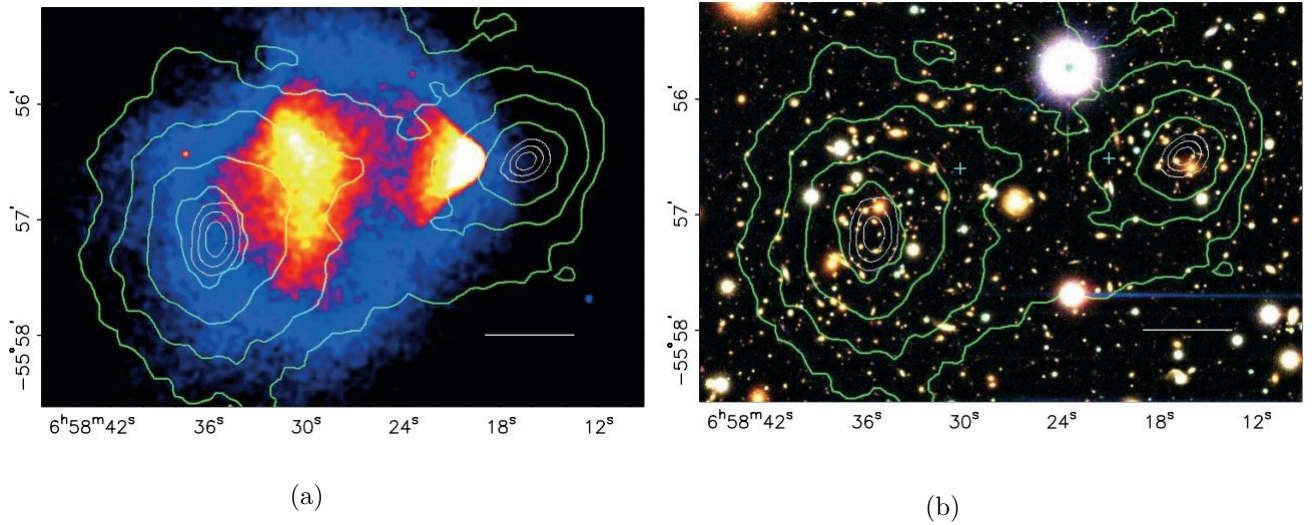


FIGURE 2.4: *Bullet Cluster* produced by the merger of two galaxies. In green is the map of the gravitational well of the system. (a) X-Ray image. (b) Optical image. Images taken from Clowe *et. al.* [8].

Until now, we have seen direct empirical evidence of the existence of dark matter (DM) at galactic scales. But why stop there? Why not consider larger scales? What about universal scales? We have evidence for that as well. The Cosmic Microwave Background (CMB) was discovered in 1964 by Penzias and Wilson [9]. It is known as a 2.73 K background radiation that originates from approximately 400,000 years after the Big Bang, when electrons and nucleons lost enough energy to form atoms—i.e., neutral particles—allowing photons to travel freely across the cosmos. This process is known as recombination, and the photons that were released at that time constitute the CMB.

Thanks to this discovery, astronomers and cosmologists were ³ able to study the nature of this primordial plasma. Projects such as the Differential Microwave Radiometers (DMR), launched as part of the Cosmic Background Explorer (COBE) in 1992, revealed in just one year of data collection that, although the CMB is highly uniform and can be modeled as a black body, it exhibits tiny fluctuations and anisotropies. Cosmologists believe these were the first overdensity seeds that created potential wells, eventually leading to the formation of the first galaxies and large-scale structures in the universe. However, the difference between the energies of the photons that emerged from these potential wells, compared to the mean photon energy of the baryon-photon bath, is on the order of 10^{-6} . This is too small to account for the large-scale structures observed today [10].

To reinforce this idea, let us discuss the Wilkinson Microwave Anisotropy Probe (WMAP). This satellite was launched in 2001 with the mission of studying the temperature fluctuations of the CMB with a precision of 10^{-7} degrees [11]. Its seven-year data collection is represented by the red dots in Figure 2.5, showing the power spectrum of the CMB as a function of the multipole moment. The data from WMAP estimated a total matter density of $\Omega_m h^2 = 0.1334 \pm 0.0054$ and a baryonic matter density of $\Omega_b h^2 = 0.0226 \pm 0.0005$. This implies that the missing dark component constitutes about 84% of the total mass density of the universe. Figure 2.5 also demonstrates that the CMB power spectrum is highly sensitive to the baryonic density, yielding results significantly different from those obtained by WMAP [9].

³And they continue to do so today.

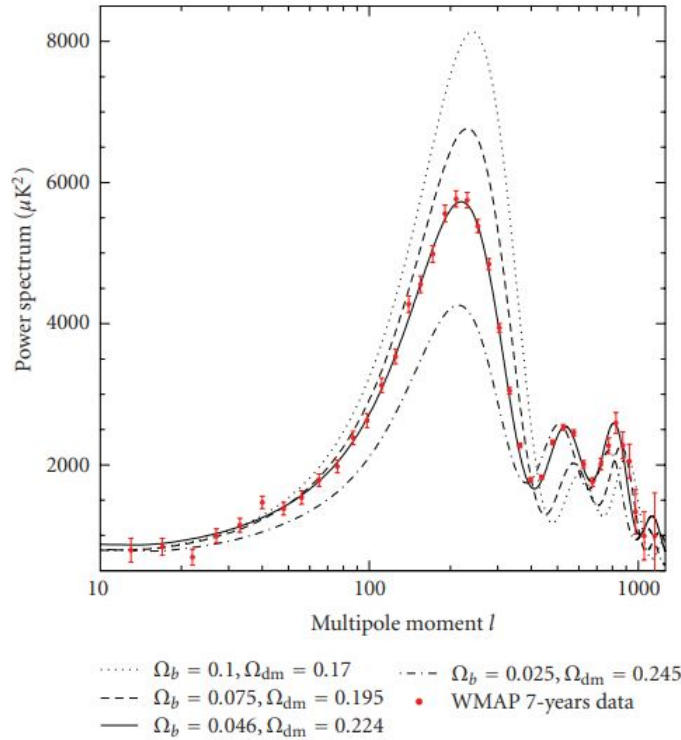


FIGURE 2.5: CMB's power spectrum for different values of the baryonic (Ω_b) and Dark matter (Ω_{dm}) densities. Image taken from Garret *et. al.* [9].

2.1.2 Structures and large-scale simulations

As we have seen in the last section, astronomical observations show that mass has a linear dependence on the galactic radius, $M(r) \sim r$ (see Figure 2.3(a)). This indicates that the density profile of a hypothetical spherically symmetric DM halo follows:

$$\rho = \frac{M(r)}{V} = \frac{3M(r)}{4\pi r^3} \longrightarrow \rho \sim \frac{1}{r^2} \quad (2.2)$$

In contrast to baryonic matter, dark matter does not collapse into a disk. This implies that it cannot have non-gravitational self-interactions that dissipate its energy, or at least, such interactions must be extremely weak. This density profile presents a problem: it "blows up"⁴ as $r \rightarrow 0$, meaning that the profile predicts an infinite amount of mass at the core of the galaxy. Furthermore, many would argue that modeling dark matter halos as perfect spheres is overly simplistic, raising the question: How far is this profile valid? Commonly, the maximum radius r_{max} where the profile remains valid is when the density reaches the critical density of the universe, ρ_c [12]:

$$\frac{M_{model}(r_{max})}{\frac{4}{3}\pi r^3} = \Delta_c \rho_c \quad (2.3)$$

where Δ_c is a the threshold value related to the critical density. Some researchers use the virial radius r_v as r_{max} , which represents the maximum distance where the Virial Theorem holds. One of the first well-accepted models, developed through the study of 19 halos with masses ranging from 10^{11} to 10^{15} solar masses, based on DM-only N-body simulations, is the Navarro-Frenk-White (NFW) profile [13]:

$$\frac{\rho_{NFW}(r)}{\rho_c} = \frac{\delta_c}{(r/r_s)(1 + r/r_s)^2} \quad (2.4)$$

⁴I like to call it that way.

where r_s is the 'scale radius', and δ_c is the characteristic overdensity, a free parameter. This model is purely phenomenological and was motivated by the lack of correlation between luminosity and the rotational dynamics of stars in galaxies. Figure 2.6(a) indicates a correlation between overdensity and halo mass (black dots). Less massive halos are denser, reinforcing the hierarchical clustering scenario, where lighter halos collapsed first at higher redshifts. In this framework, these halos gradually merged to form less dense but more massive structures. This corresponds to the Cold Dark Matter (CDM) scenario, which requires DM to have low kinetic energy. In other words, it must be non-relativistic to allow sufficient time and energy low enough for collapse into these denser halos.

Assuming this scenario, Navarro defined the formation redshift as the epoch when half of the final halo's mass M was contained in progenitors, each exceeding some fraction f of M . Assuming that the characteristic overdensity simply reflects the density of the universe at the time of formation, he proposed [13]:

$$\delta_c(M) \sim (1 + z_{\text{form}}(M))^3 \quad (2.5)$$

This is plotted in Figure 2.6(a) for different fractions f of the final mass M , supporting the conclusion that the characteristic overdensity depends on the halo's formation time. Additionally, Navarro plotted the rotation curves of disk galaxies (see Figure 2.6(b)) using the mass-to-light ratio and the circular velocity of the halo at the virial radius to determine the halo mass⁵. First, comparing these with observations, we see that the circular velocity profiles predicted by ρ_{NFW} match well, at least at the outer radii⁶. Furthermore, faster-rotating galaxies tend to be brighter, but as seen in the dashed lines of Figure 2.6(b), the halo circular velocity is significantly lower than that of the disk. At higher speeds, the halo mass remains almost unchanged, differing only by roughly 10% [13]. This suggests that in brighter spiral galaxies, the amount of baryonic matter might be comparable to that of dark matter.

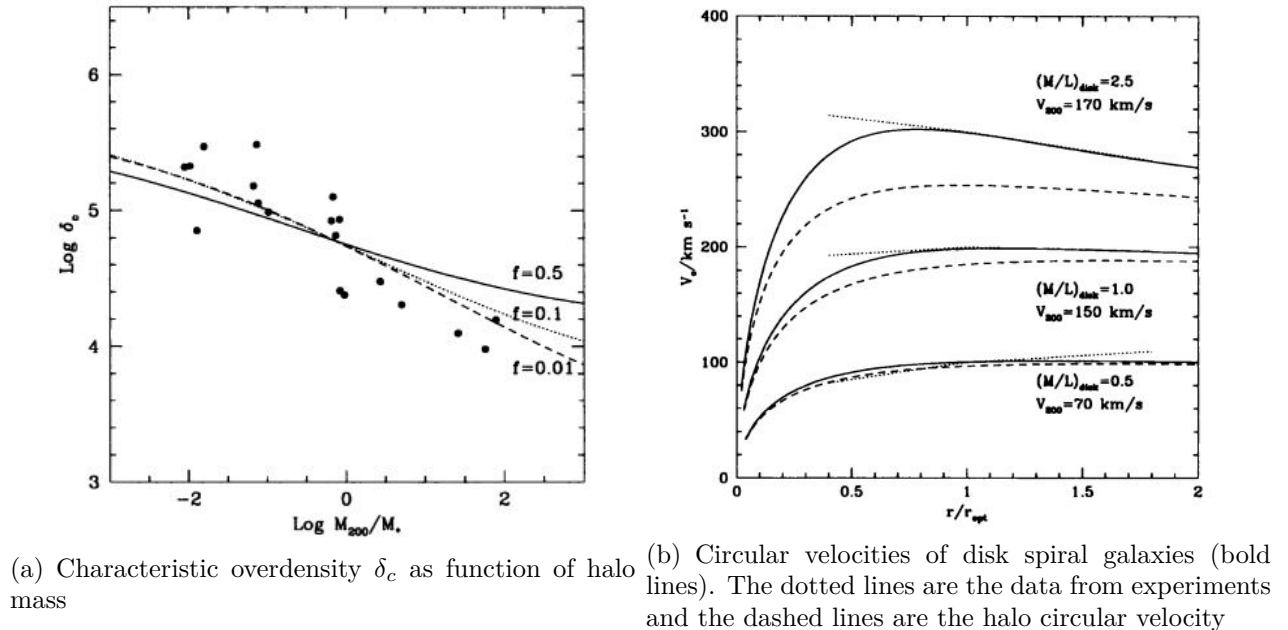


FIGURE 2.6: Images taken from Navarro [13].

This relation between luminosity and dark matter (DM) was also found by Persic *et al.* [6], who were attempting to find a universal rotational curve for spiral galaxies. They compared the slopes of approximately 1100

⁵Different definitions of the maximum radius lead to different definitions of halo mass.

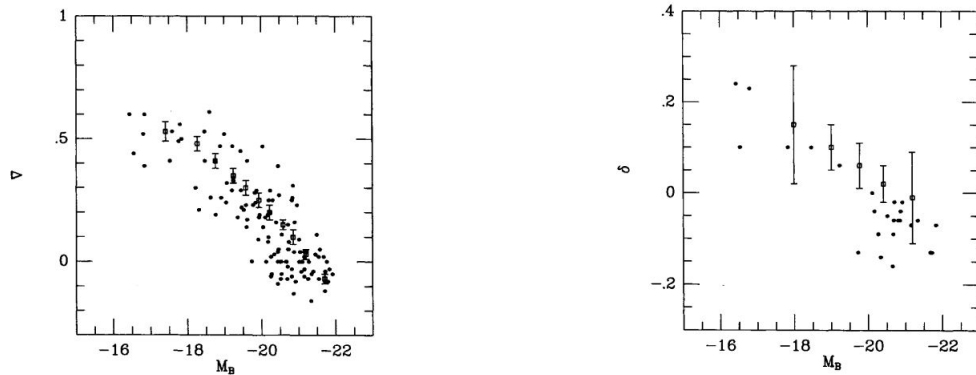
⁶In units of the optical radius

rotational curves derived from the brightness profile with the actual slopes, determined by the system's dynamics, labeled as ∇_{lum} and ∇ , respectively. This comparison was made at the radius R_{opt} . A model-independent quantity is defined as the difference between the two slopes. If $\nabla - \nabla_{lum}$ is less than 0.04, then the light can trace the mass of the galaxy. However, if the difference exceeds 0.2, there is a non-luminous component involved.

The luminous slope was approximately $\nabla_{lum} = -0.24 \pm 0.03$, setting the model-independent quantity to $\nabla + 0.25$, which indicates the presence of a dark component within R_{opt} . The slopes are plotted against luminosity in Figure 2.7(a), showing that for faint galaxies, the slope of the curve is steep, while it flattens for brighter galaxies. With this, they were able to calculate the dark matter component, which constitutes about 15% of the mass in brighter (larger) galaxies and can account for up to 85% in fainter (smaller) galaxies.

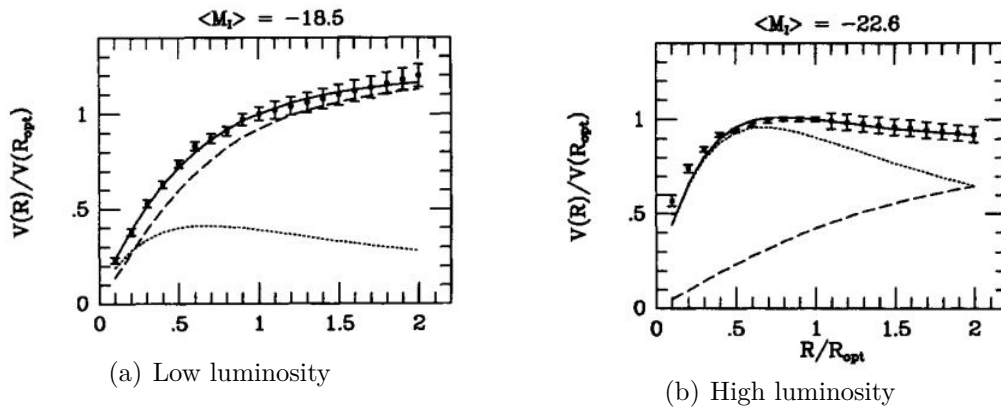
A similar quantity is defined in the range $[R_{opt}, 2R_{opt}]$, known as the fractional velocity variation:

$$\delta = \frac{V(2R_{opt}) - V(R_{opt})}{V(R_{opt})}$$



(a) Slope of the rotational curve against luminosity. (b) Fractional velocity variation against luminosity.

FIGURE 2.7: In the graphics we must take the absolute value of the luminosity M_B . Images taken from Persic et. al. [6].



(a) Low luminosity

(b) High luminosity

FIGURE 2.8: Dots: Data. Bold line: Universal RC. Dashed line: Halo RC. Dotted line: Disk RC. In the graphics we must take the absolute value of the luminosity M_B . Images taken from Persic et. al. [6].

Then, the quantity $\delta - \delta_{lum}$ indicates the mass discrepancy in the working range. With $\delta_{lum} = -0.25 \pm 0.02$, the difference is $\delta + 1/4$. Following the criteria as before, luminous matter is completely unable to trace the

mass, and we can conclude that dark matter (DM) is more abundant in the outer regions of the galaxy. In fact, Persic *et al.* [6] concluded that DM increases with radius and decreases with luminosity. The plot for δ against luminosity is shown in Figure 2.7(b). Finally, we show in Figure 2.8 the universal rotational curve found by them, plotted for both low and high luminosity, providing support for our conclusions so far. For faint galaxies, the transition between a gravitational well dominated by luminous matter and one dominated by dark matter occurs well inside the galaxy. In contrast, for larger and brighter galaxies, DM dominates only in the outer regions.

Returning to dark matter profiles, Navarro's results gave rise to the hypothesis that the DM halos surrounding spiral galaxies exhibit similar behavior to that of Cold Dark Matter (CDM) halos. Of course, the ρ_{NFW} profile is not the only density profile found in the literature. Some of the others are listed below [14, 15, 16]:

$$\begin{aligned}\rho_{gNFW}(r) &= \frac{\rho_s}{(r/r_s)^{\gamma_{gNFW}} (1 + r/r_s)^{3-\gamma_{gNFW}}} && \text{(Generalized NFW profile)} \\ \rho_{ein}(r) &= \rho_s \exp \left[-\frac{2}{\alpha_s} \left(\left(\frac{r}{r_s} \right)^{\alpha_s} - 1 \right) \right] && \text{(Einasto profile)} \\ \rho_{burk}(r) &= \rho_0 \frac{r_0^3}{(r + r_0)(r^2 + r_0^2)} && \text{(Burkert profile)}\end{aligned} \tag{2.6}$$

As shown, all of these profiles are spherically symmetric, although they differ in their dependence on the galactocentric radius r . Researchers use these profiles to model the DM halo of the Milky Way, comparing them to actual experimental data or DM-only simulations. For example, using the NFW profile, they found the parameters that best fit the Milky Way (MW) halo to be $r_s \approx 15$ kpc and $\rho_0 \approx 1.1 \times 10^7 M_\odot/\text{kpc}^3$. For the Einasto profile, the scale is $r_s \approx 20$ kpc and $\alpha_s \approx 0.17$, with a normalization parameter of $\rho_s \approx 2 \times 10^3 M_\odot/\text{kpc}^3$, and so on.

In Figure 2.9, we can see that all the profiles, including 12 additional ones derived from the FIRE-2 simulation, match in the outer regions of the galaxy. However, as we approach the galactic center, they begin to yield different values [12]. Modeling the inner parts of the galaxy is a difficult task because, as we saw earlier, the potential well becomes dominated by baryons. Some researchers use the above profiles and attempt to extrapolate to the inner regions using simulations that incorporate baryons into the dynamics of the system. However, these efforts have led to a variety of different profiles.

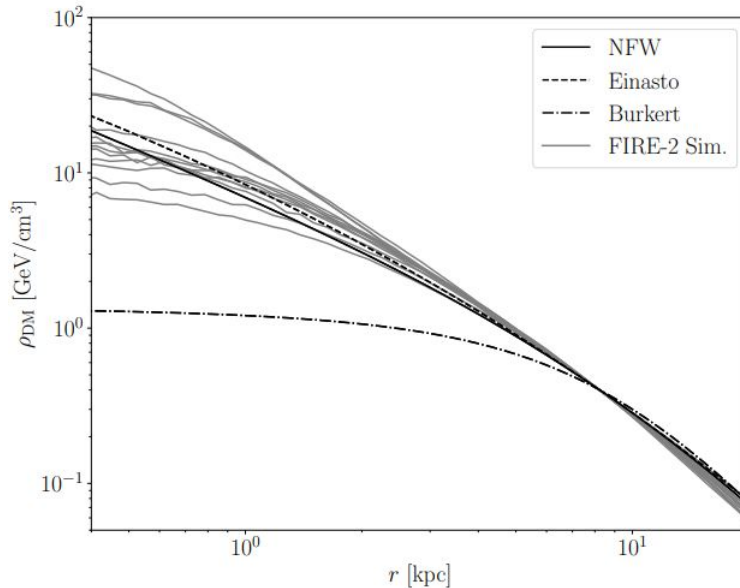


FIGURE 2.9: Milky Way fitting models for different DM halo profiles. Image taken from Safdi [12]

Despite the difficulties, the majority of dwarf galaxies are well modeled by the NFW profile, including galaxies such as ultra-faint⁷ dwarfs and classical dwarfs. However, the next step—bright dwarfs—are strongly influenced by baryonic effects [12]. Moreover, the ultra-faint and classical galaxies seem to be the same DM-dominated type Sa galaxies identified by Rubin [7] (see Figure 2.1(a)). These halos surrounding dwarf galaxies are embedded within the host halo of the Milky Way and other similar galaxies. Indeed, there are DM substructures within the halo, commonly known as subhalos, due to hierarchical clustering. N-body simulations predict subhalos that are not massive enough to form stars, with masses on the scale of the moon or asteroids. The number of subhalos within a host halo depends on the halo mass, as described by [17]:

$$\frac{dN_{sh}}{dM_{halo}} \sim \frac{1}{M_{halo}^2} \quad (2.7)$$

2.1.3 N-body simulations

In the previous section, we discussed different halo density profiles and how they are tested in N-body simulations. These simulations are significant computational achievements that allow researchers to create various scenarios to study the behavior of DM halos, bringing the early universe into 'our labs'. Without further ado, let us now discuss one of the first and most important DM-only N-body simulations, known as the Millennium Simulation (MS), and, in particular, its successor, the Millennium-II Simulation (MS-II).

The MS simulation published its results in 2005, using 10 billion particles in a volume of $(500 h^{-1} Mpc)^3$, which allowed the study of the formation of different halos and the implementation of physical models. However, as mentioned earlier, it is theorized that large structure formation occurred through hierarchical clustering, where low-mass halos merged together. To study the complex substructures present in main halos, such as subhalos hosting dwarf galaxies, smaller volumes and higher mass resolution are needed, which results in significant computational costs.

The MS-II reduces the volume of the simulation to a box with a side length of $100 h^{-1} Mpc$, using 2160^3 particles. This increases the mass resolution of its predecessor by a factor of 125, while using the same parameters for dark and baryonic matter fractional density. To give an idea of the high computational cost, the CPU hours required to take the initial conditions at redshift $z = 127$ and evolve them to the present were approximately 1.4 million, with only 10% of these hours used to evolve the simulation to $z = 6$ [18].

⁷The smaller halos that can pull enough dust and matter to form stars.

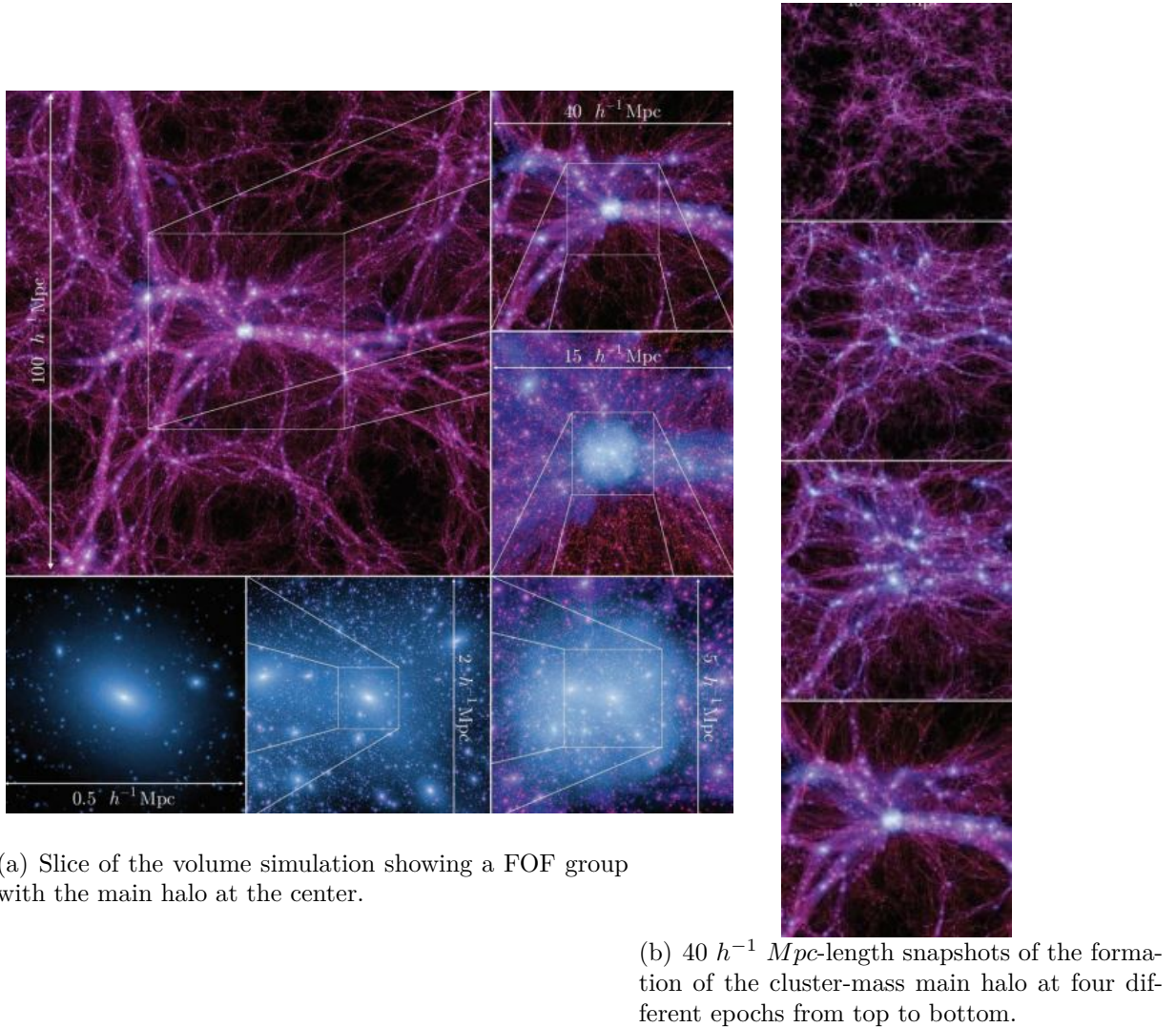


FIGURE 2.10: Some snapshots of MS-II simulation. Images taken from Boylan-Kolchin *et. al.* [18].

Figure 2.10(a) shows a slice of thickness $15 h^{-1} Mpc$ of the simulated box, centered on the largest friend-of-friends group in the simulation, which is composed of a cluster-mass halo containing approximately 119 million particles. In the upper-left panel, we can observe the filamentous structure known as the 'Cosmic Web', which represents the organization of galaxies and galaxy clusters on the largest scales. Moving clockwise through the panels, we zoom into the rich substructures contained within the main halo.

To emphasize, the main halo has a diameter of approximately $5 h^{-1} Mpc$ (bottom-right panel) and is composed solely of dark matter (DM) particles. As we delve deeper into the main halo, we find numerous subhalos, even as small as 1/10th of its diameter (bottom-left panel). This highlights the importance of higher mass resolution to study these substructures, especially when at least 99% of the mass of some of these subhalos has been stripped away by tidal forces as they merge, causing their masses to decrease as redshift decreases [18].

The last point is illustrated in Figure 2.10(b). This shows a slice of length $40 h^{-1} Mpc$ at four different redshifts: $z = 6.20$, $z = 2.07$, $z = 0.99$, and $z = 0$, from top to bottom, showing the formation of the cluster-mass main halo that we observed in Figure 2.10(a). This is done using merger trees, an algorithm that relates every subhalo between subsequent snapshots in the simulation based on their binding energies. Each merger

tree contains around 590 million subhalos. As redshift decreases, the blue-glowing points become brighter⁸, indicating that smaller halos are merging into more massive ones and reinforcing the idea that heavier halos form at lower redshifts.

For Milky Way-mass halos, the simulation is unable to resolve all of the substructures. One possible solution is to focus the simulation on a single galaxy halo, increasing the mass resolution, but sacrificing the statistical representativeness of the sample. To address this and other issues, higher mass resolutions for smaller volumes are required [18].

2.2 Properties and popular candidates

Alright, with the compilation of evidence that we have just seen, we can answer questions like, where is the DM? How much there is? But, we do not know with absolute clarity, what is the DM? For certain, we can make some statements about the properties of DM, based on the observations that we've seen:

1. It must live up to its name, in other words, be Dark, invisible. It should not interact with light, so it should be neutral.
2. Interacts almost only through gravity, hence it should be massive.
3. As we see with the coma cluster, DM does not have means to dissipate energy. Hence, it must interact weakly⁹ with it-self and SM particles and almost be collisionless.
4. Dark matter is though to be the seeds for every large structure in the universe, so it should be massive enough to create this potential wells.
5. There should be a constant amount of DM to preserve the large structures. In that it must be an stable particle.

There are tons of candidates proposed in the literature, ranging from baryonic objects like Primordial Black Holes (PBH) [20] to particles which lives in so tiny extra-dimensions, in a way that we can see them [9]. This implies a large range of masses. A lower limit we can set to the mass the particle is trying to define its spin-statistic nature. If it's a boson, then we can put any amount of particles in the same place, and due to the large occupation number, it behaves as a classical field. Its wave function then would be expanded for large distances, with a De Broglie wavelength comparative with some galactic radius and its stability ensured by the Uncertainty principle. With a lower mass limit of 10^{-22} eV, this kind of ultra-light DM it is known as Fuzzy Dark Matter. Some works claim that, treating this as a classical field with the Klein-Gordon equation, giving some wave modes k that are above Jeans mode (the inverse of the Jeans radius), which suppress the amount of tiny substructures that can be formed. This is a possible solution to the Cups-Core problem [21]. The most known candidate among ultra-light DM is the Axion, proposed in first place by Peccei and Quinn in 1977 as a solution for the strong CP problem [22]. A complete review of axion physics and its cosmological implications can be found in [9].

Talking about ultra-light particles, what happen to the neutrinos? With a first glance on its properties, one can argue that they are the obvious candidate. Furthermore, we now that neutrino have masses due to neutrino oscillations [23]. Very tiny ones, but they have it anyways. Moreover, the mass of the neutrinos is one of the issues of the SM, having to add an sterile right-handed neutrino to form a Yukawa term and give mass to it, so consider them as DM could bring some light to beyond the SM physics.

⁸Do not be confused; this is a DM-only simulation.

⁹There are some theories that postulate and strongly-interacting DM known as SIMPS, which below some scale it confines like QCD [19].

The problem with this elusive particle, is that it is relativistic, due to its tiny mass, entering in the category of Hot Dark Matter (HDM). This led some part of the community to think, that the structures we see today would not had time to form, but actually, some works claim that if neutrinos are the dominant form of DM in the universe, they could even slow down the processes of nucleosynthesis, and the Jeans mass $M_{J\nu}$, the amount of mass necessary for gravitational collapse, would have evolved to form large structure first and then smaller ones (see Figure ??), following a top-bottom structure formation in the early universe [24], which is exactly the opposite of what we see in simulations.

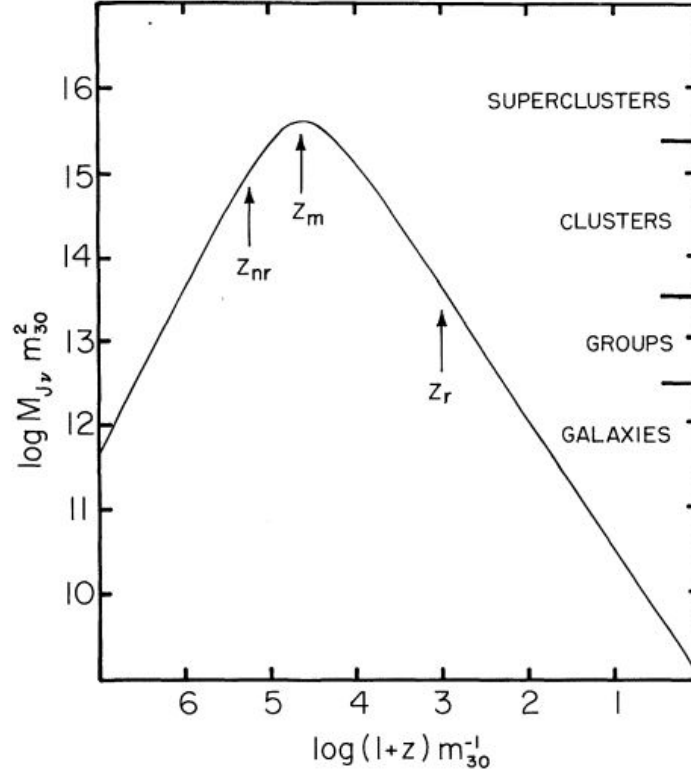


FIGURE 2.11: *Logarithm of the Jeans mass $M_{j\nu}$ for massive neutrinos as function of redshift. Image taken from Bond et. al. [24].*

2.2.1 Weakly Interactive Massive Particles

With the only Standard Model candidate ruled out, let us now discuss heavier particles—specifically, a class of candidates that are among the most promising in the literature. These are the Weakly Interacting Massive Particles (WIMPs), which interact only through the weak interaction with the SM sector, in addition to gravity. The reason behind their success is the so-called *WIMP miracle*, which essentially states that certain WIMP theories predict the exact velocity-averaged cross-section¹⁰ required to achieve the observed fractional energy density of dark matter, as measured by WMAP. This cross-section is approximately $\langle v\sigma \rangle \sim 10^{-26} \text{ cm}^3/\text{s}$ for masses in the range of 1 GeV to 1 TeV [12, 25]. The model presented in this thesis, along with its dark matter candidates, belongs to the WIMP category.

Three of the main WIMP-like candidates arise from supersymmetry (SUSY), an extension of the SM that introduces an additional symmetry relating bosons and fermions. In the Minimal Supersymmetric Standard

¹⁰We will define this quantity in the next section.

Model (MSSM), the number of SM particles is effectively doubled, as each known particle gains a superpartner of opposite spin-statistics. One such candidate is the sneutrino $\tilde{\nu}$, the bosonic superpartner of the neutrino. Sneutrinos can annihilate among themselves and co-annihilate with the superpartner of the electron via tree-level interactions involving other supersymmetric particles. However, sneutrinos have been excluded as viable dark matter candidates due to constraints from direct detection experiments (to be discussed later) and their insufficient relic density [26].

The other two prominent SUSY candidates are the Higgsino, the superpartner of the Higgs boson, and the neutralino. The Higgsino appears in the context of Minimal Dark Matter Models (MDMM), where it is treated as a doublet in the fundamental representation of $SU(2)_L$, consisting of a charged fermion and a neutral one. At first, these states are degenerate, but after electroweak symmetry breaking, a mass splitting of approximately 400 MeV arises between them. For a mass of 1 TeV, the Higgsino in this representation has been ruled out as a dark matter candidate by the LZ experiment, which excludes cross-sections above 10^{-46} cm^2 , while the Higgsino's predicted cross-section is on the order of 10^{-40} cm^2 [12].

The most promising supersymmetric dark matter candidate is the neutralino, a mixture of the photino, Zino, and neutral Higgsino—the superpartners of the photon, Z boson, and Higgs boson, respectively. The neutralino has received significant attention in the literature as a leading WIMP candidate [27, 9].

2.3 Freeze-out and relic density

The Cold Dark Matter (CDM) scenario¹¹ and WIMP-like models are founded on the following principles. After cosmic inflation and the Big Bang, the universe was a hot, dense plasma composed of a variety of particles, well described by the Friedmann equations. In the first few minutes of the early universe, it entered a radiation-dominated era, during which the formation of heavy dark matter particles is believed to have occurred—long before nucleosynthesis. From that moment on, these DM particles began forming the gravitational wells necessary for hierarchical large-scale structure formation.

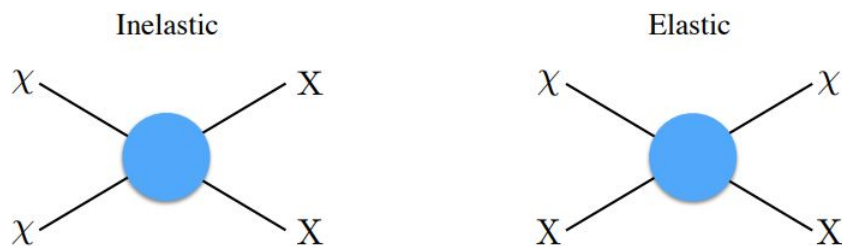


FIGURE 2.12: Pictorial diagram of the scattering processes that were happening in the early universe between DM (χ) and SM (X) particles. The inelastic scattering can be read from left to right for annihilation, and in the other direction for production of DM. Image taken from [25].

DM was initially in thermodynamic equilibrium with the rest of the plasma and the photon bath through elastic and inelastic collisions among themselves and SM particles (see Figure 2.12). However, as the universe expanded, DM began to decouple from the rest of the plasma. The standard assumption is that DM particles are heavier than SM ones, so for a brief period, the rate of DM particle creation matched its annihilation rate.

¹¹CDM refers to the Cold Dark Matter component in the Λ CDM model, which is the standard cosmological framework on which most modern theories and simulations are based.

However, as the universe expanded, the plasma cooled down, and SM particles no longer had sufficient energy to produce DM particles. Additionally, it became increasingly difficult for a DM particle to encounter another one to annihilate with. At this point, the condition $\Gamma_{in} \ll H$ is imposed, where Γ_{in} is the inelastic scattering rate, and H is the Hubble constant. Consequently, DM departs from thermal equilibrium due to the suppression of inelastic interactions, and its number density n_χ becomes "frozen" in time, persisting unchanged until the present day and beyond. This process, in which the DM density stabilizes over time, is known as freeze-out (see Figure 2.13).

Mathematically, this process is described using Statistical Mechanics. Any particle, whether bosonic or fermionic, in thermal equilibrium follows the distribution function:

$$f_i(p) = \frac{1}{\exp\{(E_i - \mu_i)/T\} \pm 1} \quad (2.8)$$

where $E_i = \sqrt{p_i^2 + m_i^2}$ is the energy of the species, μ_i is its chemical potential, and the plus sign corresponds to bosons, following Bose-Einstein statistics, while the minus sign corresponds to fermions, following Fermi-Dirac statistics. This distribution can be derived using the Grand Canonical Ensemble. By taking an ensemble average, we can compute the number density and energy density of the species as:

$$n_i = g_i \int \frac{d^3p}{(2\pi)^3} f_i(p) ; \quad \rho_i = g_i \int \frac{d^3p}{(2\pi)^3} E_i f_i(p) \quad (2.9)$$

where g_i is the number of degrees of freedom of the particle. In the relativistic limit, where $T \gg m$, these quantities scale as T^3 and T^4 , respectively. In contrast, in the non-relativistic limit, the number density follows $n \sim e^{-m/T}$, which corresponds to Maxwell-Boltzmann statistics, while the energy density remains constant. The statistical evolution of a species in the system can be described by the Boltzmann equation:

$$L[f] = C[f] \quad (2.10)$$

where $C[f]$ is an operator accounting for interactions, and $L[f]$ is the Liouville operator, which can be written in covariant form as [25]:

$$L[f] = E \frac{\partial f}{\partial t} - \frac{\dot{a}}{a} |\mathbf{p}|^2 \frac{\partial f}{\partial E} \quad (2.11)$$

By integrating this operator over phase space and applying integration by parts, we obtain:

$$g_i \int \frac{d^3p}{(2\pi)^3} L[f] = \frac{1}{a^3} \frac{d}{dt} (na^3) = \frac{dn}{dt} + 3Hn \quad (2.12)$$

which describes the time evolution of the number density of the species.

On the interaction side, the annihilation rate can be expressed as $\Gamma_{in} = n_\chi \langle \sigma v \rangle$, which can be interpreted as a DM particle traveling at velocity v and attempting to scatter off another DM particle with number density n_χ and cross-section σ . In equilibrium, two such terms—one for creation and one for annihilation—cancel out exactly. Out of equilibrium, after carefully treating the interaction operator $C[f]$ with a suitable level of approximation (see [12, 25]), we obtain the evolution equation for the number density:

$$\frac{dn}{dt} + 3Hn = \langle \sigma v \rangle (n^2 - n_{eq}^2) \quad (2.13)$$

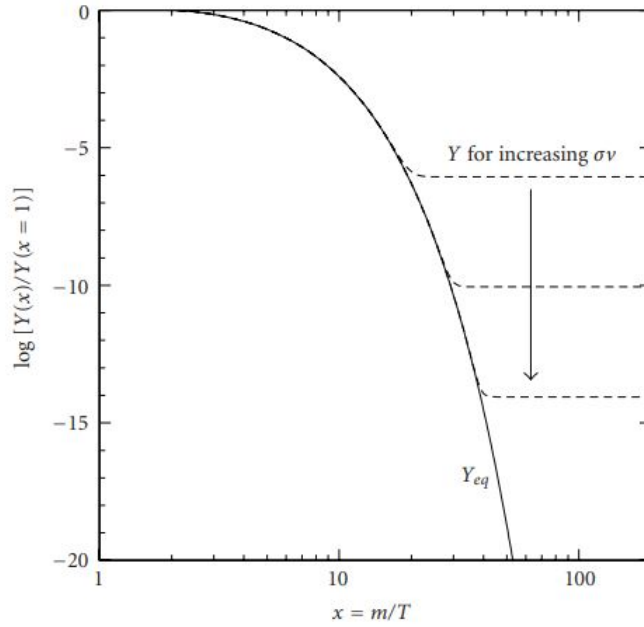


FIGURE 2.13: Co-moving number density as function of the re-scaled time variable x . Image taken from Garret et. al. [9].

All the information necessary to compute the so-called relic density is contained in the factor $\langle\sigma v\rangle$, which must include all annihilation and coannihilation processes of DM. Moreover, we can rewrite the number density in terms of the total entropy density s to factor out the effects of the universe's expansion, defining $Y = n/s$ and the rescaled time variable $x = m_\chi/T$. This leads to the equation [9]:

$$\frac{dY}{dx} = \sqrt{\frac{\pi}{45G}} \frac{g_i^{1/2} m_\chi}{x^2} \langle\sigma v\rangle (Y_{eq}^2 - Y^2) \quad (2.14)$$

By solving the equation numerically, we can integrate from $x = 0$ to $x_0 = m_\chi/T_0$, where T_0 is the temperature of the universe at freeze-out, to obtain Y_0 and ultimately compute the relic density, given by [9]:

$$\Omega_\chi h^2 = \frac{h^2 m_\chi s_0 Y_0}{\rho_c} \quad (2.15)$$

2.4 Detection

There are three approaches to detecting the elusive DM: direct detection, indirect detection, and collider searches. Although none of these methods directly ‘see’ DM, they differ in how they interact with the detector. In this section, we provide a brief overview of the framework and examples of the first two approaches.

2.4.1 Direct detection

Imagine you are walking down the street, and suddenly, a DM particle hits you in the head. Would it hurt? Probably not. Similar to neutrinos, DM is expected to interact too weakly with ordinary matter to be noticeable by us. But, what if DM actually manages to collide with something, say, an atomic nucleus? This is the fundamental idea behind direct detection. For a DM particle with a mass of the order of $m_\chi \sim 100$ [GeV] and velocities around $v \sim 200$ [km/s], the upper limit for the energy transfer in a collision with a heavy nucleus, assuming the DM particle is completely stopped, is $E_R \sim 30$ [9]. Meanwhile, the typical binding energy of a nucleus is up to $\sim 1 - 10$ [MeV], which is about a thousand times larger [25]. Given these numbers, we can say

that DM "sees" the nucleus as a whole and does not interact directly with quarks.

With this in mind, let's set up a detector. Typically, these detectors consist of large tanks filled with non-radioactive materials¹², often noble gases such as Xenon ($m_N \sim 120$ GeV [25]), as in XENON10 [28] and LUX [29]. Others, like CDMS-II, use Germanium and Silicon cooled to 50 mK, which allows them to distinguish between electrons and WIMPs [30]. Most of these detectors are located deep underground with thick shielding to protect them from unwanted background signals. They rely on highly sensitive thermometers capable of detecting tiny perturbations generated by nuclear recoils, which propagate through the detector material. Some setups use an electric field to guide emitted electrons, which gain enough energy to escape their binding potential, producing an ionization event. Others take advantage of Xenon's scintillation properties: when an electron transitions to an excited state and subsequently emits a photon to return to its ground state, this photon is captured by photomultipliers and converted into an electric signal [9]. A more extensive and detailed review of the various detectors and techniques can be found in [31, 32].

Unfortunately, no experiment has yet provided a conclusive signal confirming the detection of such recoils. However, most of these experiments have set upper limits on the cross-section of these events. Some of these exclusion limits are shown in Figure 2.14. In particular, the graph on the right highlights what is known as the neutrino floor (orange-yellow lower region), which marks the sensitivity threshold at which experiments become affected by coherent neutrino scattering off nuclei.

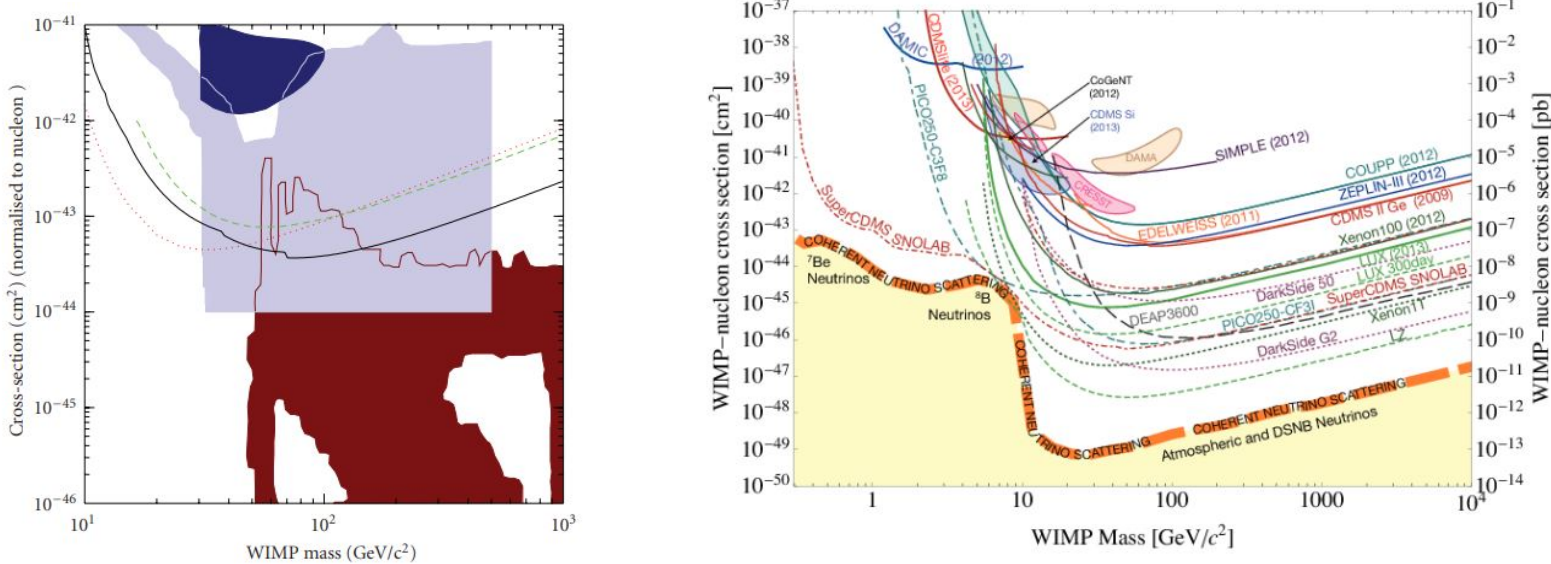


FIGURE 2.14: (a) Exclusion curves coming from ZEPLIN III (dashed), CDMS (Solid) and XENON10 (Dotted). The shaded regions are zones of the parameter space where different theories predicts DM. Image taken from [9]. (b) Exclusion curves and neutrino floor. Image taken from [31].

The exclusion lines are set based on the criterion that, at some value of the cross-section and DM mass, the expected number of events (N) exceeds the number of measured events. Let us compute N within a recoil energy range $E_1 - E_2$:

$$N = \sum_i \int_{E_1}^{E_2} \frac{dR_i}{dE_R} \mathcal{E}_i(E_R) dE_R \quad (2.16)$$

where the index i runs over all nuclear species. The factor $\mathcal{E}_i(E_R)$ is the exposure factor, which depends on the

¹²Radioactivity has an energy of the order of MeV.

experiment. The particle physics contribution appears in the differential rate per nucleus mass, given by:

$$\frac{dR_i}{dE_R} = \frac{n_\chi}{m_N} \left\langle v \frac{d\sigma_i}{dE_R} \right\rangle = \frac{\rho_{DM}}{m_\chi m_N} \int_{v_{min}}^{v_{max}} d^3v v \frac{d\sigma_i}{dE_R} \tilde{f}(\vec{v}, t) \quad (2.17)$$

where ρ_{DM} is the local DM density profile, usually assumed to follow the Standard Halo Model. $\tilde{f}(\vec{v}, t)$ is the Maxwell-Boltzmann phase-space distribution in the lab frame. Since DM is non-relativistic, we can perform a Galilean boost to the galactic reference frame (assumed to be at rest), yielding $\tilde{f}(\vec{v}, t) = f(\vec{v} + \vec{v}_{obs}(t))$, where $\vec{v}_{obs}(t) = \vec{v}_\odot + \vec{v}_\oplus(t)$, with the Sun's velocity in the Milky Way (~ 220 [km/s]) and the Earth's velocity (~ 30 [km/s]). Expanding this, we obtain:

$$\begin{aligned} \vec{v}_{obs}(t) &\approx \vec{v}_\odot (1 + \epsilon \cos(\omega(t - t_0))) \\ \longrightarrow f &\approx \underbrace{f(\vec{v} + \vec{v}_\odot)}_{A_0} + \underbrace{\epsilon \cos(\omega(t - t_0)) f'(\vec{v} + \vec{v}_\odot)}_{A_1} \end{aligned} \quad (2.18)$$

where ϵ is the Sun-Earth velocity ratio and $\omega = 2\pi/\text{year}$. The second term, A_1 , is called the Annual Modulation, encapsulating the effects of Earth's motion around the Sun. In the galactic frame, DM has no preferred direction of motion, but in the lab frame, there is a DM "wind" opposite to the Sun's motion¹³. During June, Earth moves toward this "wind," increasing the DM flux, whereas in December, it moves away, decreasing the flux. There is also gravitational focusing when Earth is in front of or behind the Sun relative to the DM wind. Further reading can be found in [25] and references therein.

To complete the computation of the differential rate, we need to define the integration limits and the differential cross-section. v_{max} is set as the DM escape velocity from the gravitational potential, $v_{esc} \sim 500$ [km/s]. v_{min} is determined by analyzing the kinematics of the process $\chi N \rightarrow \chi' N$ in the lab frame, where the nucleus is initially at rest and χ' is an excited state with mass $m_\chi + \delta$, classified as inelastic scattering. Using the non-relativistic limit, we obtain:

$$v_{min} = \frac{1}{\sqrt{2m_N E_R}} \left| \frac{E_R(m_\chi + m_N)}{m_\chi} + \delta \right| \quad (2.19)$$

For $\delta \rightarrow 0$, we recover the elastic scattering case, and the velocity simplifies to:

$$v_{min} = \sqrt{\frac{m_N E_R}{2\mu^2}} \quad (2.20)$$

where μ is the system's reduced mass. Notably, for increasing DM mass, the minimum velocity decreases. Finally, we compute the differential cross-section using the effective Lagrangian:

$$\mathcal{L}_{eff} = g(q^2, m_\phi^2) \bar{\chi} \Gamma_\chi \chi \bar{Q} \Gamma_Q Q \quad (2.21)$$

where $g(q^2, m_\phi^2)$ is the coupling constant associated with a Dirac spinor χ interacting with a quark Q , mediated by a vector or scalar particle ϕ . Γ_i can take various forms, such as $I, \gamma^\mu, \gamma^5, \sigma^{\mu\nu}$, among others. To transition from quark to nucleon fields, we use results from [33]:

$$\mathcal{M} = f_p \bar{\chi} \chi \bar{p} p + f_n \bar{\chi} \chi \bar{n} n \quad (2.22)$$

where $f_{p,n}$ denotes the DM coupling to protons and neutrons, given by:

$$f_p = \sum_{q=u,d,s} m_p \frac{g_\phi}{m_q} f_{T_q}^p + \frac{2}{27} \left(1 - \sum_{q=u,d,s} f_{T_q}^p \right) \sum_{c,b,t} m_p \frac{g_\phi}{m_q} \quad (2.23)$$

¹³To visualize this, think of the solar system as a large vessel partially submerged in an ocean, moving forward. The crew inside can observe water currents approaching and passing by through a window. From the water's perspective (the galactic frame), the submarine is in motion.

where $f_{T_q}^p$ represents the fraction of the proton mass carried by quark q . A similar expression holds for f_n . Rewriting the amplitude for the entire nucleus:

$$\mathcal{M} = [Zf_p + (A - Z)f_n]\bar{\chi}\chi\bar{N}\Gamma_N N \quad (2.24)$$

where Z is the atomic number, A is the mass number, and Γ_N incorporates nuclear form factors $F(q)$, known as the Helm factor [25]. The resulting differential cross-section is:

$$\frac{d\sigma}{dE_R} = \frac{2m_N}{\pi v^2} [Zf_p + (A - Z)f_n]^2 F^2(q) \quad (2.25)$$

For $f_p \approx f_n$, the differential cross-section scales as A^2 , meaning that heavier nuclei provide stronger scattering signals due to coherent DM-nucleus coupling. This type of interaction is spin-independent, as σ does not depend on nuclear spin. A comprehensive review of spin-dependent interactions can be found in [34].

As shown above, the differential cross-section scales as $1/v^2$, leading to a differential rate proportional to:

$$\frac{dR}{dE_R} \sim \int_{v_{min}}^{v_{esc}} d^3v \frac{f(\vec{v} + \vec{v}_\odot)}{v} \sim \int_{v_{min}}^{v_{esc}} dv v e^{-v^2/v_0^2} \sim e^{-E_R/E_0} \quad (2.26)$$

which exhibits an exponential decay with recoil energy [25].

2.4.2 Indirect detection

If, in direct detection, DM interacts directly with the detector, the alternative approach is to detect particles produced by the annihilation of DM particles—in other words, indirect detection. Although DM annihilation is strongly suppressed after freeze-out, it can still occur, and its signals can be enhanced by observing regions of the sky with high DM density. In this approach, we seek products that can travel long distances without being disrupted and that we can also detect. Some of these products include neutrinos, antimatter, and the preferred candidate: gamma rays.

Neutrinos

WIMPs can be attracted by potential wells, increasing their density until the annihilation rate reaches half of its maximum value, leading to equilibrium. This process produces a stable and constant source of neutrinos. An example of such a neutrino source is our Sun. The neutrino flux from WIMP annihilation is given by:

$$\frac{dN_\nu}{dE_\nu} = \frac{\Gamma_A}{4\pi D^2} \sum_f B_X^f \frac{dN_\nu^f}{dE_\nu} \quad (2.27)$$

where D is the distance to the detector, Γ_A is the WIMP annihilation rate, and f represents each final state of the annihilation. The terms B_X^f and $\frac{dN_\nu^f}{dE_\nu}$ correspond to the branching ratio and the neutrino flux of the final states, respectively. Depending on the mass of the DM particle, annihilation can lead to different products, which subsequently decay into neutrinos [9].

To set up a neutrino detector, we need to follow an approach similar to direct detection. Neutrinos can pass through the detector walls and interact with the atoms in the medium, producing high-energy electrons, muons, and taus. These particles travel faster than the speed of light in the medium, emitting Cherenkov radiation. Some neutrino telescopes, such as AMANDA-II, located 2000 [m] beneath the Antarctic ice [35], or Super-Kamiokande, situated in the Kamioka-Mozumi mine in Japan [36], have not detected any excess in the muon flux beyond the background, but they have placed strong limits on it.

Antimatter

Antimatter is very rare from a cosmological perspective, so an excess in an antimatter signal, beyond the well-known production mechanisms, can be a good tracer for DM. These signals can be observed in the form of charged cosmic rays. Antiprotons can be produced during hadronization, just after the formation of a $q\bar{q}$ pair. Additionally, positrons can be detected as secondary products from $W^\pm$ and Z decays via $e^+ \nu_e$. The main issue with this method is that these particles are charged, meaning they interact with the magnetic fields of astrophysical objects, causing a complete loss of information about the original location of the DM annihilation. Consequently, instead of focusing on very dense regions, we analyze the galaxy as a whole.

An antimatter detector must be positioned very high in the atmosphere to reduce background noise from other cosmic rays and particle showers. The HEAT collaboration observed an excess of positrons at around 10 [GeV] in 1994 [37]. However, to match the observed results, a boost factor must be applied to the WIMP annihilation rate in a smooth halo. Similarly, PAMELA detected an increase in the positron fraction between 1.5 and 100 GeV [38], but its interpretation requires heavy DM masses and a boost factor associated with the presence of clumps in cuspy profiles [17]. In conclusion, more data is needed to clarify these excesses of antimatter and determine whether they originate from DM annihilation or local astrophysical objects [9].

Gamma-rays

The most promising way to indirectly detect dark matter is through the production of highly energetic photons. Unlike charged cosmic rays, gamma rays travel in straight lines, passing unperturbed through other objects. Although the direct production of gamma rays, such as $\chi\chi \rightarrow \gamma\gamma$ or $\chi\chi \rightarrow \gamma Z$, is loop-suppressed in many models, it can manifest as a sharp gamma-ray line in the spectrum with energy $E_\gamma \sim m_\chi$ (see Figure 2.15, right, in blue). This signal is enhanced if there is Bremsstrahlung (see Figure 2.15, right, in red). Additionally, gamma rays can also be produced as secondary products of SM particle interactions or as part of the energy spectrum resulting from the hadronization of quark-antiquark pairs. The different energy contributions can be determined using a simple toy model of the process $\chi\chi \rightarrow \gamma X$, where $X = H, Z, \gamma$, or any other neutral state, applying energy conservation in the non-relativistic limit [25]:

$$2m_\chi = E_\gamma + \sqrt{E_\gamma^2 + m_X^2} \longrightarrow E_\gamma \approx m_\chi \left(1 - \frac{m_X^2}{4m_\chi^2}\right) \quad (2.28)$$

To fully characterize the gamma-ray signal, we need to determine the photon flux, which is given by the annihilation rate per particle for each annihilation channel, multiplied by the total number of DM particles in a volume dV and by the number of photons at a given energy E_γ for the i -th channel $\frac{dN_i}{dE_\gamma}$, integrated over the volume [25]:

$$\frac{d\Phi}{dE_\gamma} = \sum_i \int \Gamma_i^{Ann} \frac{dV}{4\pi r^2} \frac{dN_i}{dE_\gamma} \quad (2.29)$$

where Γ_i^{Ann} is the annihilation rate, defined as [12]:

$$\Gamma_i^{Ann} = \frac{1}{2} n_{DM}^2 \langle \sigma_i v \rangle = \frac{\langle \sigma_i v \rangle}{2m_\chi^2} \rho_{DM}^2(r) \quad (2.30)$$

Then, the photon flux is:

$$\frac{d\Phi}{dE_\gamma} = \sum_i \frac{\langle \sigma_i v \rangle}{8\pi m_\chi^2} \frac{dN_i}{dE_\gamma} J(\psi) \quad (2.31)$$

where,

$$J(\psi) = \int d\Omega dr \rho_{DM}^2(r) \quad (2.32)$$

is the J-factor, which depends on the angle ψ away from the galactic plane and absorbs all astrophysical uncertainties. In particular, the J-factor increases in the inner regions of the Milky Way, reaching values of $\sim 10^{22-25} [\text{GeV}/\text{cm}^5]$, whereas dwarf galaxies exhibit enhancements of only $\sim 10^{19-20} [\text{GeV}/\text{cm}^5]$ [25]. In the left panel of Figure 2.15, we show the J-factor for different DM density profiles found in the literature and discussed in this chapter. We see that the Einasto profile appears “brighter,” extending deeper into the galactic plane. However, all profiles diverge significantly in their values as we approach the galactic center.

Why do we look at other astrophysical objects beyond our own galaxy? Despite the fact that dwarf galaxies have a lower J-factor, they are entirely DM-dominated and provide cleaner targets with fewer stars, dust, and astrophysical backgrounds than the Milky Way. Additionally, we can also study supermassive galaxy clusters, which provide an extreme boost factor to the signal [12].

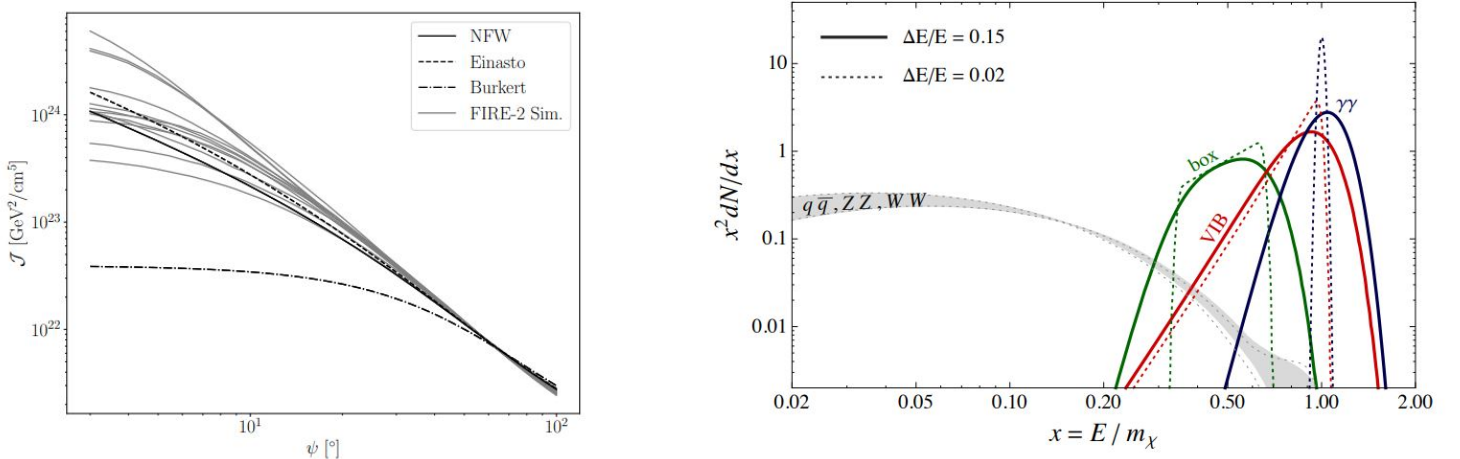


FIGURE 2.15: (a) J-factor for different DM halo profiles like Einasto, NFW and Burkert. (b) Photon energy spectrum for the different channels of production. The blue lines corresponds to a pure loop-production and the red lines to and extra Bremsstrahlung. The green lines corresponds to production in secondary process.

To detect these gamma rays, we can either launch a detector onboard a satellite into space or set up a telescope on Earth. Space-based detectors, such as the Fermi Large Area Telescope (LAT), which is part of the Fermi Gamma-ray Space Telescope [39], use alternating layers of high- Z materials like tungsten and silicon. The tungsten is used to convert incoming gamma rays into electron-positron pairs, while the silicon layers infer the direction of the incoming photon. Additionally, a calorimeter is used to determine the energy of the gamma ray. The advantage of these telescopes is that they can observe a large fraction of the sky with an energy range of approximately 100 MeV – 2 TeV [12]. However, a major drawback is the need to overcome Earth’s gravity to place them in orbit, which significantly increases costs. To mitigate expenses, the effective observation area is limited to about 1 m^2 .

Ground-based detectors, on the other hand, aim to capture the fast-traveling particles produced in the upper atmosphere when gamma rays interact with it. Particles such as electrons (e^-), positrons (e^+), or even muons are detected by ground-based telescopes, similar to neutrino detectors. Additionally, these particles generate what is known as a ‘particle shower’, producing secondary photons along their path, which then decay into more high-energy particles with energies starting from approximately 100 GeV and beyond [12]. Both the original and secondary particles travel faster than the speed of light in the medium, emitting Cherenkov radiation, which can be detected by ground-based telescopes such as HESS, MAGIC, and VERITAS (see [40] for an overview).

An upcoming observatory, the Southern Wide-Field Gamma-Ray Observatory (SWGGO), will be located in the Atacama Astronomical Park in Chile, at an altitude of 4 km above sea level [\[41\]](#).

Fermion triplet model

3.1 Lagrangian

Our model is an extension of the Standard Model, replicating its gauge-scalar sector structure with a dark Higgs field doublet ($\tilde{\varphi}$) that mixes with the Standard Higgs (H) field, acting as a Higgs portal. Additionally, it includes three dark gauge bosons (the carriers of a hypothetical fifth force) living in the adjoint representation of the gauge group $SU(2)_D$ ¹. The main difference is the introduction of three dark fermions, forming a triplet in the adjoint representation of the same group, acting as a vector-like partner. This setup allows a mass term for the dark fermion triplet without explicitly breaking the symmetry. The Lagrangian is:

$$\begin{aligned}\mathcal{L} = & \mathcal{L}_{SM} - \frac{1}{2} \text{Tr}\{V_{\mu\nu}V^{\mu\nu}\} + i\bar{\psi}\not{D}\psi - M_\psi\bar{\psi}\psi \\ & + (D_\mu\tilde{\varphi})^\dagger D^\mu\tilde{\varphi} - \mu_D^2\tilde{\varphi}^\dagger\tilde{\varphi} - \lambda_D(\tilde{\varphi}^\dagger\tilde{\varphi})^2 \\ & - \lambda_{h\phi}(\tilde{H}^\dagger\tilde{H})(\tilde{\varphi}^\dagger\tilde{\varphi})\end{aligned}\tag{3.1}$$

where $\lambda_{h\phi}$ is the coupling constant of the mixing between the two Higgs fields. μ_D and λ_D are the constants associated with the dark sector of the scalar potential.

Thus, the full gauge group of the model is $SU(2)_L \times U(1)_Y \times SU(2)_D$ ². There is also a global $U(1)_{DF}$ symmetry corresponding to the conservation of dark fermion number. The covariant derivatives are defined as follows:

$$\begin{aligned}D_\mu\psi &= \partial_\mu\psi - ig_D[V_\mu, \psi] = T^a\partial_\mu\psi^a - ig_D[T^a, T^b]V_\mu^a\psi^b = T^a\partial_\mu\psi^a + g_D\epsilon^{cab}T^cV_\mu^a\psi^b \\ D_\mu\tilde{\varphi} &= \partial_\mu\tilde{\varphi} - ig_DV_\mu\tilde{\varphi} = \partial_\mu\tilde{\varphi} - ig_DT^aV_\mu^a\tilde{\varphi}\end{aligned}\tag{3.2}$$

The fermion covariant derivative follows the adjoint representation, as expected. The dark gauge field strength tensor is given by:

$$V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - ig_D[V_\mu, V_\nu]$$

In component form:

$$V_{\mu\nu}^a = \partial_\mu V_\nu^a - \partial_\nu V_\mu^a + g_De^{abc}V_\mu^bV_\nu^c\tag{3.3}$$

where g_D is the dimensionless dark coupling constant.

¹which is not chiral.

²The $SU(3)_C$ group of QCD is left implicit.

3.2 Gauge invariance

Like we mentioned before, the fermion field ψ lives in the adjoint representation of $SU(2)_D$, so it should transform as:

$$\psi' = U\psi U^{-1} \quad (3.4)$$

and consequently, its covariant derivative should transform in the same way:

$$(D_\mu \psi)' = U(D_\mu \psi)U^{-1} \quad (3.5)$$

where $U = U(x)$ is some matrix of the group $SU(2)_D$. Now, let's examine how $\bar{\psi} = \psi^\dagger \gamma^0$ transforms. Taking the Hermitian conjugate of the transformation of ψ :

$$(\psi')^\dagger = (\psi^\dagger)' = (U\psi U^{-1})^\dagger = (U^{-1})^\dagger (U\psi)^\dagger = U\psi^\dagger U^{-1}$$

Multiplying by γ^0 from the right, we obtain:

$$\begin{aligned} (\psi^\dagger)' \gamma^0 &= (\psi^\dagger \gamma^0)' = \bar{\psi}' = U\psi^\dagger \gamma^0 U^{-1} \\ &\longrightarrow \bar{\psi}' = U\bar{\psi} U^{-1} \end{aligned} \quad (3.6)$$

Thus, $\bar{\psi}$ transforms in the same way as ψ . Now, let's analyze how the kinetic and mass terms transform:

$$i\bar{\psi}' (\not{D}\psi)' = iU\bar{\psi} U^{-1} U \not{D}\psi U^{-1} = U\bar{\psi} \not{D}\psi U^{-1} \neq \bar{\psi} \not{D}\psi \quad (3.7)$$

As we can see, this term breaks the symmetry, as does the mass term:

$$M_F \bar{\psi}' \psi' = M_F U\bar{\psi} U^{-1} U\psi U^{-1} = U\bar{\psi} \psi U^{-1} \neq \bar{\psi} \psi \quad (3.8)$$

One way to fix this is to enclose these terms within a trace:

$$\begin{aligned} i \text{Tr} \{ \bar{\psi}' (\not{D}\psi)' \} &= i \text{Tr} \{ U\bar{\psi} \not{D}\psi U^{-1} \} = i \text{Tr} \{ U^{-1} U\bar{\psi} \not{D}\psi \} = i \text{Tr} \{ \bar{\psi} \not{D}\psi \} \\ M_F \text{Tr} \{ \bar{\psi}' \psi' \} &= M_F \text{Tr} \{ U\bar{\psi} \psi U^{-1} \} = M_F \text{Tr} \{ \bar{\psi} \psi \} \end{aligned} \quad (3.9)$$

Thus, the Lagrangian is modified as follows:

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{SM} - \frac{1}{2} \text{Tr} \{ V_{\mu\nu} V^{\mu\nu} \} + i \text{Tr} \{ \bar{\psi} \not{D}\psi \} - M_\psi \text{Tr} \{ \bar{\psi} \psi \} \\ &\quad + (D_\mu \tilde{\varphi})^\dagger D^\mu \tilde{\varphi} - \mu_D^2 \tilde{\varphi}^\dagger \tilde{\varphi} - \lambda_D (\tilde{\varphi}^\dagger \tilde{\varphi})^2 \\ &\quad - \lambda_{h\phi} (\tilde{H}^\dagger \tilde{H}) (\tilde{\varphi}^\dagger \tilde{\varphi}) \end{aligned} \quad (3.10)$$

For the kinetic term of V_μ , we know that $V_{\mu\nu}$ transforms as:

$$\begin{aligned} V'_{\mu\nu} &= U V_{\mu\nu} U^{-1} \\ \longrightarrow \text{Tr} \{ V'_{\mu\nu} V^{\mu\nu'} \} &= \text{Tr} \{ U V_{\mu\nu} U^{-1} U V^{\mu\nu'} U^{-1} \} = \text{Tr} \{ U^{-1} U V_{\mu\nu} V^{\mu\nu'} \} = \text{Tr} \{ V_{\mu\nu} V^{\mu\nu'} \} \end{aligned} \quad (3.11)$$

confirming that it is gauge invariant. Finally, it is straightforward to verify that the scalar sector respects the symmetry under the transformations $\tilde{\varphi}' = U\tilde{\varphi}$ and $\tilde{H}' = \mathcal{U}\tilde{H}$, where $\mathcal{U} = \mathcal{U}(x)$ belongs to the gauge group $SU(2)_L$ of the Standard Model.

3.3 Spontaneous symmetry breaking

In a manner similar to the electroweak symmetry breaking in the Standard Model, our dark Higgs field acquires a vacuum expectation value (v.e.v) given by:

$$\langle \tilde{\varphi} \rangle = \begin{pmatrix} 0 \\ \frac{v_D}{\sqrt{2}} \end{pmatrix} \quad (3.12)$$

Similarly, the Standard Model Higgs field acquires its own v.e.v:

$$\langle \tilde{H} \rangle = \begin{pmatrix} 0 \\ \frac{v_S}{\sqrt{2}} \end{pmatrix} \quad (3.13)$$

3.3.1 Higgs Portal

To determine expressions for v_D and v_S , we must consider the entire scalar potential, i.e., the potential for the dark and Standard Model Higgs fields, including their mixing term:

$$V(\tilde{\varphi}, \tilde{\varphi}^\dagger, H, \tilde{H}^\dagger) = \mu_D^2 \tilde{\varphi}^\dagger \tilde{\varphi} + \lambda_D (\tilde{\varphi}^\dagger \tilde{\varphi})^2 + \mu_S^2 \tilde{H}^\dagger \tilde{H} + \lambda_S (\tilde{H}^\dagger \tilde{H})^2 + \lambda_{h\phi} (\tilde{\varphi}^\dagger \tilde{\varphi}) (\tilde{H}^\dagger \tilde{H}) \quad (3.14)$$

Evaluating the v.e.v.'s in the potential, we obtain:

$$V(v_D, v_S) = \frac{\mu_D^2}{2} v_D^2 + \frac{\lambda_D}{4} v_D^4 + \frac{\mu_S^2}{2} v_S^2 + \frac{\lambda_S}{4} v_S^4 + \frac{\lambda_{h\phi}}{4} v_D^2 v_S^2 \quad (3.15)$$

Minimizing, we obtain the following 2×2 system of equations:

$$\begin{aligned} \frac{\partial V}{\partial v_D} &= v_D \left(\lambda_D v_D^2 + \frac{\lambda_{h\phi}}{2} v_S^2 + \mu_D^2 \right) = 0, \\ \frac{\partial V}{\partial v_S} &= v_S \left(\lambda_S v_S^2 + \frac{\lambda_{h\phi}}{2} v_D^2 + \mu_S^2 \right) = 0. \end{aligned} \quad (3.16)$$

This yields the solutions:

$$v_D = \pm \frac{\sqrt{2} \sqrt{\lambda_{h\phi} \mu_S^2 - 2\lambda_S \mu_D^2}}{\sqrt{4\lambda_S \lambda_D - \lambda_{h\phi}^2}}, \quad v_S = \pm \frac{\sqrt{2} \sqrt{\lambda_{h\phi} \mu_D^2 - 2\lambda_D \mu_S^2}}{\sqrt{4\lambda_D \lambda_S - \lambda_{h\phi}^2}}, \quad (v_D, v_S) = (0, 0). \quad (3.17)$$

Since μ_D^2 and μ_S^2 are negative, the trivial solution is a maximum. We choose the positive solution for each v.e.v. as the minimum of our potential and expand around it:

$$\tilde{\varphi}(x) = e^{\frac{i}{v_D} T^a \pi_D^a} \begin{pmatrix} 0 \\ \frac{\tilde{\phi}(x) + v_D}{\sqrt{2}} \end{pmatrix}, \quad \tilde{H}(x) = e^{\frac{i}{v_S} T^a \pi_S^a} \begin{pmatrix} 0 \\ \frac{\tilde{h}(x) + v_S}{\sqrt{2}} \end{pmatrix}$$

where $\tilde{\phi}$ and $\tilde{h}$ are the dark and Standard Model Higgs bosons, respectively, and π^a are the Goldstone bosons for each sector. For simplicity in this derivation, we work in the unitary gauge. Thus, the expansions become:

$$\tilde{\varphi} = \begin{pmatrix} 0 \\ \frac{\tilde{\phi} + v_D}{\sqrt{2}} \end{pmatrix}, \quad \tilde{H} = \begin{pmatrix} 0 \\ \frac{\tilde{h} + v_S}{\sqrt{2}} \end{pmatrix}. \quad (3.18)$$

Finally, we can determine μ_D^2 and μ_S^2 from the v.e.v.'s:

$$\begin{aligned}\mu_D^2 &= -\lambda_D v_D^2 - \frac{\lambda_{h\phi}}{2} v_S^2, \\ \mu_S^2 &= -\frac{\lambda_{h\phi}}{2} v_D^2 - \lambda_S v_S^2.\end{aligned}\tag{3.19}$$

The final form of the potential is:

$$\begin{aligned}V(\tilde{\phi}, \tilde{h}) &= -\frac{v_D^4 \lambda_D}{4} - \frac{v_S^4 \lambda_S}{4} - \frac{v_S^2 v_D^2 \lambda_{h\phi}}{4} \\ &\quad + v_D^2 \lambda_D \tilde{\phi}^2 + v_S v_D \lambda_{h\phi} \tilde{h} \tilde{\phi} + v_S^2 \lambda_S \tilde{h}^2 \\ &\quad + v_D \lambda_D \tilde{\phi}^3 + \frac{1}{2} v_D \lambda_{h\phi} \tilde{\phi} \tilde{h}^2 + \frac{1}{2} v_S \lambda_{h\phi} \tilde{\phi}^2 \tilde{h} + v_S \lambda_S \tilde{h}^3 \\ &\quad + \frac{\lambda_D}{4} \tilde{\phi}^4 + \frac{\lambda_{h\phi}}{4} \tilde{\phi}^2 \tilde{h}^2 + \frac{\lambda_S}{4} \tilde{h}^4.\end{aligned}\tag{3.20}$$

We identify three constant terms, additional interaction terms, and, more importantly, mass terms for the Higgs bosons, including a quadratic mixing term. This poses a problem because it indicates that the mass matrix $\mathbb{M}$ is not diagonal:

$$v_D^2 \lambda_D \tilde{\phi}^2 + v_S v_D \lambda_{h\phi} \tilde{h} \tilde{\phi} + v_S^2 \lambda_S \tilde{h}^2 = \frac{1}{2} \begin{pmatrix} \tilde{\phi} & \tilde{h} \end{pmatrix} \begin{pmatrix} 2v_D^2 \lambda_D & v_S v_D \lambda_{h\phi} \\ v_S v_D \lambda_{h\phi} & 2v_S^2 \lambda_S \end{pmatrix} \begin{pmatrix} \tilde{\phi} \\ \tilde{h} \end{pmatrix}, \quad \mathbb{M} = \begin{pmatrix} 2v_D^2 \lambda_D & v_S v_D \lambda_{h\phi} \\ v_S v_D \lambda_{h\phi} & 2v_S^2 \lambda_S \end{pmatrix}.$$

This implies that the original Higgs bosons are not the physical states; we must rotate them. This is achieved as follows:

$$\begin{pmatrix} \tilde{\phi} \\ \tilde{h} \end{pmatrix} = \mathcal{O} \begin{pmatrix} \phi \\ h \end{pmatrix}, \quad \mathcal{O} = \begin{pmatrix} \cos(\theta_D) & -\sin(\theta_D) \\ \sin(\theta_D) & \cos(\theta_D) \end{pmatrix}.$$

Thus, the gauge Higgs bosons, in terms of the physical states, are:

$$\tilde{\phi} = \cos(\theta_D) \phi - \sin(\theta_D) h, \quad \tilde{h} = \sin(\theta_D) \phi + \cos(\theta_D) h.\tag{3.21}$$

Rotating the mass matrix, we obtain:

$$\mathbb{M}' = \mathcal{O}^T \mathbb{M} \mathcal{O}.$$

$$\mathbb{M}'_{11} = 2 \left(v_D^2 \lambda_D \cos^2(\theta_D) - v_S v_D \lambda_{h\phi} \sin(\theta_D) \cos(\theta_D) + v_S^2 \lambda_S \sin^2(\theta_D) \right)\tag{3.22}$$

$$\mathbb{M}'_{12} = \mathbb{M}'_{21} = v_D v_S \lambda_{h\phi} \cos(2\theta_D) + (v_D^2 \lambda_D - v_S^2 \lambda_S) \sin(2\theta_D)\tag{3.23}$$

$$\mathbb{M}'_{22} = 2 \left(v_S^2 \lambda_S \cos^2(\theta_D) + v_S v_D \lambda_{h\phi} \sin(\theta_D) \cos(\theta_D) + v_D^2 \lambda_D \sin^2(\theta_D) \right)\tag{3.24}$$

Requiring the off-diagonal terms to vanish, we obtain:

$$\tan(2\theta_D) = \frac{v_D v_S \lambda_{h\phi}}{(v_S^2 \lambda_S - v_D^2 \lambda_D)}\tag{3.25}$$

Finally, the Higgs boson masses are:

$$\begin{aligned}m_\phi^2 &= \lambda_D v_D^2 (1 + \sec(2\theta_D)) + \lambda_S v_S^2 (1 - \sec(2\theta_D)), \\ m_h^2 &= \lambda_D v_D^2 (1 - \sec(2\theta_D)) + \lambda_S v_S^2 (1 + \sec(2\theta_D)).\end{aligned}\tag{3.26}$$

3.3.2 Gauge-Scalar sector

We will develop the covariant derivative $D_\mu \tilde{\varphi}$ and expand it around the vacuum:

$$\begin{aligned} D_\mu \tilde{\varphi} &= \begin{pmatrix} \partial_\mu - i\frac{g_D}{2}V_\mu^3 & -i\frac{g_D}{2}(V_\mu^1 - iV_\mu^2) \\ -i\frac{g_D}{2}(V_\mu^1 + iV_\mu^2) & \partial_\mu + i\frac{g_D}{2}V_\mu^3 \end{pmatrix} \begin{pmatrix} 0 \\ \frac{\tilde{\phi} + v_D}{\sqrt{2}} \end{pmatrix} \\ &= \begin{pmatrix} -ig_D \frac{(V_\mu^1 - iV_\mu^2)(\tilde{\phi} + v_D)}{2^{3/2}} \\ \frac{\partial_\mu \tilde{\phi}}{\sqrt{2}} + ig_D \frac{V_\mu^3(\tilde{\phi} + v_D)}{2^{3/2}} \end{pmatrix} = \begin{pmatrix} -ig_D \frac{V_\mu^+(\tilde{\phi} + v_D)}{2} \\ \frac{\partial_\mu \tilde{\phi}}{\sqrt{2}} + ig_D \frac{V_\mu^3(\tilde{\phi} + v_D)}{2^{3/2}} \end{pmatrix} \end{aligned}$$

where we have defined $V_\mu^\pm = \frac{1}{\sqrt{2}}(V_\mu^1 \mp V_\mu^2)$. Taking the Hermitian conjugate of the above expression and multiplying both terms:

$$\begin{aligned} (D_\mu \tilde{\varphi})^\dagger D^\mu \tilde{\varphi} &= \frac{g_D^2 v_D^2}{4} V_\mu^- V^{\mu+} + \frac{g_D^2 v_D^2}{8} V_\mu^0 V^{\mu 0} \\ &\quad + \frac{g_D^2 v_D}{2} V_\mu^- V^{\mu+} \tilde{\phi} + \frac{g_D^2 v_D}{4} V_\mu^0 V^{\mu 0} \tilde{\phi} \\ &\quad + \frac{g_D^2}{4} V_\mu^- V^{\mu+} \tilde{\phi}^2 + \frac{g_D^2}{8} V_\mu^0 V^{\mu 0} \tilde{\phi}^2 \\ &\quad + \frac{1}{2} \partial_\mu \tilde{\phi} \partial^\mu \tilde{\phi} \end{aligned} \tag{3.27}$$

where we set $V_\mu^3 = V_\mu^0$. We observe the presence of three-point and four-point interaction terms, as well as mass terms for the vector bosons, confirming that the $SU(2)_D$ symmetry is completely broken. Additionally, the three bosons are degenerate, possessing the same mass:

$$M_{V^0} = M_{V^\pm} = M_V = \frac{g_D v_D}{2} \tag{3.28}$$

Finally, we rewrite the dark Higgs boson in terms of the physical fields. Now, interactions arise between the dark gauge bosons and both Higgs bosons, dark and Standard Model, establishing a connection to the Standard Model sector:

$$\begin{aligned} (D_\mu \varphi)^\dagger D^\mu \varphi &= \frac{g_D^2 v_D^2}{4} V_\mu^- V^{\mu+} + \frac{g_D^2 v_D^2}{8} V_\mu^0 V^{\mu 0} \\ &\quad + \frac{g_D^2 v_D \cos(\theta_D)}{4} V_\mu^0 V^{\mu 0} \phi + \frac{g_D^2 v_D \cos(\theta_D)}{2} V_\mu^+ V^{\mu-} \phi \\ &\quad + \frac{g_D^2 v_D \sin(\theta_D)}{4} V_\mu^0 V^{\mu 0} h + \frac{g_D^2 v_D \sin(\theta_D)}{2} V_\mu^+ V^{\mu-} h \\ &\quad + \frac{g_D^2 \cos^2(\theta_D)}{8} V_\mu^0 V^{\mu 0} \phi^2 + \frac{g_D^2 \cos^2(\theta_D)}{4} V_\mu^+ V^{\mu-} \phi^2 \\ &\quad + \frac{g_D^2 \sin^2(\theta_D)}{8} V_\mu^0 V^{\mu 0} h^2 + \frac{g_D^2 \sin^2(\theta_D)}{4} V_\mu^+ V^{\mu-} h^2 \\ &\quad + \frac{g_D^2 \cos(\theta_D) \sin(\theta_D)}{4} V_\mu^0 V^{\mu 0} \phi h + \frac{g_D^2 \cos(\theta_D) \sin(\theta_D)}{2} V_\mu^+ V^{\mu-} \phi h + \text{kinetic terms} \end{aligned}$$

3.3.3 Gauge-Fermion interactions

Remember that the covariant derivative acting on the fermion field is given in the form of the adjoint representation:

$$\not{D}\psi = \not{\partial}\psi + ig_D [\not{X}, \psi] = T^a \not{\partial}\psi^a - ig_D \not{X}^b \psi^c [T^b, T^c] = T^a \not{\partial}\psi^a + g_D \epsilon^{bcd} \not{X}^b \psi^c T^d$$

Multiplying by $i\bar{\psi} = i\bar{\psi}^a T^a$ and taking the trace:

$$\begin{aligned} i \operatorname{Tr}\{\bar{\psi} \not{D} \psi\} &= i \operatorname{Tr}\{\bar{\psi}^a T^a (T^b \not{D} \psi^b + g_D \epsilon^{bcd} \mathcal{V}^b \psi^c T^d)\} = i \operatorname{Tr}\{T^a T^b\} \bar{\psi}^a \not{D} \psi^b + i g_D \operatorname{Tr}\{T^a T^d\} \epsilon^{bcd} \bar{\psi}^a \mathcal{V}^b \psi^c \\ &= \frac{i}{2} \bar{\psi}^a \not{D} \psi^a + \frac{i}{2} g_D \epsilon^{abc} \bar{\psi}^a \mathcal{V}^b \psi^c \end{aligned} \quad (3.29)$$

Then,

$$\begin{aligned} i \operatorname{Tr}\{\bar{\psi} \not{D} \psi\} &= \frac{i}{2} \bar{\psi}^1 \not{D} \psi^1 + \frac{i}{2} \bar{\psi}^2 \not{D} \psi^2 + \frac{i}{2} \bar{\psi}^3 \not{D} \psi^3 \\ &\quad + i \frac{g_D}{2} \bar{\psi}^1 (\mathcal{V}^2 \psi^3 - \mathcal{V}^3 \psi^2) + i \frac{g_D}{2} \bar{\psi}^2 (\mathcal{V}^3 \psi^1 - \mathcal{V}^1 \psi^3) + i \frac{g_D}{2} \bar{\psi}^3 (\mathcal{V}^1 \psi^2 - \mathcal{V}^2 \psi^1) \end{aligned} \quad (3.30)$$

Using the fact that $\psi^3 = \psi^0$, $V^3 = V^0$, and:

$$\begin{aligned} \psi^1 &= \frac{1}{\sqrt{2}} (\psi^+ + \psi^-) ; \quad \psi^2 = \frac{i}{\sqrt{2}} (\psi^+ - \psi^-) \\ V^1 &= \frac{1}{\sqrt{2}} (V^+ + V^-) ; \quad V^2 = \frac{i}{\sqrt{2}} (V^+ - V^-) \end{aligned} \quad (3.31)$$

We obtain,

$$\begin{aligned} i \operatorname{Tr}\{\bar{\psi} \not{D} \psi\} &= \frac{i}{2} \bar{\psi}^0 \not{D} \psi^0 + \frac{i}{2} \bar{\psi}^- \not{D} \psi^- + \frac{i}{2} \bar{\psi}^+ \not{D} \psi^+ \\ &\quad + \frac{i g_D}{2} \bar{\psi}^+ \mathcal{V}^+ \psi^0 - \frac{i g_D}{2} \bar{\psi}^0 \mathcal{V}^- \psi^- \\ &\quad + \frac{i g_D}{2} \bar{\psi}^0 \mathcal{V}^- \psi^+ - \frac{i g_D}{2} \bar{\psi}^- \mathcal{V}^+ \psi^0 \\ &\quad + \frac{i g_D}{2} \bar{\psi}^- \mathcal{V}^0 \psi^- - \frac{i g_D}{2} \bar{\psi}^+ \mathcal{V}^0 \psi^+ \end{aligned} \quad (3.32)$$

Meanwhile, for the mass terms, using the same relations between dark-charged particles:

$$M_F \operatorname{Tr}\{\bar{\psi} \psi\} = \frac{M_F}{2} (\bar{\psi}^1 \psi^1 + \bar{\psi}^2 \psi^2 + \bar{\psi}^3 \psi^3) = \frac{M_F}{2} (\bar{\psi}^0 \psi^0 + \bar{\psi}^- \psi^- + \bar{\psi}^+ \psi^+)$$

where we observe that the dark fermions are degenerate.

3.3.4 Dark gauge sector

Let the Lagrangian term be:

$$-\frac{1}{2} \operatorname{Tr}\{V_{\mu\nu} V^{\mu\nu}\} = -\frac{1}{4} V_{\mu\nu}^a V^{\mu\nu a}$$

where $V_{\mu\nu}^a = \partial_\mu V_\nu^a - \partial_\nu V_\mu^a + i g_D \epsilon^{abc} V_\mu^b V_\nu^c$. The product above will give terms of the form:

$$-g_D \epsilon^{abc} V_\mu^a V_\nu^b \partial^\mu V^{\nu c}$$

which results in 3-line vertices with 4-momentum dependence. Lastly, we also have:

$$\frac{g_D^2}{4} e^{fab} e^{fcd} V_\mu^a V_\nu^b V^{\mu c} V^{\nu d}$$

giving rise to 4-line vertices. The complete Feynman rules for each vertex, including the other sectors, can be found in Appendix A.

3.4 Dark matter candidates

The structure of the Lagrangian introduces two types of discrete symmetries, based on the group Z_2 : dark charge (DC) and dark fermion number (DF). With this, there are four particles charged under these quantum numbers, which define three different species of dark matter, as shown in Table 3.1. The conservation of these quantum numbers forbids the decay of these particles, which means that they are stable, a necessary condition for being a dark matter candidate. Depending on the mass regime, any of them could be our DM particle.

Particle	Z_2^{DC}	Z_2^{DF}
$\psi^\pm$	1	1
ψ^0	0	1
$V^\pm$	± 1	0

TABLE 3.1: *Dark matter species and their quantum numbers belonging to corresponding discrete symmetry.*

Passarino-Veltman functions

4.1 A brief introduction

The Passarino-Veltman functions were introduced by G. Passarino and M. Veltman in the late 1970s [42, 43] as a way to encapsulate one-loop integrals with N propagators, which appear in diagrams for radiative corrections, such as the ones shown in Figure 4.1. They defined two types of integrals: scalar and tensorial. The functions T_0^N are integrals with N propagators and no loop momenta in the numerator, meaning they cannot have free Lorentz indices. As an introduction, we present the first four Passarino-Veltman functions for $N = 1, 2, 3, 4$:

$$\begin{aligned}
 A_0(m_0^2) &= \frac{1}{i\pi^2} \int \frac{d^D k}{k^2 - m_0^2} \quad (\text{Tadpole}) \\
 B_0(p_1^2, m_0^2, m_1^2) &= \frac{1}{i\pi^2} \int \frac{d^D k}{(k^2 - m_0^2)((k + p_1)^2 - m_1^2)} \quad (\text{Correction to propagator}) \\
 C_0(p_1^2, (p_1 - p_2)^2, p_2^2, m_0^2, m_1^2, m_2^2) &= \frac{1}{i\pi^2} \int \frac{d^D k}{(k^2 - m_0^2)((k + p_1)^2 - m_1^2)((k + p_2)^2 - m_2^2)} \quad (\text{Figure 4.1(a)}) \quad (4.1) \\
 D_0(p_1^2, (p_1 - p_2)^2, (p_2 - p_3)^2, p_3^2, p_2^2, (p_1 - p_3)^2, m_0^2, m_1^2, m_2^2, m_3^2) &= \frac{1}{i\pi^2} \int \frac{d^D k}{(k^2 - m_0^2)((k + p_1)^2 - m_1^2)((k + p_2)^2 - m_2^2)((k + p_3)^2 - m_3^2)} \quad (\text{Figure 4.1(b)})
 \end{aligned}$$



FIGURE 4.1: *Different one-loop diagrams*

Notice that the integrals are in D dimensions, so they can be evaluated using dimensional regularization and other techniques, such as Feynman parameters and Wick rotation.

The tensor integrals can be decomposed into scalar ones using the Passarino-Veltman reduction algorithm, which we will describe below. To do this, we first need to introduce some notation. This chapter is based on [44].

4.2 Passarino-Veltman Reduction

4.2.1 Some notation

Let's define an N -point tensor integral of rank¹ P as:

$$T^{N;\mu_1\cdots\mu_P}(p_1^2, \dots, p_{N-1}^2, m_0^2, m_1^2, \dots, m_{N-1}^2) = \frac{1}{i\pi^2} \int d^D k \frac{\prod k^{\mu_i}}{N_0 \cdots N_{N-1}} \quad (4.2)$$

where $N_i = (k + p_i)^2 - m_i^2$ is the i -th propagator, and thus, $p_0 = 0$ to get $N_0 = k^2 - m_0^2$. If $P = 0$, we obtain the scalar integrals presented in the introduction.

Whenever we have brackets $\{g \cdots p \cdots\}$ with letters inside, it denotes symmetrization with respect to the involved Lorentz indices. For example:

$$\begin{aligned} \{p \cdots p\}_{i_1 \cdots i_P}^{\mu_1 \cdots \mu_P} &= p_{i_1}^{\mu_1} \cdots p_{i_P}^{\mu_P} \\ \{gp\}_{i_1}^{\mu\nu\rho} &= g^{\mu\nu} p_{i_1}^\rho + g^{\mu\rho} p_{i_1}^\nu + g^{\nu\rho} p_{i_1}^\mu \\ \{gg\}^{\mu\nu\alpha\beta} &= g^{\mu\nu} g^{\alpha\beta} + g^{\mu\alpha} g^{\nu\beta} + g^{\mu\beta} g^{\nu\alpha} \end{aligned} \quad (4.3)$$

The subscript i_r ($r = 1, \dots, N-1$) refers to the r -th time an external momentum $p_i^{\mu_i}$ appears in the decomposition. Additionally, when decomposing an integral, we will cancel the N_i propagator, reducing the function to an integral with $N-1$ propagators. This will be written as:

$$T^{N-1, \mu_1 \cdots \mu_P}(k) \quad (4.4)$$

For example, the 3-point function C_0 multiplied by N_1 :

$$N_1 C_0 = B_0(1) = \frac{1}{i\pi^2} \int d^D k \frac{N_1}{N_0 N_1 N_2} = \frac{1}{i\pi^2} \int d^D k \frac{1}{(k^2 - m_0^2)((k + p_2)^2 - m_2^2)} = B_0(p_2^2, m_0^2, m_1^2)$$

Finally, we define the factor f_i as:

$$f_i = p_i^2 - m_i^2 + m_0^2 \quad (4.5)$$

Note that the term "external" momentum refers to 4-momenta that are not part of the loop but are not necessarily on-shell. In that case, $f_i = m_0^2$.

4.2.2 Tensor integral decomposition (TID)

The N -point tensor integral of rank P can be decomposed using the general formula:

$$\begin{aligned} T^{N;\mu_1\cdots\mu_P} &= \sum_{n=0}^{\lfloor \frac{P}{2} \rfloor} \sum_{i_{2n+1}, \dots, i_P=1}^{N-1} \left\{ \underbrace{g \cdots g p \cdots p}_n \right\}_{i_{2n+1}, \dots, i_P}^{\mu_1 \cdots \mu_P} T_{\underbrace{0 \cdots 0}_{2n}}^N{}_{i_{2n+1} \cdots i_P} \\ &= \sum_{i_1 \cdots i_P}^{N-1} p_{i_1}^{\mu_1} \cdots p_{i_P}^{\mu_P} T_{i_1 \cdots i_P}^N + \sum_{i_3, \dots, i_P=1}^{N-1} \{gp \cdots p\}_{i_3 \cdots i_P}^{\mu_1 \cdots \mu_P} T_{00 i_3 \cdots i_P}^N + \cdots \\ &+ \begin{cases} \sum_{i_P=1}^{N-1} \{g \cdots gp\}_{i_P}^{\mu_1 \cdots \mu_P} T_{\underbrace{0 \cdots 0}_{P-1}}^N{}_{i_P} & ; P \text{ odd} \\ \{g \cdots g\}^{\mu_1 \cdots \mu_P} T_{\underbrace{0 \cdots 0}_{P=2n}}^N & ; P \text{ even} \end{cases} \end{aligned} \quad (4.6)$$

¹Number of indices, i.e., loop momenta in the numerator.

As we can see, there is an i_r subscript for each p_{i_r} , and there is a pair of 0's for each metric $g^{\mu_i \mu_{i+1}}$. The functions $T_{\underbrace{0 \dots 0}_{2n} i_{2n+1} \dots i_P}^N$ are known as Passarino-Veltman coefficient functions. For example, if we set $P = 1, 2, 3$, we get:

$$\begin{aligned} T^{N,\mu} &= \sum_{i_1=1}^{N-1} p_{i_1}^\mu T_{i_1}^N \quad (P=1) \\ T^{N,\mu\nu} &= \sum_{i_1, i_2=1}^{N-1} p_{i_1}^\mu p_{i_2}^\nu T_{i_1 i_2}^N + g^{\mu\nu} T_{00}^N \quad (P=2) \\ T^{N,\mu\nu\rho} &= \sum_{i_1, i_2, i_3=1}^{N-1} p_{i_1}^\mu p_{i_2}^\nu p_{i_3}^\rho T_{i_1 i_2 i_3}^N + \sum_{i_1=1}^{N-1} \{gp\}_{i_1}^{\mu\nu\rho} T_{00 i_1}^N \quad (P=3) \end{aligned} \quad (4.7)$$

4.2.3 The algorithm: Building equation systems

Given a tensor integral $T^{N;\mu_1 \dots \mu_P}$, we can perform the tensor decomposition shown above. However, we need a method to determine the coefficient functions $T_{\underbrace{0 \dots 0}_{2n} i_{2n+1} \dots i_P}^N$. For example, for $P = 1$, we can contract $T^{N;\mu}$ with p_1^μ to eliminate the indices:

$$p_{1\mu} T^{N;\mu} = \sum_{i_1=1}^{N-1} (p_1 \cdot p_{i_1}) T_{i_1}^N \quad (4.8)$$

On the other hand, taking the explicit expression of the N -point function of rank 1 and contracting it with the same 4-momentum:

$$p_{1\mu} T^{N;\mu} = \frac{1}{i\pi^2} \int d^D k \frac{p \cdot k}{N_0 N_1 \dots N_{N-1}} \quad (4.9)$$

Now, expanding the N_1 propagator:

$$N_1 = (k + p_1)^2 - m_1^2 = k^2 + 2k \cdot p_1 + p_1^2 - m_1^2 + m_0^2 - m_0^2$$

where we added zero in the form of $m_0^2 - m_0^2$. Recalling that $N_0 = k^2 - m_0^2$ and defining $f_1 = p_1^2 - m_1^2 + m_0^2$, we obtain:

$$2k \cdot p_1 = N_1 - N_0 - f_1 \quad (4.10)$$

Multiplying by two and substituting into $p_{1\mu} T^{N;\mu}$:

$$2p_{1\mu} T^{N;\mu} = \frac{1}{i\pi^2} \int d^D k \frac{N_1 - N_0 - f_1}{N_0 N_1 \dots N_{N-1}} = T_0^{N-1}(1) - T_0^{N-1}(0) - f_1 T_0^N \quad (4.11)$$

Matching these two expressions, we obtain an equation for $T_{i_1}^N$:

$$\sum_{i_1=1}^{N-1} 2(p_1 \cdot p_{i_1}) T_{i_1}^N = T_0^{N-1}(1) - T_0^{N-1}(0) - f_1 T_0^N \quad (4.12)$$

We need $N - 1$ equations to determine each coefficient function, which means contracting $T^{N;\mu}$ with the $N - 1$ momenta. In general, for any rank, contracting with the k -th 4-momentum gives:

$$2p_k^{\mu_1} T_{\mu_1 \dots \mu_P}^N = T_{\mu_2 \dots \mu_P}^{N-1}(k) - T_{\mu_2 \dots \mu_P}^{N-1}(0) - f_k T_{\mu_2 \dots \mu_P}^N \quad (4.13)$$

Now, consider the tensor decomposition for $P = 2$:

$$T^{N,\mu\nu} = \sum_{i_1, i_2=1}^{N-1} p_{i_1}^\mu p_{i_2}^\nu T_{i_1 i_2}^N + g^{\mu\nu} T_{00}^N \quad (4.14)$$

Now, we have N coefficient functions to determine: the $N - 1$ functions $T_{i_1 i_2}^N$ and T_{00}^N . We obtain the required $N \times N$ system of equations by contracting with each of the $N - 1$ 4-momenta using the relation above and contracting with the metric $g^{\mu\nu}$. For the metric, we have:

$$g_{\mu\nu} T^{N;\mu\nu} = \sum_{i_1, i_2=1}^{N-1} (p_{i_1} \cdot p_{i_2}) T_{i_1 i_2}^N + D T_{00}^N$$

where we used $g^{\mu\nu} g_{\mu\nu} = D$ in D dimensions. On the other hand, contracting the metric with the explicit form of the N -point function of rank 2:

$$g_{\mu\nu} T^{N;\mu\nu} = \frac{1}{i\pi^2} \int d^D k \frac{k^2}{N_0 N_1 \cdots N_{N-1}}$$

Since $N_0 = k^2 - m_0^2$, we have $k^2 = N_0 + m_0^2$, leading to:

$$g_{\mu\nu} T^{N;\mu\nu} = \frac{1}{i\pi^2} \int d^D k \frac{N_0 + m_0^2}{N_0 N_1 \cdots N_{N-1}} = T_0^{N-1}(0) + m_0^2 T_0^N \quad (4.15)$$

which results in:

$$\sum_{i_1, i_2=1}^{N-1} (p_{i_1} \cdot p_{i_2}) T_{i_1 i_2}^N + D T_{00}^N = T_0^{N-1}(0) + m_0^2 T_0^N \quad (4.16)$$

This provides the final equation needed to complete the system. In general, contracting a metric with a P -rank tensor integral gives:

$$g^{\mu_1 \mu_2} T_{\mu_1 \cdots \mu_P}^N = T_{\mu_3 \cdots \mu_P}^{N-1}(0) + m_0^2 T_{\mu_3 \cdots \mu_P}^N \quad (4.17)$$

With these two relations, we can construct the system of equations step by step, starting with $N = 1$, $P = 1$, then $P = 2$, and so on, progressing to $N = 2$ with $P = 1$, and so forth. If we have A_1 , we can obtain A_{00} ; if we have B_1 , we can determine B_{11} , B_{00} , and so on. There exist recursion relations for different N -point functions of ranks up to $N = 5, 6$, which can be found in [44, 45]. However, they will not be necessary for the calculations in this thesis, as we mainly deal with almost C^ν and D^ν functions (3- and 4-point functions of rank 1).

4.3 Some examples of the reduction algorithm

In the following, we present examples of the determination of the coefficient functions for 1- and 2-point functions, up to rank 2.

4.3.1 1-point functions

Starting with $P = 1$, the decomposition leads to:

$$A^\mu = \sum_{i_1=1}^{1-1} p_{i_1}^\mu A_1 = 0$$

There is nothing to sum over. Recall that every tensor decomposition of upper rank involves summations up to $N - 1$. However, since $N = 1$, all these sums vanish. Only the even-rank functions, $P = 2n$, survive for $N = 1$ because they contain an extra term that is not summed and is proportional to the symmetrization of the n metrics. Consequently, only the functions A_{00} , A_{0000} , and so on, exists. In general:

$$A^{\mu_1 \cdots \mu_{2n}} = \underbrace{\{g \cdots g\}}_n^{\mu_1 \cdots \mu_{2n}} \underbrace{A_0 \cdots 0}_{2n} \quad (4.18)$$

In the particular case of $P = 2$ ($n = 1$), we obtain the decomposition:

$$A^{\mu\nu} = g^{\mu\nu} A_{00}$$

Since there are no momenta, we only need to contract with the metric:

$$g_{\mu\nu} A^{\mu\nu} = D A_{00} \quad (4.19)$$

Applying the contraction relation with the metric:

$$g_{\mu\nu} A^{\mu\nu} = g_{\mu\nu} T^{1;\mu\nu} = T_0^0(0) + m_0^2 T_0^1 = m_0^2 A_0 \quad (4.20)$$

where any function $T^{N=0}$ is zero by definition. Thus, we obtain:

$$A_{00} = \frac{m_0^2}{D} A_0 \quad (4.21)$$

Now, for the rank-4 function:

$$\begin{aligned} g_{\mu\nu} A^{\mu\nu\rho\sigma} &= g_{\mu\nu} \{gg\}^{\mu\nu\rho\sigma} A_{0000} \\ g_{\mu\nu} A^{\mu\nu\rho\sigma} &= m_0^2 A^{\rho\sigma} = m_0^2 g^{\rho\sigma} A_{00} \\ \longrightarrow g_{\mu\nu} \{gg\}^{\mu\nu\rho\sigma} A_{0000} &= m_0^2 g^{\rho\sigma} A_{00} \end{aligned} \quad (4.22)$$

Notice that:

$$g_{\mu\nu} \{gg\}^{\mu\nu\rho\sigma} = g_{\mu\nu} (g^{\mu\nu} g^{\rho\sigma} + g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho}) = (D+2) g^{\rho\sigma}$$

Thus, we obtain:

$$A_{0000} = \frac{m_0^2}{D+2} A_{00} = \frac{(m_0^2)^2}{D(D+2)} A_0 \quad (4.23)$$

Each iteration with a higher rank introduces an additional factor of m_0^2 . On the other hand, considering the rank-6 case and focusing only on the symmetrization term while contracting it with one metric:

$$\begin{aligned} g_{\mu\nu} \{ggg\}^{\mu\nu\rho\sigma\lambda\epsilon} &= g_{\mu\nu} (g^{\mu\nu} \{gg\}^{\rho\sigma\lambda\epsilon} + g^{\mu\rho} \{gg\}^{\nu\sigma\lambda\epsilon} + g^{\mu\sigma} \{gg\}^{\nu\rho\lambda\epsilon} + g^{\mu\lambda} \{gg\}^{\nu\rho\sigma\epsilon} + g^{\mu\epsilon} \{gg\}^{\nu\rho\sigma\lambda}) \\ &= (D+4) \{gg\}^{\rho\sigma\lambda\epsilon} \end{aligned} \quad (4.24)$$

We observe that, compared to the previous rank, the prefactor D increases by two, while the symmetrization loses one metric. Therefore, for a rank- $2n$ function, we can recursively derive:

$$g_{\mu_1\mu_2} \underbrace{\{g \cdots g\}}_n^{\mu_1 \cdots \mu_{2n}} = (D+2(n-1)) \underbrace{\{g \cdots g\}}_{n-1}^{\mu_3 \cdots \mu_{2n}} \quad (4.25)$$

Thus, for the rank- $2n$ function, we obtain:

$$\begin{aligned} g_{\mu_1\mu_2} A^{\mu_1 \cdots \mu_{2n}} &= g_{\mu_1\mu_2} \underbrace{\{g \cdots g\}}_n^{\mu_1 \cdots \mu_{2n}} \underbrace{A_0 \cdots 0}_{2n} = (D+2(n-1)) \underbrace{\{g \cdots g\}}_{n-1}^{\mu_3 \cdots \mu_{2n}} \underbrace{A_0 \cdots 0}_{2n} \\ g_{\mu_1\mu_2} A^{\mu_1 \cdots \mu_{2n}} &= m_0^2 A^{\mu_3 \cdots \mu_{2n}} = m_0^2 \underbrace{\{g \cdots g\}}_{n-1}^{\mu_3 \cdots \mu_{2n}} \underbrace{A_0 \cdots 0}_{2(n-1)} \end{aligned}$$

$$\underbrace{A_0 \cdots 0}_{2n} = \frac{m_0^2}{(D+2(n-1))} \underbrace{A_0 \cdots 0}_{2(n-1)} \quad (4.26)$$

By recursively substituting all the lower-rank coefficient functions until reaching $n = 0$, we obtain a general formula for the coefficient function of any even rank in terms of A_0 :

$$A_{\underbrace{0 \dots 0}_{2n}} = \left(\prod_{i=1}^n \frac{1}{D + 2(i-1)} \right) m_0^{2n} A_0 \quad (4.27)$$

Furthermore, using dimensional regularization and setting $D = 4 - \epsilon$, we expand the term in parentheses up to zeroth order in ϵ :

$$\prod_{i=1}^n \frac{1}{D + 2(i-1)} = \prod_{i=1}^n \frac{1}{2(i+1) - \epsilon} = \prod_{i=1}^n \frac{1}{2(i+1)} = \frac{2^{-n}}{(n+1)!}$$

Thus, we obtain:

$$A_{\underbrace{0 \dots 0}_{2n}} = \frac{m_0^{2n}}{2^n(n+1)!} A_0 \quad (4.28)$$

4.3.2 2-point functions

For the 2-point function, we use the notation $T^N = B$. Thus, for rank 1, the tensor decomposition is given by:

$$B^\mu = \sum_{i_1=1}^{2-1} p_{i_1}^\mu B_{i_1} = p_1^\mu B_1. \quad (4.29)$$

In this case, we simply contract with $p_{1\mu}$, obtaining:

$$\begin{aligned} 2p_{1\mu} B^\mu &= 2p_1^2 B_1, \\ 2p_{1\mu} B^\mu &= A_0(1) - A_0(0) - f_1 B_0, \end{aligned} \quad (4.30)$$

where $A_0(1) = A_0(m_0^2)$ and $A_0(0) = A_0(m_1^2)$. Then, we obtain:

$$B_1(p_1^2, m_0^2, m_1^2) = \frac{1}{2p_1^2} [A_0(m_0^2) - A_0(m_1^2) - (p_1^2 - m_1^2 + m_0^2) B_0(p_1^2, m_0^2, m_1^2)]. \quad (4.31)$$

For rank 2, we have:

$$B^{\mu\nu} = \sum_{i_1, i_2=1}^{2-1} p_{i_1}^\mu p_{i_2}^\nu B_{i_1 i_2} + \{g\}^{\mu\nu} B_{00} = p_1^\mu p_1^\nu B_{11} + g^{\mu\nu} B_{00}. \quad (4.32)$$

Now, contracting with p_1^μ and the metric tensor, we obtain equations for the coefficient functions B_{11} and B_{00} :

$$\begin{aligned} 2p_{1\mu} B^{\mu\nu} &= 2p_1^2 p_1^\nu B_{11} + p_1^\nu B_{00}, \\ 2p_{1\mu} B^{\mu\nu} &= A^\nu(1) - A^\nu(0) - f_1 B^\nu = -f_1 p_1^\nu B_1, \end{aligned}$$

where we set the A^ν functions to zero based on the results obtained in the previous section. Next, contracting with the metric tensor:

$$\begin{aligned} g_{\mu\nu} B^{\mu\nu} &= p_1^2 B_{11} + D B_{00}, \\ g_{\mu\nu} B^{\mu\nu} &= A_0(0) + m_0^2 B_0. \end{aligned}$$

Thus, the resulting 2×2 system of equations is:

$$\left. \begin{aligned} 2(p_1^2 B_{11} + B_{00}) &= -f_1 B_1 \\ p_1^2 B_{11} + D B_{00} &= A_0(0) + m_0^2 B_0 \end{aligned} \right\}. \quad (4.33)$$

Solving for B_{11} and B_{00} :

$$\longrightarrow \begin{cases} B_{11}(p_1^2, m_0^2, m_1^2) = \frac{1}{2(D-1)p_1^2} [2m_0^2 B_0(p_1^2, m_0^2, m_1^2) + A_0(m_1^2) - (p_1^2 - m_1^2 + m_0^2) B_1(p_1^2, m_0^2, m_1^2)] \\ B_{00}(p_1^2, m_0^2, m_1^2) = \frac{1}{2(D-1)} [2A_0(m_1^2) + 2m_0^2 B_0(p_1^2, m_0^2, m_1^2) + (p_1^2 - m_1^2 + m_0^2) B_1(p_1^2, m_0^2, m_1^2)] \end{cases} \quad (4.34)$$

By substituting B_1 , we can express both functions purely in terms of scalar functions. However, for higher ranks and higher N -point functions, the resulting expressions become increasingly complex.

Amplitudes involved in the calculations

In this chapter, we will develop almost all the amplitudes needed to compute the relevant quantities for testing our model. Regarding relic density, not all annihilation and co-annihilation processes involving only the dark sector are possible at tree level. Due to the absence of a Yukawa term, the first diagrams for dark fermion annihilation into dark scalars appear at next-to-leading order.

As a first step, we present three tree-level examples of processes involving only dark fermions in the initial and final states, processes involving only bosons, and mixed processes. All of these contribute to relic density calculations. In the second part, we move into the one-loop regime, where we decompose all these diagrams into Passarino-Veltman functions. These diagrams include processes required for computing direct detection signals, which can only occur at one-loop order, as well as dark fermion annihilations into Standard Model particles.

All diagrams for these processes were visualized, and their amplitudes extracted, using *FeynArts* [46]. Additionally, we used *FeynCalc* [47] to perform some simplifications of gamma matrices, traces, and spinor properties, such as Dirac's equation and summation over polarizations. These last two are packages designed as an extension to *Wolfram Mathematica* [48].

5.1 Tree-Level Amplitudes

5.1.1 Co-annihilation of dark fermions

We are going to consider the process $\psi^+ \bar{\psi}^0 \rightarrow \psi^+ \bar{\psi}^0$, whose only diagram involved is:

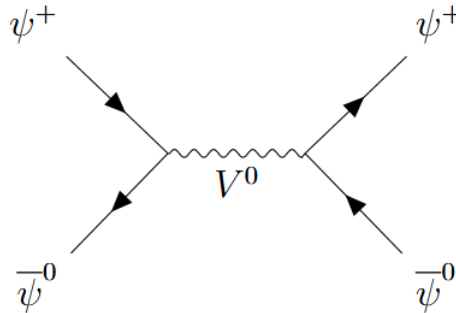


FIGURE 5.1: *Diagrams that contribute to the process*

Which gives the amplitude:

$$\begin{aligned} i\mathcal{M} &= \bar{v}(p_2)(-ig_D\gamma^\mu)u(p_1)\left(\frac{-ig_{\mu\nu}}{q^2 - M_V^2}\right)\bar{u}(p_3)(-ig_D\gamma^\nu)v(p_4) \\ &= i\frac{g_D^2}{s - M_V^2}\bar{v}(p_2)\gamma^\mu u(p_1)\bar{u}(p_3)\gamma^\nu v(p_4)g_{\mu\nu} \end{aligned} \quad (5.1)$$

Taking the complex conjugate:

$$(i\mathcal{M})^* = -i\frac{g_D^2}{s - M_V^2}\bar{v}(p_4)\gamma^\alpha u(p_3)\bar{u}(p_1)\gamma^\beta v(p_2)g_{\alpha\beta}$$

and the squared amplitude:

$$|\mathcal{M}|^2 = \frac{g_D^4}{(s - M_V^2)^2}\bar{v}(p_2)\gamma^\mu u(p_1)\bar{u}(p_3)\gamma^\nu v(p_4)\bar{v}(p_4)\gamma^\alpha u(p_3)\bar{u}(p_1)\gamma^\beta v(p_2)g_{\mu\nu}g_{\alpha\beta} \quad (5.2)$$

We sum over all possible final spin states and average over all initial ones. Additionally, we make the spin indices explicit to regroup them:

$$\begin{aligned} |\overline{\mathcal{M}}|^2 &= \frac{1}{4} \sum_{spins} |\mathcal{M}|^2 = \sum_{spins} \frac{g_D^4}{4(s - M_V^2)^2} \bar{v}_a(p_2)\gamma_{ab}^\mu u_b(p_1)\bar{u}_c(p_3)\gamma_{cd}^\nu v_d(p_4)\bar{v}_e(p_4)\gamma_{ef}^\alpha u_f(p_3)\bar{u}_g(p_1)\gamma_{gh}^\beta v_h(p_2)g_{\mu\nu}g_{\alpha\beta} \\ &= \sum_{spins} \frac{g_D^4}{4(s - M_V^2)^2} \gamma_{ab}^\mu u_b(p_1)\bar{u}_g(p_1)\gamma_{gh}^\beta v_h(p_2)\bar{v}_a(p_2)\gamma_{cd}^\nu v_d(p_4)\bar{v}_e(p_4)\gamma_{ef}^\alpha u_f(p_3)\bar{u}_c(p_3)g_{\mu\nu}g_{\alpha\beta} \\ &= \frac{g_D^4}{4(s - M_V^2)^2} \gamma_{ab}^\mu (\not{p}_1 + M_F)_{bg} \gamma_{gh}^\beta (\not{p}_2 - M_F)_{ha} \gamma_{cd}^\nu (\not{p}_4 - M_F)_{de} \gamma_{ef}^\alpha (\not{p}_3 + M_F)_{fc} g_{\mu\nu}g_{\alpha\beta} \\ &= \frac{g_D^4}{4(s - M_V^2)^2} \text{Tr}\{\gamma^\mu(\not{p}_1 + M_F)\gamma^\beta(\not{p}_2 - M_F)\} \text{Tr}\{\gamma^\nu(\not{p}_4 - M_F)\gamma^\alpha(\not{p}_3 + M_F)\} g_{\mu\nu}g_{\alpha\beta} \end{aligned}$$

where we have used the property of spinors, summed over all spins:

$$\sum_{\sigma} u(p, \sigma)\bar{u}(p, \sigma) = \not{p} + m ; \quad \sum_{\sigma} v(p, \sigma)\bar{v}(p, \sigma) = \not{p} - m \quad (5.3)$$

and the definition of the trace. Next, we calculate one of the traces:

$$\begin{aligned} \text{Tr}\{\gamma^\nu(\not{p}_4 - M_F)\gamma^\alpha(\not{p}_3 + M_F)\} &= \text{Tr}\{\gamma^\nu\gamma^\rho\gamma^\alpha\gamma^\sigma\}p_{\rho 4}p_{\sigma 3} - M_F^2 \text{Tr}\{\gamma^\nu\gamma^\alpha\} \\ &= 4(p_4^\nu p_3^\alpha - g^{\nu\alpha}(p_3 \cdot p_4) + p_3^\nu p_4^\alpha - M_F^2 g^{\nu\alpha}) \end{aligned}$$

where we have applied the properties of the trace of the gamma matrices:

$$\text{Tr}\{\gamma^\nu\gamma^\alpha\} = 4g^{\nu\alpha} ; \quad \text{Tr}\{\gamma^\nu\gamma^\alpha\gamma^\beta\} = 0 ; \quad \text{Tr}\{\gamma^\nu\gamma^\rho\gamma^\alpha\gamma^\sigma\} = 4(g^{\nu\rho}g^{\alpha\sigma} - g^{\nu\alpha}g^{\rho\sigma} + g^{\nu\sigma}g^{\rho\alpha}) \quad (5.4)$$

We proceed similarly with the other trace, yielding:

$$\text{Tr}\{\gamma^\mu(\not{p}_1 + M_F)\gamma^\beta(\not{p}_2 - M_F)\} = 4(p_1^\mu p_2^\beta - g^{\mu\beta}(p_1 \cdot p_2) + p_1^\mu p_2^\beta - M_F^2 g^{\mu\beta})$$

Then, the amplitude takes the form:

$$|\overline{\mathcal{M}}|^2 = \frac{g_D^4}{4(s - M_V^2)} (p_1^\mu p_2^\beta - g^{\mu\beta}(p_1 \cdot p_2) + p_1^\mu p_2^\beta - M_F^2 g^{\mu\beta}) (p_4^\nu p_3^\alpha - g^{\nu\alpha}(p_3 \cdot p_4) + p_3^\nu p_4^\alpha - M_F^2 g^{\nu\alpha}) g_{\mu\nu}g_{\alpha\beta}$$

Contracting this, we obtain:

$$|\overline{\mathcal{M}}|^2 = \frac{8g_D^4}{(s - M_V^2)^2} [(p_1 \cdot p_3)(p_2 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3) + M_F^2[(p_1 \cdot p_2) + (p_3 \cdot p_4)] + 2M_F^2]$$

Using the Mandelstam variables $s = (p_1 + p_2)^2 = 2(M_F^2 + (p_1 \cdot p_2))$, $t = (p_1 - p_3)^2 = 2(M_F^2 - (p_1 \cdot p_3))$, and $u = (p_2 - p_3)^2 = 2(M_F^2 - (p_2 \cdot p_3))$, we can simplify it to:

$$|\overline{\mathcal{M}}|^2 = \frac{2g_D^4}{(s - M_V^2)^2} (24M_F^4 + 8M_F^2(t + u) + t^2 + u^2) \quad (5.5)$$

5.1.2 Dark fermions into dark bosons

For this annihilation process, we consider $\psi^-\bar{\psi}^- \rightarrow V^+V^-$. There are two diagrams involved:

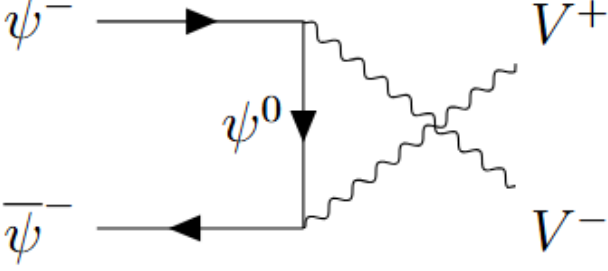


FIGURE 5.2: *s*-channel diagram

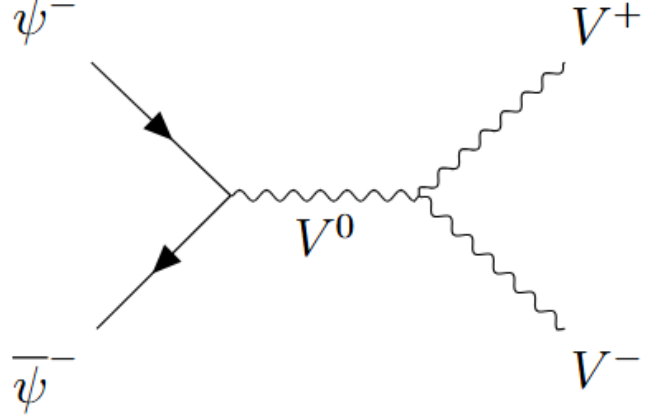


FIGURE 5.3: *u*-channel diagram

We start with the diagram of Figure 5.2, using $k^\mu = (p_3 + p_4)^\mu$:

$$\begin{aligned} i\mathcal{M}_a &= \bar{v}(p_2)(-ig_D\gamma^\mu)u(p_1)\left(-i\frac{g_{\mu\nu}}{s-M_V^2}\right)ig_D[g^{\rho\sigma}(p_4-p_3)^\nu + g^{\rho\nu}(k+p_3)^\sigma - g^{\sigma\nu}(p_4+k)^\rho]\epsilon_\rho^*(p_3)\epsilon_\sigma^*(p_4) \\ &= -i\frac{g_D^2}{s-M_V^2}\bar{v}(p_2)\gamma^\mu u(p_1)\left[g^{\rho\sigma}(p_4-p_3)_\mu + \delta_\mu^\rho(p_4+2p_3)^\sigma - \delta_\mu^\sigma(2p_4+p_3)^\rho\right]\epsilon_\rho^*(p_3)\epsilon_\sigma^*(p_4) \end{aligned} \quad (5.6)$$

We take the complex conjugate and multiply it by the original to obtain the squared amplitude. Then, we sum over all polarizations of the final states and average over all initial spin states:

$$\begin{aligned} |\overline{\mathcal{M}}_a|^2 &= \frac{g_D^4}{4(s-M_V^2)^2} \text{Tr}\{\gamma^\mu(\not{p}_1 + M_F)\gamma^\alpha(\not{p}_2 - M_F)\}\left(-g_{\rho\beta} + \frac{p_{3\rho}p_{3\beta}}{M_V^2}\right)\left(-g_{\sigma\lambda} + \frac{p_{4\sigma}p_{4\lambda}}{M_V^2}\right) \\ &\quad \times \left[g^{\rho\sigma}(p_4-p_3)_\mu + \delta_\mu^\rho(p_4+2p_3)^\sigma - \delta_\mu^\sigma(2p_4+p_3)^\rho\right]\left[g^{\beta\lambda}(p_4-p_3)_\alpha + \delta_\alpha^\beta(p_4+2p_3)^\lambda - \delta_\alpha^\lambda(2p_4+p_3)^\rho\right] \end{aligned}$$

where we have used the property of the polarization vectors of massive bosons:

$$\sum_\sigma \epsilon_\mu(p, \sigma)\epsilon_\nu^*(p, \sigma) = -g_{\mu\nu} + \frac{p_\mu p_\nu}{M_V^2} \quad (5.7)$$

The trace gives:

$$\text{Tr}\{\gamma^\mu(\not{p}_1 + M_F)\gamma^\alpha(\not{p}_2 - M_F)\} = 4(p_1^\mu p_2^\alpha - g^{\mu\alpha}(p_1 \cdot p_2) + p_1^\alpha p_2^\mu - M_F^2 g^{\mu\alpha})$$

Contracting all indices and simplifying using the Mandelstam variables, we obtain:

$$\begin{aligned} |\overline{\mathcal{M}}_a|^2 &= -\frac{g_D^4}{2M_V^4(s-M_V^2)^2} \left[(12M_V^4 - 4M_V^2s + s^2)M_F^4 + (24M_V^6 + 12M_V^4(s-2u) - 2M_V^2s(s-4u) + \dots \right. \\ &\quad \left. \dots - s^2(s+2u))M_F^2 + 12M_V^8 - 4(s+6u)M_V^6 + (17s^2 + 20su + 12u^2)M_V^4 + \dots \right. \\ &\quad \left. \dots - 2s(2s^2 + 3su + 2u^2)M_V^2 + s^2u(s+u) \right] \end{aligned}$$

On the other hand, the diagram in Figure 5.3 gives the amplitude:

$$\begin{aligned} i\mathcal{M}_b &= \bar{v}(p_2)(ig_D\gamma^\mu)\left(-i\frac{\not{K} + M_F}{u - M_F^2}\right)(ig_D\gamma^\nu)u(p_1)\epsilon_\nu^*(p_3)\epsilon_\mu^*(p_4) \\ &= i\frac{g_D^2}{u - M_F^2}\bar{v}(p_2)\gamma^\mu(\not{K} + M_F)\gamma^\nu u(p_1)\epsilon_\nu^*(p_3)\epsilon_\mu^*(p_4) \end{aligned} \quad (5.8)$$

Following an analogous process to the one described above, we obtain the squared amplitude:

$$\begin{aligned} |\overline{\mathcal{M}}_b|^2 &= \frac{g_D^4}{4(u - M_F^2)^2} \text{Tr}\left\{\gamma^\mu(\not{K} + M_F)\gamma^\nu(\not{p}_1 + M_F)\gamma^\alpha(\not{K} + M_F)\gamma^\beta(\not{p}_2 - M_F)\right\} \\ &\quad \left(-g_{\mu\alpha} + \frac{p_{4\mu}p_{4\alpha}}{M_V^2}\right)\left(-g_{\nu\beta} + \frac{p_{3\nu}p_{3\beta}}{M_V^2}\right) \end{aligned}$$

The trace can be decomposed into different traces of an even number of gamma matrices, up to eight in a single trace. Contracting and simplifying as before:

$$|\overline{\mathcal{M}}_b|^2 = \frac{g_D^4}{2} \left(7 + \frac{12M_F^2 - 4t}{M_F^2 - u} - \frac{4(2M_F^2 + M_V^2)^2}{(M_F^2 - u)^2} + \frac{u(t - M_F^2 - 4M_V^2)}{M_V^4} + \frac{M_F^4 + (16M_V^2 - t)M_F^2 - 4M_V^2t}{M_V^4} \right)$$

However, we must sum over all possible amplitudes. Thus, the total amplitude of the process is:

$$|\overline{\mathcal{M}}|^2 = \frac{1}{4} \sum_{spins, \sigma} (|\mathcal{M}_a|^2 + |\mathcal{M}_b|^2 + \mathcal{M}_a\mathcal{M}_b^* + \mathcal{M}_a^*\mathcal{M}_b)$$

After extensive algebraic manipulation, using the relation $s + t + u = 2M_F^2 + 2M_V^2$ for this process¹, we obtain:

$$\begin{aligned} |\overline{\mathcal{M}}|^2 &= \frac{g_D^4}{2(u - M_F^2)^2(s - M_V^2)^2} \left[-23M_F^8 - (8M_V^2 + t + 3u)M_F^6 \right. \\ &\quad + (27M_V^4 - 4(7t + 17u)M_V^2 + 12t^2 + 59tu + 15u^2)M_F^4 \\ &\quad + (8M_V^6 - 2(6t + 49u)M_V^4) + 8(t^2 + 14tu + 4u^2)M_V^2 - 4t^3 - 28t^2u - 31tu^2 - 5u^3)M_F^2 \\ &\quad \left. - 4M_V^8 + 8(t - 4u)M_V^6 + (12u^2 - 52tu - 4t^2)M_V^4 - 4u(2t + u)(3t + u)M_V^2 + 4t^3u + 5tu^3 \right] \end{aligned} \quad (5.9)$$

5.1.3 Co-annihilation to produce a scalar

We will consider the process $\psi^+V^- \rightarrow \psi^0h$, which produces one Higgs boson and has a net lepton number equal to 1. The only diagram is the one shown in Figure 5.4:

$$i\mathcal{M} = \bar{u}(p_3)\left(\frac{g_D}{2}\gamma^\mu\right)u(p_1)\left(\frac{-ig_{\mu\nu}}{t - M_V^2}\right)(-\sin(\theta_D)M_Vg_Dg^{\nu\alpha})\epsilon_\alpha^*(p_2) = i\frac{M_Vg_D^2\sin(\theta_D)}{2(t - M_V^2)}\bar{u}(p_3)\gamma^\mu u(p_1)\epsilon_\mu^*(p_2) \quad (5.10)$$

The amplitude squared, summed and averaged over all the polarizations is:

$$|\overline{\mathcal{M}}|^2 = \frac{M_V^2g_D^4}{16(t - M_V^2)}\left(-g_{\mu\nu} + \frac{p_{2\mu}p_{2\nu}}{M_V^2}\right)\text{Tr}\{(\not{p}_1 + M_F)\gamma^\mu(\not{p}_3 + M_F)\gamma^\nu\} \quad (5.11)$$

¹The general expression is

$$s + t + u = \sum_i m_i^2$$

Applying trace properties:

$$|\overline{\mathcal{M}}|^2 = \frac{M_V^2 g_D^4}{16(t - M_V^2)} \left(-g_{\mu\nu} + \frac{p_{2\mu} p_{2\nu}}{M_V^2} \right) (2t g^{\mu\nu} + 4p_1^\mu p_3^\nu + 4p_3^\mu p_1^\nu) \quad (5.12)$$

And contracting and replacing all scalar products by Mandelstam variables, and on-shell invariant mass relations:

$$|\overline{\mathcal{M}}|^2 = \frac{g_D^4}{8(t - M_V^2)} [M_F^4 + M_F^2(6M_V^2 - s - u) + M_V^4 - M_V^2(s - t + u) + su] \quad (5.13)$$

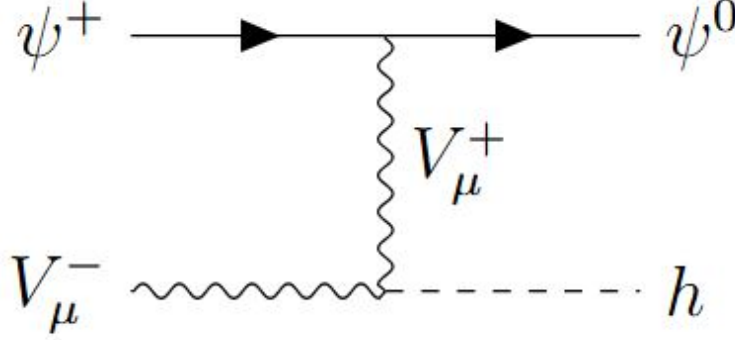


FIGURE 5.4: *Diagram involved*

5.2 Next-to-leading-order diagrams

For direct detection and the relic density, we need to compute diagrams involving loops with triangles and boxes. Due to the fermion propagators in the diagrams, Passarino-Veltman functions of rank up to three will appear, with three or even four propagators, depending on the number of particles in the loop. These functions are very difficult to compute explicitly, which can make the calculation of amplitudes and cross sections a complicated task.

In light of this, we will introduce an effective Lagrangian to the theory, composed of operators of higher dimensions, which will bring effective vertices with them. The task here is, first, to compute these effective vertices and define form factors that will be the coefficients multiplying certain momenta, inner products of gamma matrices and momenta, inner products of two momenta, etc., which will contribute to a particular process. Second, we will compute the loop diagrams for the selected process and define the form factors as before. Finally, we will match the form factors of each Lagrangian. The majority of the diagrams computed below involve only dark fermions and scalars.

As we will see, the introduction of this operator will bring corrections to the effective vertices and even propagators, corresponding to next-to-leading-order corrections. However, we will find that we can only implement the triangle loop in a low-energy limit, where some functions have an analytical form, exclusively for direct detection. The rest of the diagrams will have to be evaluated numerically.

5.2.1 Effective lagrangian

To build an effective Lagrangian, we need to include every possible operator that can produce vertices with the on-shell particles in Figures 5.6 and 5.7. These operators must be gauge invariant under $SU(2)_D$, Lorentz

invariant, and Hermitian. The effective Lagrangian is as follows:

$$\begin{aligned}\mathcal{L}_{\text{eff}} = & \frac{k_1}{M_V} \text{Tr}\{\bar{\psi}\psi\}\tilde{\varphi}^\dagger\tilde{\varphi} \\ & + \frac{k_2}{M_V^2} \text{Tr}\{\bar{\psi}\gamma^\mu\psi\}\tilde{\varphi}^\dagger D_\mu\tilde{\varphi} + \frac{k_3}{M_V^2} \text{Tr}\{\bar{\psi}\not{D}\psi\}\tilde{\varphi}^\dagger\tilde{\varphi} \\ & + \frac{k_4}{M_V^3} \text{Tr}\{\bar{\psi}\psi\}(D_\mu\tilde{\varphi})^\dagger D^\mu\tilde{\varphi} + \frac{k_5}{M_V^3} \text{Tr}\{\bar{\psi}D_\mu\psi\}\tilde{\varphi}^\dagger D^\mu\tilde{\varphi}\end{aligned}\quad (5.14)$$

where we keep the k_i constants dimensionless and divide by the appropriate power of masses to ensure that each term has four dimensions.

5.2.2 Triangular vertex

Effective part

Extracting only the terms of the effective Lagrangian with 2 fermions and 1 scalar, we obtain:

$$\mathcal{L}_3 = \frac{v_D k_1}{2M_V} \bar{\psi}^a \psi^a \tilde{\phi} + \frac{v_D k_2}{2M_V^2} \bar{\psi}^a \not{\partial} \tilde{\phi} \psi^a + \frac{v_D k_3}{2M_V^2} \bar{\psi}^a \not{\partial} \psi^a \tilde{\phi} + \frac{v_D k_5}{2M_V^3} \bar{\psi}^a \partial_\mu \psi^a \partial^\mu \tilde{\phi} \quad (5.15)$$

We will consider that all the particles are going into the vertex of Figure 5.5 to derive its Feynman rules. Replacing all fields with incident plane waves and using:

$$V^{(3)} = \frac{\partial^3(i\mathcal{L}_3)}{\partial \bar{\psi}^a(p_2) \partial \psi^a(p_1) \partial \tilde{\phi}}$$

we obtain:

$$V^{(3)} = i \frac{v_D k_1}{2M_V} + \frac{v_D k_2}{2M_V^2} \not{p}_1 + \frac{v_D k_3}{2M_V^2} \not{p}_2 - i \frac{v_D k_5}{2M_V^3} p_1 \cdot q$$

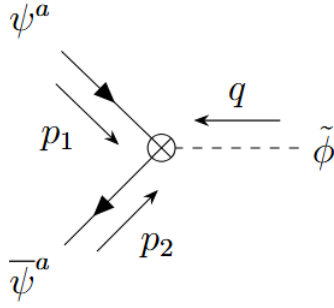
Using that $q^\mu = (p_1 + p_2)^\mu$, we can obtain a vertex that depends on $\not{p}_1$, $\not{p}_2$, and $p_1 \cdot q$:

$$\begin{aligned}V^{(3)} = & i \frac{v_D k_1}{2M_V} + \frac{v_D}{2M_V^2} (k_2 + k_3) \not{p}_1 + \frac{v_D k_2}{2M_V^2} \not{p}_2 - i \frac{v_D k_5}{2M_V^3} p_1 \cdot q \\ = & iF_1 + F_2 \not{p}_1 + F_3 \not{p}_2 + iF_4 p_1 \cdot q\end{aligned}\quad (5.16)$$

where we obtain the form factors:

$$\begin{aligned}F_1 = \frac{v_D k_1}{2M_V} \quad ; \quad F_2 = \frac{v_D}{2M_V^2} (k_2 + k_3) \\ F_3 = \frac{v_D k_2}{2M_V^2} \quad ; \quad F_4 = -\frac{v_D k_5}{2M_V^3}\end{aligned}\quad (5.17)$$

The form factors have dimensions ranging from zero down to -2 .

FIGURE 5.5: *Mono-Higgs effective vertex*

This particular vertex will be needed to some direct detection processes where, if we rotate the vertex in 90 degrees and set the Higgs boson as a virtual particle that could be connected to two electrons or other particles, so we are setting the dark Higgs boson to be off-shell, while the fermions remain on-shell².

Loop part

Now we compute the vertex with the loop, using the gauge state of the dark Higgs boson³:

$$i\mathcal{M} = \int \frac{d^4k}{(2\pi)^4} \bar{v}(p_2) \left(\frac{g_D}{2} \gamma^\mu \right) \frac{i(\not{k} + M_F)}{k^2 - M_F^2} \frac{-ig_{\mu\alpha}}{(k - p_1)^2 - M_V^2} M_V g_D g^{\alpha\beta} \frac{-ig_{\beta\nu}}{(k + p_2)^2 - M_V^2} \left(\frac{g_D}{2} \gamma^\nu \right) u(p_1)$$

where the momenta of the propagators are set as shown in Figure 5.6. Contracting the indices, we get:

$$i\mathcal{M} = -i \frac{M_V g_D^3}{4(2\pi)^4} \int d^4k \frac{\bar{v}(p_2) \gamma_\mu (\not{k} + M_F) \gamma^\mu u(p_1)}{(k^2 - M_F^2)((k - p_1)^2 - M_V^2)((k + p_2)^2 - M_V^2)} \quad (5.18)$$

Using the identities:

$$\gamma_\mu \not{k} \gamma^\mu = -2\not{k} ; \quad \gamma_\mu \gamma^\mu = 4$$

we obtain:

$$i\mathcal{M} = i \frac{M_V g_D^3}{4(2\pi)^4} \left[2\bar{v}(p_2) \gamma_\nu u(p_1) \int d^4k \frac{k^\nu}{(k^2 - M_F^2)((k - p_1)^2 - M_V^2)((k + p_2)^2 - M_V^2)} - 4M_F \bar{v}(p_2) u(p_1) \int d^4k \frac{1}{(k^2 - M_F^2)((k - p_1)^2 - M_V^2)((k + p_2)^2 - M_V^2)} \right]$$

where we recognize the form of the Passarino-Veltman 3-functions of rank 1 and zero:

$$i\mathcal{M} = i \frac{M_V g_D^3}{4(2\pi)^4} i\pi^2 [2\bar{v}(p_2) \gamma_\nu u(p_1) C^\nu - 4M_F \bar{v}(p_2) u(p_1) C_0] \quad (5.19)$$

where

$$C^{\nu,0} = C^{\nu,0}(p_1^2, q^2, p_2^2, M_F^2, M_V^2, M_V^2) \quad (5.20)$$

²Flipping the vertex also means that we have to change the signs of the 4-momenta, which will change the sign of some form factors.

³Despite setting the dark Higgs off-shell, we are going to "cut" its propagator because we are only concerned with how the loop establishes the form factors of the effective vertex. When we compute an amplitude with this vertex, we will then include the Higgs propagator.

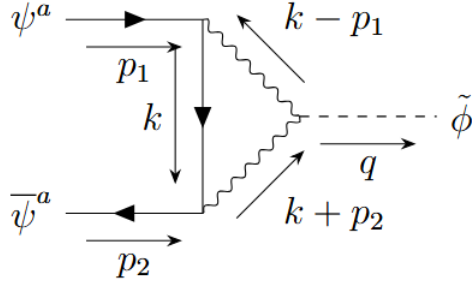


FIGURE 5.6: Loop vertex with the corresponding momenta.

Now we need to perform the Passarino-Veltman reduction to express C^ν in terms of scalar integrals. The tensor decomposition yields:

$$C^\nu = p_2^\nu C_2 - p_1^\nu C_1$$

Following the algorithm, we obtain the system of equations:

$$\begin{aligned} 2p_1 \cdot p_2 C_2 - 2p_1^2 C_1 &= -B_0(1) + B_0(0) + f_1 C_0 \\ 2p_2^2 C_2 - 2p_1 \cdot p_2 C_1 &= B_0(2) - B_0(0) - f_2 C_0 \end{aligned} \quad (5.21)$$

where we have $f_i = p_i^2 - M_V^2 + M_F^2$, for $i = 1, 2$, and the scalar 2-point functions are given by:

$$B_0(0) = B_0(q^2, M_V^2, M_V^2) ; B_0(1) = B_0(p_2^2, M_F^2, M_V^2) ; B_0(2) = B_0(p_1^2, M_F^2, M_V^2) \quad (5.22)$$

Solving this system, we find:

$$\begin{aligned} C_1 &= \frac{1}{2((p_1 \cdot p_2)^2 - p_1^2 p_2^2)} \left[(p_1 \cdot p_2 + p_2^2) B_0(0) - p_2^2 B_0(1) - (p_1 \cdot p_2) B_0(2) + (f_1 p_2^2 + f_2 (p_1 \cdot p_2)) C_0 \right] \\ C_2 &= \frac{1}{2((p_1 \cdot p_2)^2 - p_1^2 p_2^2)} \left[(p_1 \cdot p_2 + p_1^2) B_0(0) - p_1^2 B_0(2) - (p_1 \cdot p_2) B_0(1) + (f_2 p_1^2 + f_1 (p_1 \cdot p_2)) C_0 \right] \end{aligned}$$

Note that these coefficient functions are symmetric under the interchange $1 \longleftrightarrow 2$. Setting $p_1^2 = p_2^2 = M_F^2$ and $p_1 \cdot p_2 = q^2/2 - M_F^2$, we obtain:

$$\begin{aligned} C_1 &= \frac{1}{q^2 - 4M_F^2} \left[B_0(0) - \frac{2M_F^2}{q^2} B_0(1) + \frac{2M_F^2 - q^2}{q^2} B_0(2) + (2M_F^2 - M_V^2) C_0 \right] \\ C_2 &= \frac{1}{q^2 - 4M_F^2} \left[B_0(0) + \frac{2M_F^2 - q^2}{q^2} B_0(1) - \frac{2M_F^2}{q^2} B_0(2) + (2M_F^2 - M_V^2) C_0 \right] \end{aligned} \quad (5.23)$$

Now, observe that $B_0(1)$ and $B_0(2)$ are symmetric under the interchange $p_1 \longleftrightarrow -p_2$. Furthermore, the integral result depends only on $p_1^2 = p_2^2 = M_F^2$. Thus, we can conclude that $B_0(1) = B_0(2) = B_0(p_2^2, M_F^2, M_V^2)$, and hence, $C_1 = C_2$:

$$C_1 = \frac{1}{q^2 - 4M_F^2} \left[B_0(0) - B_0(1) + (2M_F^2 - M_V^2) C_0 \right] \quad (5.24)$$

The original integrals C^ν and C_0 are finite. Therefore, the amplitude is:

$$\mathcal{M} = -i \frac{M_V g_D^3}{64\pi^2} \bar{v}(p_2) [4M_F C_0 + C_1 (\not{p}_1 - \not{p}_2)] u(p_1) \quad (5.25)$$

which gives the following form factors:

$$\begin{pmatrix} F_1 = \frac{M_V g_D^3}{64\pi^2} 4M_F C_0 & F_2 = \frac{2M_V g_D^3}{64\pi^2} C_1 \\ F_3 = -F_2 & F_4 = 0 \end{pmatrix} \quad (5.26)$$

Furthermore, in the case of direct detection, where $q^2 = 0$, the form factors reduce to:

$$F_1 = \frac{M_V g_D^3}{64\pi^2} C_0^{(dd)} \quad F_2 = \frac{M_V g_D^3}{64\pi^2} C_1^{(dd)} \quad F_3 = -F_2 \quad (5.27)$$

where $C_0^{(dd)} = 4M_F C_0$ has dimensions of inverse mass (so F_1 is dimensionless, consistent with the effective part), and $C_1^{(dd)} = 2C_1$, which has dimensions of inverse mass squared (so F_2 and F_3 have dimensions of inverse mass). The explicit analytical form of these functions is calculated using the *Mathematica* package *FeynHelpers* [49], particularly the package *Package-X* [50]:

$$\begin{aligned} C_0^{(dd)} &= \frac{2M_V^2(4M_F^2 - M_V^2) \ln\left(\frac{M_F^2}{M_V^2}\right) - 4(M_V^2 - 2M_F^2) \sqrt{M_V^2(M_V^2 - 4M_F^2)} \ln\left(\frac{\sqrt{M_V^2(M_V^2 - 4M_F^2)} + M_V^2}{2M_F M_V}\right)}{M_F M_V^2(4M_F^2 - M_V^2)} \\ C_1^{(dd)} &= \frac{1}{2M_F^4 M_V^2(4M_F^2 - M_V^2)} \left[2M_F^2 M_V^2(4M_F^2 - M_V^2) + M_V^2(M_V^4 - 6M_F^2 M_V^2 + 8M_F^4) \ln\left(\frac{M_F^2}{M_V^2}\right) \right. \\ &\quad \left. + 2\sqrt{M_V^2(M_V^2 - 4M_F^2)}(2M_F^4 - 4M_F^2 M_V^2 + M_V^4) \ln\left(\frac{\sqrt{M_V^2(M_V^2 - 4M_F^2)} + M_V^2}{2M_F M_V}\right) \right] \end{aligned} \quad (5.28)$$

The dd superscript denotes the 3-point functions in the direct detection limit with $q^2 = 0$. An interesting observation is that when we transition to the kinematic regime where $V^\pm$ is the lighter particle (and thus the DM candidate), the logarithms acquire an imaginary value, so we need to program this carefully. Another thing to note is that these functions diverge when $M_V = 2M_F$, but the limit gives:

$$\lim_{M_V \rightarrow 2M_F} C_i^{(dd)} \sim -\frac{1}{M_F}$$

Finally, performing the matching between the loop part and the effective part, we find that in this limit, the k -factors are:

$$\begin{pmatrix} k_1 = \frac{M_V^2 g_D^3}{32\pi^2 v_D} C_0^{(dd)} & k_2 = -\frac{M_V^3 g_D^3}{32\pi^2 v_D} C_1^{(dd)} \\ k_3 = -2k_2 & k_5 = 0 \end{pmatrix} \quad (5.29)$$

A small disclaimer: when considering the physical states, which must be accounted for in different processes, these form factors should be multiplied by $\cos(\theta_D)$ or $-\sin(\theta_D)$, depending on whether it's a dark Higgs or an SM Higgs, respectively.

5.2.3 Box-loop vertex

Effective part

From the effective Lagrangian, the terms that contribute to this vertex are:

$$\mathcal{L}_4 = \frac{k_1}{4M_V} \bar{\psi}^a \psi^a \tilde{\phi}^2 + \frac{k_2}{2M_V^2} \bar{\psi}^a \tilde{\phi} \not{\partial} \tilde{\phi} \psi^a + \frac{k_3}{4M_V^2} \bar{\psi}^a \not{\partial} \psi^a \tilde{\phi}^2 + \frac{k_4}{4M_V^3} \bar{\psi}^a \psi^a \partial_\mu \tilde{\phi} \partial^\mu \tilde{\phi} + \frac{k_5}{2M_V^3} \bar{\psi}^a \partial_\mu \psi^a \partial^\mu \tilde{\phi} \tilde{\phi}$$

However, with our previous result, we now know that $k_5 = 0$. Therefore, we have:

$$V^{(4)} = i \frac{k_1}{4M_V} + \frac{1}{4M_V^2} (k_2 + k_3) \not{p}_1 + \frac{k_2}{4M_V^2} \not{p}_2 - i \frac{k_4}{4M_V^3} (p_3 \cdot p_4) \quad (5.30)$$

where $q = p_1 + p_2 = p_3 + p_4$. Thus, the form factors are:

$$\begin{aligned} G_1 &= \frac{k_1}{4M_V} & G_2 &= \frac{k_2 + k_3}{4M_V^2} \\ G_3 &= \frac{k_2}{4M_V^2} & G_4 &= -\frac{k_4}{4M_V^3} \end{aligned} \quad (5.31)$$

Loop part

Now, from the perspective of the loops, we have the diagrams in Figure 5.7 and 5.9. The box loop (a) gives the following amplitude (in four dimensions):

$$i\mathcal{M}_a = \int \frac{d^4k}{(2\pi)^4} \frac{\bar{v}(p_2) \left(\frac{1}{2} g_D \gamma^\mu \right) (i)(\mathcal{K} + M_F)(-ig_{\mu\alpha}) M_V g_D g^{\alpha\beta} (-ig_{\beta\rho}) M_V g_D g^{\rho\lambda} (-ig_{\lambda\nu}) \left(\frac{1}{2} g_D \gamma^\nu \right) u(p_1)}{(k^2 - M_F^2)((k - p_1)^2 - M_V^2)((k + p_2)^2 - M_V^2)((k + p_3 - p_1)^2 - M_V^2)}$$

Contracting and using the properties of the gamma matrices in four dimensions:

$$\mathcal{M}_a = \frac{M_V^2 g_D^4}{32\pi^2} \bar{v}(p_2) \left[2M_F D_0^{(a)} - \gamma_\nu D^{\nu,(a)} \right] u(p_1) \quad (5.32)$$

From the tensor decomposition, we obtain:

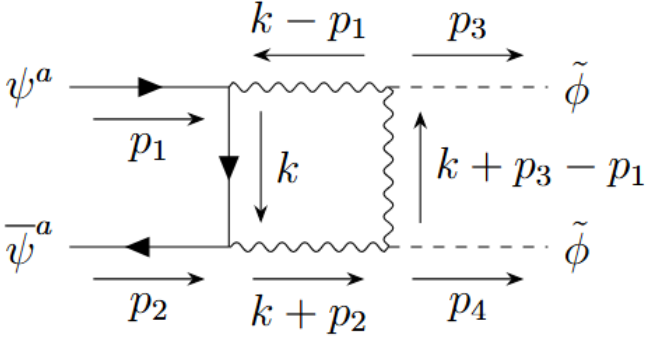


FIGURE 5.7: *t-channel*

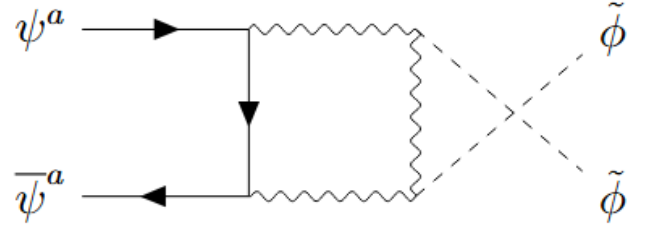


FIGURE 5.8: *u-channel*

$$D^{\nu,(a)} = -p_1^\nu D_1^{(a)} + p_2^\nu D_2^{(a)} + (p_3 - p_1)^\nu D_3^{(a)}$$

By applying the reduction algorithm, we get the following system of equations:

$$\begin{aligned} -2p_1^2 D_1^{(a)} + 2(p_1 \cdot p_2) D_2^{(a)} + 2p_1 \cdot (p_3 - p_1) D_3^{(a)} &= C_0^{(a)}(0) - C_0^{(a)}(1) + f_1 D_0^{(a)} \\ -2(p_1 \cdot p_2) D_1^{(a)} + 2p_2^2 D_2^{(a)} + 2p_1 \cdot (p_3 - p_1) D_3^{(a)} &= C_0^{(a)}(2) - C_0^{(a)}(0) - f_2 D_0^{(a)} \\ -2(p_3 - p_1) \cdot p_1 D_1^{(a)} + 2(p_3 - p_1) \cdot p_2 D_2^{(a)} + 2(p_3 - p_1)^2 D_3^{(a)} &= C_0^{(a)}(3) - C_0^{(a)}(0) - f_3 D_0^{(a)} \end{aligned} \quad (5.33)$$

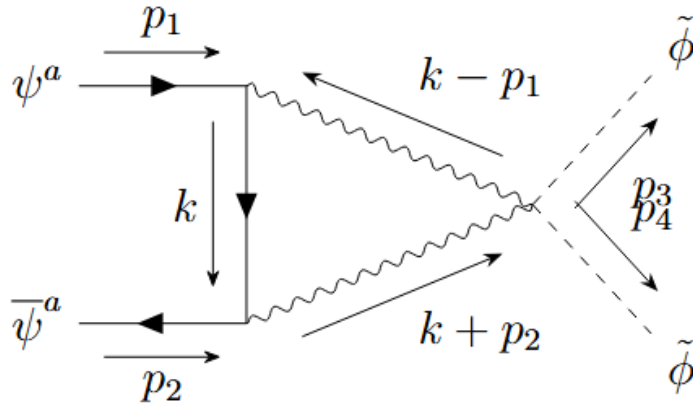


FIGURE 5.9: *Triangular channel*

By setting $p_1^2 = p_2^2 = M_F^2$, $p_3^2 = p_4^2 = M_\phi^2$, and using the relations $C_0^{(a)}(2) = C_0^{(a)}(1)$ and the Mandelstam variables, we obtain the following coefficient functions, solved and simplified with the aid of *FeynCalc* and *Wolfram Mathematica*⁴:

$$D_1^{(a)} = D_2^{(a)} = \frac{M_\phi^2 - M_F^2 + t}{2(M_F^2 - M_\phi^2)^2 - 2tu} C_0^{(a)}(0) + \frac{M_F^2 - M_\phi^2 + t}{2(M_F^2 - M_\phi^2)^2 - 2tu} C_0^{(a)}(3) \\ - \frac{t}{(M_F^2 - M_\phi^2)^2 - 2tu} C_0^{(a)}(1) \\ - \frac{M_F^2 (M_F^2 - M_V^2 - M_\phi^2 - 2t) + M_V^2 (M_\phi^2 + t) - t(M_\phi^2 - t)}{2(M_F^2 - M_\phi^2)^2 - 2tu} D_0^{(a)} \quad (5.34)$$

$$D_3^{(a)} = \frac{u - t}{2(M_F^2 - M_\phi^2)^2 - 2tu} C_0^{(a)}(0) - \frac{2M_F^2 - 2M_\phi^2 + t + u}{2(M_F^2 - M_\phi^2)^2 - 2tu} C_0^{(a)}(3) \\ + \frac{M_F^2 - M_\phi^2 + t}{(M_F^2 - M_\phi^2)^2 - 2tu} C_0^{(a)}(1) \\ - \frac{M_F^2 (2M_F^2 - 2M_\phi^2 + t - u) - M_V^2 (t - u) - t(2M_\phi^2 - t - u)}{2(M_F^2 - M_\phi^2)^2 - 2tu} D_0^{(a)} \quad (5.35)$$

where

$$C_0^{(a)}(0) = C_0(s, p_4^2, p_3^2, M_V^2, M_V^2, M_V^2) \\ C_0^{(a)}(1) = C_0(p_2^2, p_4^2, t, M_F^2, M_V^2, M_V^2) \\ C_0^{(a)}(3) = C_0(p_1^2, s, p_2^2, M_F^2, M_V^2, M_V^2) \\ D_0^{(a)} = D_0(p_1^2, s, p_4^2, t, p_2^2, p_3^2, M_F^2, M_V^2, M_V^2, M_V^2) \quad (5.36)$$

Finally, with the coefficient functions defined, the amplitude becomes:

$$\mathcal{M}_a = \frac{M_V^2 g_D^4}{32\pi^2} \bar{v}(p_2) \left[2M_F D_0^{(a)} + D_1^{(a)} (\not{p}_1 - \not{p}_2) + D_3^{(a)} (\not{p}_1 - \not{p}_3) \right] u(p_1) \quad (5.37)$$

For the other part, due to the fact that the final particles are identical, there is another diagram in the u channel that produces almost the same coefficient functions, but with $t \longleftrightarrow u$, $p_2 \longleftrightarrow -p_1$, and p_3 having the sign flipped. Thus, we label the functions of the t channel with the superscript (a) and those of the u channel with (b) . Therefore, the amplitude for the diagram in Figure 5.8 is:

$$\mathcal{M}_b = \frac{M_V^2 g_D^4}{32\pi^2} \bar{v}(p_2) \left[2M_F D_0^{(b)} + D_1^{(b)} (\not{p}_1 - \not{p}_2) + D_3^{(b)} (\not{p}_3 - \not{p}_2) \right] u(p_1) \quad (5.38)$$

Furthermore, the two processes in Figure 5.7 have different "incarnations" depending on the dark charge of the fermion. The annihilation of $\psi^\pm \bar{\psi}^\pm$ has two versions: one with $\psi^\pm$ in the loop and another with ψ^0 . Each of them, in their respective channels, are equal. On the other hand, the $\psi^0 \bar{\psi}^0$ annihilation involves a $\psi^\pm$ in the loop, and both are equal to the others, again, in their respective channels. In conclusion, the box-loop amplitude gets a factor of 6 to account for the different diagrams.

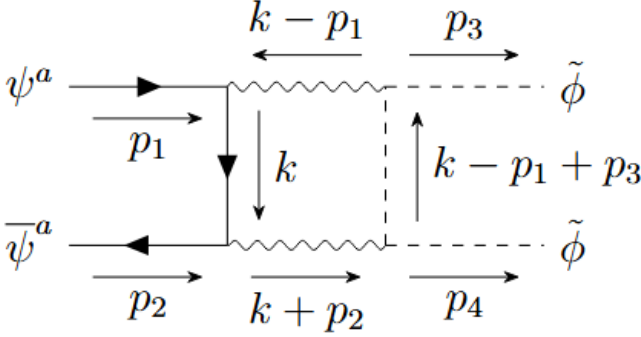
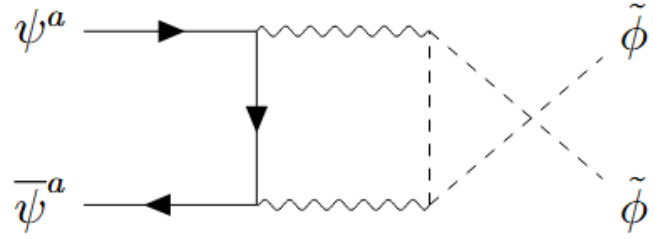
Now, for the amplitude of the diagram in Figure 5.9, we have something very similar to the $\bar{\psi}^a \psi^a \tilde{\phi}$ vertex. The only difference is that the $V^a V^a \tilde{\phi}^2$ vertex adds a factor of $g_D/2$ to the original result and removes the M_V .

⁴This is the same for almost all Passarino-Veltman reductions involved in this thesis.

Additionally, we can apply the same argument as before and multiply by 6 to account for the different particles that can form this diagram. Thus, the amplitude looks like:

$$\mathcal{M}_c = -6i \frac{g_D^4}{64\pi^2} \bar{v}(p_2) [2M_F C_0 + C_1(\not{p}_1 - \not{p}_2)] u(p_1) \quad (5.39)$$

Just as with the vertex involving one scalar, the amplitude should be multiplied by $\cos^2(\theta_D)$, $-\cos(\theta_D)\sin(\theta_D)$, or $\sin^2(\theta_D)$, depending on whether the final state is composed of 2 dark Higgs, 2 SM Higgs, or one of each, respectively.


 FIGURE 5.10: *t-channel*

 FIGURE 5.11: *u-channel*

There are also box-loop processes with the V boson connecting the two scalars, which are replaced by their corresponding Goldstone bosons. This type of diagram leads to the following amplitude:

$$i\mathcal{M}_d = \frac{g_D^4}{256\pi^4} \int d^4k \frac{\bar{v}(p_2)(\not{K} + 2\not{p}_3 - 2\not{p}_1 - \not{p}_2)(\not{K} + M_F)(\not{K} + 2\not{p}_3 - \not{p}_1)u(p_1)}{(k^2 - M_F^2)((k - p_1)^2 - M_V^2)((k + p_2)^2 - M_V^2)((k + p_3 - p_1)^2 - M_V^2)} \quad (5.40)$$

Applying some properties of the gamma matrices and performing the tensor decomposition, we obtain:

$$\begin{aligned} \mathcal{M}_d = & \frac{g_D^4}{256\pi^2} \bar{v}(p_2) \left[D_0^{(a)} \left(M_F^3 + (4f_1^{(a)} - 3M_F^2)\not{p}_1 - M_F^2\not{p}_2 + (8(f_1^{(a)} - f_3^{(a)}) + 4M_F^2)\not{p}_3 \right. \right. \\ & + M_F((\not{p}_2 - \not{p}_3)\not{p}_1 - 2(2\not{p}_1 + \not{p}_2)\not{p}_3 + 2(p_1^2 + 2p_3^2)) \Big) + C_0^{(a)}(0)(M_F + 4\not{p}_3 + \not{p}_1 - \not{p}_2) \\ & - 4C_0^{(a)}(1)(\not{p}_1 + 2\not{p}_3) + 8C_0^{(a)}(3)\not{p}_3 + C_0^{\mu,(a)}\gamma_\mu \\ & + D^{\mu,(a)} \left(M_F^2\gamma_\mu + (\not{p}_2 - 2\not{p}_3)\gamma_\mu\not{p}_1 - 2(2\not{p}_1 - \not{p}_2)\gamma_\mu\not{p}_3 - 2(p_1^2 + 2p_3^2)\gamma_\mu \right. \\ & \left. \left. + M_F((2\not{p}_3 - 2\not{p}_1 - \not{p}_2)\gamma_\mu - \gamma_\mu(\not{p}_1 - 2\not{p}_3)) \right) \right] u(p_1) \end{aligned} \quad (5.41)$$

where $f_1^{(a)} = p_1^2 - M_V^2 + M_F^2$, $f_3^{(a)} = (p_3 - p_1)^2 - M_V^2 + M_F^2$ and:

$$\begin{aligned} D^{\mu,(a)} &= -p_1^\mu D_1 + p_2^\mu D_1 + (p_3 - p_1)^\mu D_3 \\ C^{\mu,(a)}(0) &= C^\mu(s, p_4^2, p_3^2, M_V^2, M_V^2, M_V^2) = (p_1 + p_2)^\mu C_1^{(a)} + p_3^\mu C_2^{(a)} \end{aligned} \quad (5.42)$$

The coefficient functions for the tensor integral D^μ are the same as those of the box loop without Goldstones. Meanwhile, the coefficients for the C function are:

$$\begin{aligned} C_1^{(a)} &= \frac{B_0^{(a)}(1) - B_0^{(a)}(2) - M_\phi^2 C_0^{(a)}(0)}{4M_\phi^2 - s} \\ C_2^{(a)} &= \frac{-2B_0^{(a)}(1) + 2B_0^{(a)}(2) + (2M_F^2 - t - u)C_0^{(a)}(0)}{4M_\phi^2 - s} \end{aligned} \quad (5.43)$$

where,

$$\begin{aligned} B_0^{(a)}(0) &= B_0^{(a)}(1) = B_0(p_4^2, M_V^2, M_V^2) \\ B_0^{(a)}(2) &= B_0(s, M_V^2, M_V^2) \end{aligned} \quad (5.44)$$

The amplitude corresponding to the u channel is:

$$\begin{aligned} \mathcal{M}_e &= \frac{g_D^4}{256\pi^2} \bar{v}(p_2) \left[D_0^{(b)} \left(M_F^3 - (4f_2^{(b)} - 3M_F^2)p_2 + M_F^2 p_1 - (8(f_2^{(b)} - f_3^{(b)}) + 4M_F^2)p_3 \right. \right. \\ &\quad \left. \left. + M_F(p_2(p_1 - p_3) - 2p_3(2p_2 + p_1) + 2(p_2^2 + 2p_3^2)) \right) + C_0^{(b)}(0)(M_F - 4p_3 - p_2 + p_1) \right. \\ &\quad \left. + 4C_0^{(b)}(2)(p_2 + 2p_3) - 8C_0^{(b)}(3)p_3 + C_0^{\mu,(b)}\gamma_\mu \right. \\ &\quad \left. + D^{\mu,(b)} \left(M_F^2\gamma_\mu + p_2\gamma_\mu(p_1 - 2p_3) - 2p_3\gamma_\mu(p_1 - 2p_2) - (p_2^2 + 2p_3^2)\gamma_\mu \right. \right. \\ &\quad \left. \left. + M_F(\gamma_\mu(p_1 + 2p_2 - 2p_3) - (p_2 - 2p_3)\gamma_\mu) \right) \right] u(p_1) \end{aligned} \quad (5.45)$$

where $f_2^{(b)} = p_2^2 - M_V^2 + M_F^2$, $f_3^{(b)} = (p_2 - p_3)^2 - M_V^2 + M_F^2$, and

$$\begin{aligned} D^{\mu,(b)} &= -p_1^\mu D_1^{(b)} + p_2^\mu D_1^{(b)} + (p_2 - p_3)^\mu D_3^{(b)} \\ C^{\mu,(b)}(0) &= C^\mu(s, p_4^2, p_3^2, M_V^2, M_V^2) = -(p_1 + p_2)^\mu C_1^{(b)} - p_3^\mu C_2^{(b)} \end{aligned} \quad (5.46)$$

In a similar way to the box loop without Goldstones, we label the functions with the superscripts (a) and (b) for the diagrams of the t and u channels, respectively, which have the same structure but with the interchange $t \longleftrightarrow u$, $p_2 \longleftrightarrow -p_1$, and p_3 with the sign flipped, as you can check. It is easy to note that $C_i^{(a)} = C_i^{(b)}$, with $i = 1, 2$. With this result, we conclude that:

$$C^{\mu,(b)}(0) = -C^{\mu,(a)}(0)$$

Also, we have that $C_0^{(a)}(i) = C_0^{(b)}(i)$, for $i = 0, 3$, as before. Then, summing both amplitudes for the box loops with a Goldstone boson and multiplying by a factor of 6, which stands for the three pairs of processes $\bar{\psi}^a \psi^a$, one for each dark charge. As we have just shown, the Lorentz structure of the amplitudes coming from box diagrams does not match the structure of the effective vertex, due to the lack of a term proportional to p_3 , which always appears as an extra term.

5.2.4 Vanishing diagrams

There are at least ten diagrams that can produce this process individually, but they can either cancel each other out or are zero by themselves. Additionally, they must be zero because they require an effective vertex of the form $V_\mu^0 \tilde{\phi} \tilde{\phi}$ to match. This vertex can arise from some effective operator of the form:

$$(D_\mu \tilde{\phi})^\dagger V^{\mu\nu} D_\nu \tilde{\phi} \quad (5.47)$$

At first glance, this may not seem like it will vanish, but once the scalar field acquires its v.e.v, terms of the form:

$$\partial_\mu \tilde{\phi} \partial_\nu \tilde{\phi} (\partial^\mu V^{0\nu} - \partial^\nu V^{0\mu}) \quad (5.48)$$

will appear. The scalar part is symmetric under the flip of Lorentz indices, while the vector part is anti-symmetric, so the term vanishes. We do not have an effective vertex for these loops. Furthermore, the entire operator vanishes, which concludes the analysis of the effective part.

The loop part of the analysis begins with the diagrams in Figure 5.12. The corresponding amplitudes are:

$$\begin{aligned} \mathcal{M}_C &= \frac{3M_V^2 g_D^4}{(2\pi)^4 (q^2 - M_V^2)} \int d^4k \frac{\bar{v}(p_2)(2\mathcal{K} - q)u(p_1)}{(k^2 - M_V^2)((k - p_3)^2 - M_V^2)((k - q)^2 - M_V^2)} \\ \mathcal{M}_{CC} &= -\frac{3M_V^2 g_D^4}{(2\pi)^4 (q^2 - M_V^2)} \int d^4k \frac{\bar{v}(p_2)(2\mathcal{K} - q)u(p_1)}{(k^2 - M_V^2)((k - p_3)^2 - M_V^2)((k - q)^2 - M_V^2)} \end{aligned} \quad (5.49)$$

where the subscripts stand for clockwise and counter-clockwise, respectively. As you can see, the amplitudes are exactly equal and opposite, so when they are summed up, their contribution vanishes. There are other diagrams of the same form, where the vector boson connected to the Higgses is replaced by a Goldstone boson, another where V^0 decays into two Goldstones that are connected by a vector, and finally one where all the vectors in the loop are replaced by scalars. There are also versions of the diagrams in Figure 5.12 with ghost particles in the triangle. Each of these comes in pairs, similar to the ones in Figure 5.12, and each pair sums to zero as before. In summary, there are no diagrams in a triangular loop that contribute to the effective vertex $V^0\tilde{\phi}\tilde{\phi}$.

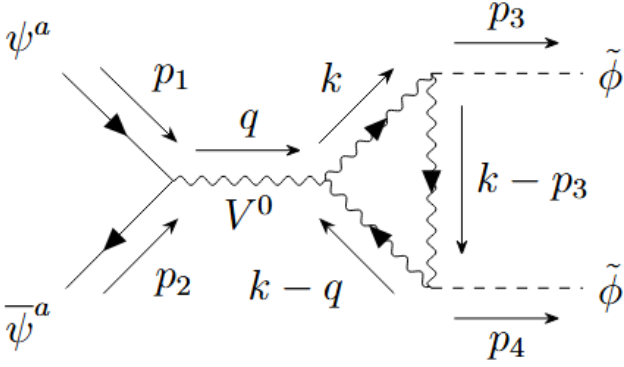


FIGURE 5.12: Type I

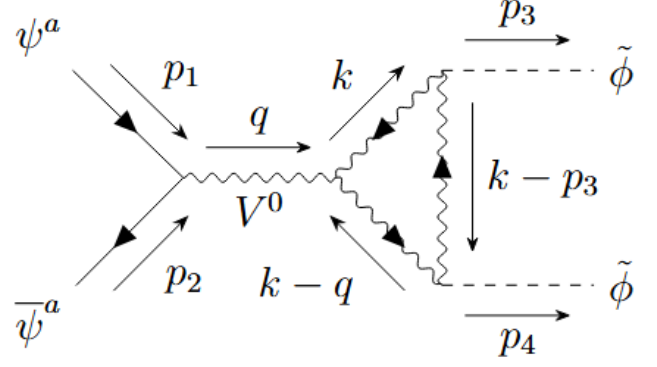


FIGURE 5.13: Type II

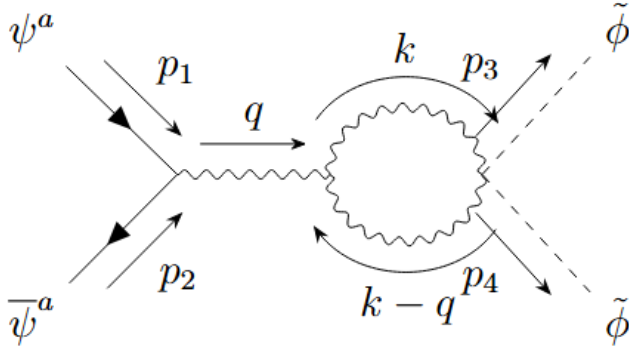


FIGURE 5.14: Type I

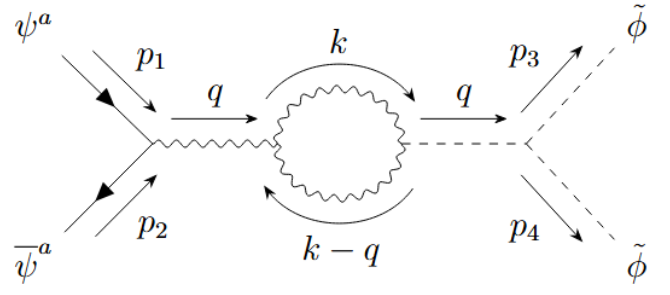


FIGURE 5.15: Type II

To complete this analysis, we have the diagrams of Figure 5.14, labeled as type I and II. Their amplitudes are as follows:

$$i\mathcal{M}_I = \int \frac{d^D k}{(2\pi)^D} \bar{v}(p_2)(ig_D\gamma^\mu)u(p_1) \frac{-ig_{\mu\nu}}{q^2 - M_V^2} \frac{-ig_{\alpha\beta}}{k^2 - M_V^2} \frac{-ig_{\rho\omega}}{(k-q)^2 - M_V^2} \frac{g_D^2}{2} g^{\beta\omega} \\ \times ig_D[(k-q+k)^\nu g^{\alpha\rho} + (-k-q)^\alpha g^{\nu\rho} + (q-k+q)^\rho g^{\nu\alpha}]$$

$$\mathcal{M}_I = \frac{g_D^4(D-1)}{2(2\pi)^D(q^2 - M_V^2)} \int d^D k \frac{\bar{v}(p_2)(2\mathcal{K} - \not{q})u(p_1)}{(k^2 - M_V^2)((k-q)^2 - M_V^2)} \\ = \frac{g_D^4 i\pi^2(D-1)}{2(2\pi)^D(q^2 - M_V^2)} [2\bar{v}(p_2)\gamma_\mu u(p_1)B^\mu - \bar{v}(p_2)\not{q}u(p_1)B_0] \quad (5.50)$$

where,

$$B^\mu = -q^\mu B_1 = -\frac{q^\mu}{2q^2} [A_0(1) - A_0(0) - f_1 B_0] \quad (5.51)$$

with $f_1 = q^2 - M_V^2 + M_V^2 = q^2$, $A_0(1) = A_0(0) = A_0(M_V^2)$, so the tensor function looks like:

$$B^\mu = \frac{q^\mu}{2} B_0 \quad (5.52)$$

As you can see, this amplitude diverges in D dimensions, but we see that $2\gamma_\mu B^\mu = \not{q} B_0$, so the amplitude vanishes exactly:

$$\mathcal{M}_I = \frac{g_D^4 i \pi^2 (D-1)}{2(2\pi)^D (q^2 - M_V^2)} [\bar{v}(p_2) \not{q} u(p_1) B_0 - \bar{v}(p_2) \not{q} u(p_1) B_0] = 0 \quad (5.53)$$

The same occurs for the type II amplitude, but to show it, we are going to "cut" two legs of the diagram, leaving only the off-shell tails:

$$\begin{aligned} \mathcal{G}^\mu &= \int \frac{d^D k}{(2\pi)^D} i g_D [(k-q+k)^\mu g^{\nu\rho} + (-k-q)^\nu g^{\mu\rho} + (q-k+k)^\rho g^{\mu\nu}] \frac{-i g_{\nu\alpha}}{k^2 - M_V^2} \frac{-i g_{\rho\beta}}{(k-q)^2 - M_V^2} M_V g_D g^{\alpha\beta} \\ &= -i \frac{M_V g_D^2}{(2\pi)^D} \int d^D k \frac{(D-1)(2k-q)^\mu}{(k^2 - M_V^2)((k-q)^2 - M_V^2)} = \frac{M_V g_D^2 \pi^2}{(2\pi)^D} [2B^\mu - q^\mu B_0] = 0 \end{aligned} \quad (5.54)$$

The same happens with the diagrams of type I and II when the bosons are interchanged for Goldstone bosons, and in the case of Figure 5.15, replaced by ghost particles. This concludes that there is no diagram that contributes to the effective vertex $V^0 \tilde{\phi} \tilde{\phi}$, as expected, corroborating with what we obtained at the beginning of this section.

5.3 Squared amplitudes

5.3.1 Relic density calculations

Scalar in the final state

For relic density calculations, we are interested in the 2-to-2 process $\bar{\psi}^a \psi^a \rightarrow \tilde{\phi} \tilde{\phi}$, which actually consists of three processes involving the physical states of the Higgs bosons: $\bar{\psi}^a \psi^a \rightarrow \phi \phi$, $\bar{\psi}^a \psi^a \rightarrow \phi h$, and $\bar{\psi}^a \psi^a \rightarrow h h$. These processes involve almost all the diagrams we have seen in the previous section, for which we will be squaring the individual amplitudes in this section.

Let's start with the process that has two dark Higgs bosons in the final state. To begin with, we consider the diagrams with an off-shell scalar in the intermediate state, which originate from the loop in Figure 5.6. We have two possibilities, which are shown in Figure 5.16. The diagrams for the process with two SM Higgs bosons and the one with each scalar, have the same structure. Furthermore, just as in the box-loop diagrams, each of the two diagrams in any of these three processes has six versions, two for each dark fermion-antifermion annihilation. This adds a factor of 6 to each amplitude.

We will summarize all the amplitudes contributing to the process $\bar{\psi}^a \psi^a \rightarrow \phi \phi$ as discussed in Section ??, multiplying each by the corresponding additional factors associated with the scalar part, and simplifying using

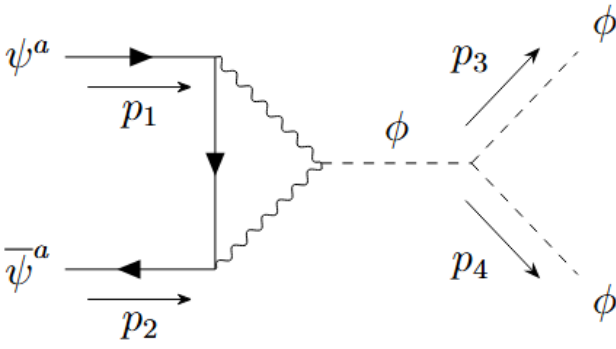


FIGURE 5.16: Mediated by a dark Higgs

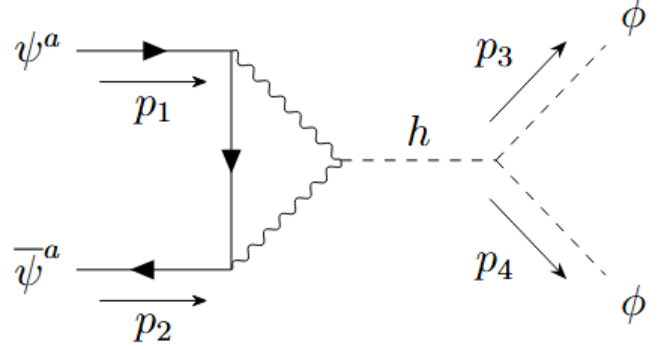


FIGURE 5.17: Mediated by an SM Higgs

the Dirac equation:

$$\begin{aligned}
 \mathcal{M}_1 &= 6 \frac{M_V g_D^3}{32\pi^2} \bar{v}(p_2) V_1 u(p_1) \frac{f_{\phi\phi\phi} \cos(\theta_D)}{s - M_\phi^2} \quad (\text{Figure 5.16}) \\
 \mathcal{M}_2 &= -6 \frac{M_V g_D^3}{32\pi^2} \bar{v}(p_2) V_1 u(p_1) \frac{f_{h\phi\phi} \sin(\theta_D)}{s - M_h^2} \quad (\text{Figure 5.17}) \\
 \mathcal{M}_3 &= -6i \frac{g_D^4}{64\pi^2} \bar{v}(p_2) V_1 u(p_1) \cos^2(\theta_D) \quad (\text{Figure 5.9}) \\
 \mathcal{M}_4 &= 6 \frac{M_V^2 g_D^4}{32\pi^2} \bar{v}(p_2) V_2 u(p_1) \cos^2(\theta_D) \quad (\text{Figure 5.7}) \\
 \mathcal{M}_5 &= 6 \frac{M_V^2 g_D^4}{32\pi^2} \bar{v}(p_2) V_3 u(p_1) \cos^2(\theta_D) \quad (\text{Figure 5.8}) \\
 \mathcal{M}_6 &= 6 \frac{g_D^4}{256\pi^2} \bar{v}(p_2) V_4 u(p_1) \cos^2(\theta_D) \quad (\text{Figure 5.10}) \\
 \mathcal{M}_7 &= 6 \frac{g_D^4}{256\pi^2} \bar{v}(p_2) V_5 u(p_1) \cos^2(\theta_D) \quad (\text{Figure 5.11})
 \end{aligned} \tag{5.55}$$

where the V_i factors are functions of the Passarino-Veltman functions resulting from the loops, and they are listed as follows:

$$\begin{aligned}
 V_1 &= 2M_F \left(C_0 + \frac{B_0(0) - B_0(1) + (2M_F^2 - M_V^2)C_0}{s - 4M_F^2} \right) \\
 V_2 &= V_{2a} - V_{2b} \not{p}_3 = M_F \left[2(D_0^{(a)} + D_1^{(a)}) + D_3^{(a)} \right] - D_3^{(a)} \not{p}_3 \\
 V_3 &= V_{3a} + V_{3b} \not{p}_3 = M_F \left[2(D_0^{(b)} + D_1^{(b)}) + D_3^{(b)} \right] + D_3^{(b)} \not{p}_3 \\
 V_4 &= V_{4a} + V_{4b} \not{p}_3 \\
 V_5 &= V_{5a} - V_{5b} \not{p}_3
 \end{aligned} \tag{5.56}$$

with

$$\begin{aligned}
 V_{4a} &= M_F \left(3C_0^{(a)}(0) - 4C_0^{(a)}(1) + 4D_0^{(a)}(f_1^{(a)} - M_F^2 + t) + 2D_1^{(a)}(3t + u) + 2D_3^{(a)}(M_F^2 - M_\phi^2 + t) \right) \\
 V_{4b} &= 4C_0^{(a)}(0) - 8C_0^{(a)}(1) + 8C_0^{(a)}(3) + C_2 + D_0^{(a)}(8f_1^{(a)} - 8f_3^{(a)} \\
 &\quad + 9M_F^2) - 4D_1^{(a)}(s - 4M_F^2) + 4D_3^{(a)}(M_F^2 - M_\phi^2) \\
 V_{5a} &= M_F \left(3C_0^{(b)}(0) - 4C_0^{(b)}(1) + 4D_0^{(b)}(f_2^{(b)} - M_F^2 + u) + 2D_1^{(b)}(2M_\phi^2 + s + 2t) + 2D_3^{(b)}(3M_\phi^2 + u) \right) \\
 V_{5b} &= 4C_0^{(b)}(0) - 8C_0^{(b)}(2) + 8C_0^{(b)}(3) + C_2 + D_0^{(b)}(8f_2^{(b)} - 8f_3^{(b)} + 9M_F^2) \\
 &\quad + 4D_1^{(b)}(s - 4M_F^2) - 2D_3^{(b)}(M_F^2 - 2M_\phi^2)
 \end{aligned} \tag{5.57}$$

Additionally, the factors f_{ijk} for $i, j, k = h, \phi$ in the first two amplitudes are combinations of trigonometric functions of the Higgs mixing angle θ_D , the different coupling constants of the scalar potential, and the vevs of each scalar. These are listed below:

$$\begin{aligned}
f_{\phi\phi\phi} &= -3 \left[2\lambda_S v_S \sin^3(\theta_D) + 2\lambda_D v_D \cos^3(\theta_D) + \lambda_{h\phi} \sin(\theta_D) \cos(\theta_D) (v_S \cos(\theta_D) + v_D \sin(\theta_D)) \right] \\
f_{hhh} &= -3 \left[2\lambda_s v_s \cos^3(\theta_D) - 2\lambda_D v_D \sin^3(\theta_D) + \lambda_{h\phi} \sin(\theta_D) \cos(\theta_D) (v_S \sin(\theta_D) - v_D \cos(\theta_D)) \right] \\
f_{hh\phi} &= v_D \lambda_{h\phi} \cos(\theta_D) (3 \sin^2(\theta_D) - 1) + v_S \lambda_{h\phi} \sin(\theta_D) (2 - 3 \sin^2(\theta_D)) \\
&\quad - 6v_D \lambda_D \cos(\theta_D) \sin^2(\theta_D) - 6v_S \lambda_S \cos^2(\theta_D) \sin(\theta_D) \\
f_{h\phi\phi} &= v_s \lambda_{h\phi} \cos(\theta_D) (3 \sin^2(\theta_D) - 1) + v_D \lambda_{h\phi} \sin(\theta_D) (2 - 3 \sin^2(\theta_D)) \\
&\quad + 6v_D \lambda_D \cos^2(\theta_D) \sin(\theta_D) - 6v_s \lambda_S \cos(\theta_D) \sin^2(\theta_D)
\end{aligned} \tag{5.58}$$

These seven amplitudes can be summed up in a more elegant and concise way. As seen, the amplitudes $\mathcal{M}_1$, $\mathcal{M}_2$, and $\mathcal{M}_3$ are simply scalar constants multiplying the spinor chain $\bar{v}(p_2)u(p_1)$. Meanwhile, the amplitudes $\mathcal{M}_4$ to $\mathcal{M}_7$ consist of scalar parts and additional terms proportional to $\not{p}_3$. Therefore, we can combine all the scalar factors into one term and all the terms proportional to $\not{p}_3$ into another, which results in the following expression:

$$\mathcal{M}_{\phi\phi} = \sum_{i=1}^7 \mathcal{M}_i = \frac{3}{16\pi^2} \bar{v}(p_2) [\alpha + \beta \not{p}_3] u(p_1) \tag{5.59}$$

where

$$\alpha = g_D^3 V_1 \alpha_1 + g_D^4 \cos^2(\theta_D) \alpha_2 \quad \beta = g_D^4 \cos^2(\theta_D) \beta_1 \tag{5.60}$$

and the sub-constants are given by

$$\begin{aligned}
\alpha_1 &= M_V \left(\frac{f_{\phi\phi\phi} \cos(\theta_D)}{s - M_\phi^2} - \frac{f_{h\phi\phi} \sin(\theta_D)}{s - M_h^2} \right) - i \frac{g_D}{2} \cos^2(\theta_D) \\
\alpha_2 &= M_V^2 (V_{2a} + V_{3a}) + \frac{V_{4a} + V_{5a}}{8} \\
\beta_1 &= M_V^2 (V_{3b} - V_{2b}) + \frac{V_{4b} - V_{5b}}{8}
\end{aligned} \tag{5.61}$$

Next, the squared amplitude is:

$$|\overline{\mathcal{M}}_{\phi\phi}|^2 = \frac{9}{512\pi^4} \left[|\alpha|^2 (s - 4M_F^2) + |\beta|^2 ((M_F^2 - t)(M_F^2 - u) - M_\phi^4) + M_F(t - u)(\alpha^* \beta + \alpha \beta^*) \right] \tag{5.62}$$

For the processes $\bar{\psi}^a \psi^a \rightarrow h h$ and $\bar{\psi}^a \psi^a \rightarrow h \phi$, the structure is almost the same; we only need to replace the constants from the scalar part of the diagrams and adjust the dependence on the Passarino-Veltman functions, by substituting the corresponding masses.

Vectors in the final state

We are also interested in the production of Standard Model bosons, such as the charged W and the neutral Z , through the annihilation of dark matter fermions. The only type of diagrams involved here are those of the triangular loop, which produces a scalar that can then decay into $W^+ W^-$ or ZZ (see Figure 5.18). There are six versions of these diagrams, each with the same amplitudes as before, accounting for the different DM fermions that can be annihilated. The amplitudes for each vector production are shown below:

$$\begin{aligned}
\mathcal{M}_W &= \frac{3}{16\pi^2} \beta_2 \bar{v}(p_2) u(p_1) g^{\mu\nu} \epsilon_\nu^*(p_3) \epsilon_\mu^*(p_4) \\
\mathcal{M}_Z &= \frac{3}{16\pi^2} \beta_3 \bar{v}(p_2) u(p_1) g^{\mu\nu} \epsilon_\nu^*(p_3) \epsilon_\mu^*(p_4)
\end{aligned} \tag{5.63}$$

where

$$\begin{aligned}\beta_2 &= M_V g_D^3 V_1 \sin(\theta_D) \cos(\theta_D) \left[\frac{1}{s - M_\phi^2} - \frac{1}{s - M_h^2} \right] \frac{e^2 v_S}{2 \sin^2(\theta_W)} ; \\ \beta_3 &= \frac{\beta_2}{\cos^2(\theta_W)}\end{aligned}\tag{5.64}$$

Taking the squared amplitude, we average over the initial spin states and sum over the polarizations of the final states, applying the property:

$$\sum_{\sigma} \epsilon_{\mu}(p, \sigma) \epsilon_{\nu}^*(p, \sigma) = -g_{\mu\nu} + \frac{p_{\mu} p_{\nu}}{M^2}\tag{5.65}$$

The squared amplitudes are:

$$\begin{aligned}|\overline{\mathcal{M}}_W|^2 &= \frac{9|\beta_2|^2}{2048\pi^2 M_W^4} (s - 4M_F^2)(12M_W^4 - 4M_W^2 s + s^2) \\ |\overline{\mathcal{M}}_Z|^2 &= \frac{9|\beta_3|^2}{2048\pi^2 M_Z^4} (s - 4M_F^2)(12M_Z^4 - 4M_Z^2 s + s^2)\end{aligned}\tag{5.66}$$

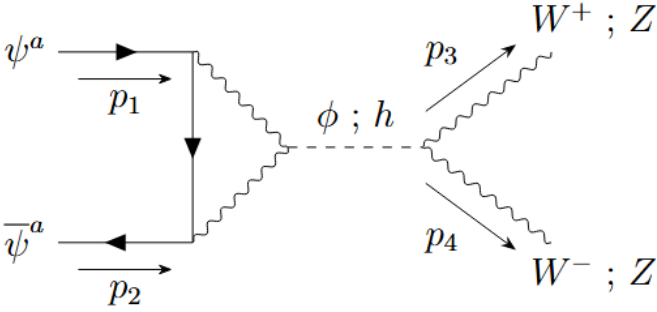


FIGURE 5.18: Vectors in the final state

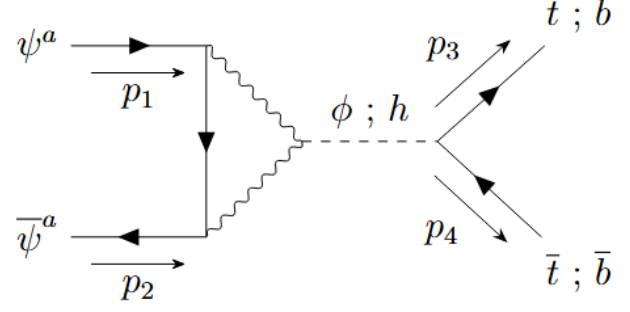


FIGURE 5.19: Quarks in the final state

Fermions in the final state

The structure of the amplitudes (see Figure 5.19) is almost the same as that of the vectors; we just need to replace the couplings and the polarization 4-vectors with the corresponding outgoing spinors. Furthermore, regardless of which fermion-antifermion pair is in the final state, the amplitude shares the same structure; we only need to change the mass of the corresponding fermion.

$$\mathcal{M}_f = \frac{3\beta_4}{16\pi^2} \bar{v}(p_2) u(p_1) \bar{u}(p_3) v(p_4)\tag{5.67}$$

where,

$$\beta_4 = M_V g_D^3 V_1 \sin(\theta_D) \cos(\theta_D) \left[\frac{1}{s - M_\phi^2} - \frac{1}{s - M_h^2} \right] \frac{M_f}{v_S}\tag{5.68}$$

squaring it:

$$|\overline{\mathcal{M}}_f|^2 = \frac{9|\beta_4|^2}{256\pi^4} (s - 4M_F^2)(s - 4M_f^2)\tag{5.69}$$

Preliminary results

6.1 Overview of the methodology

Having developed a large part of the loop diagrams in the previous chapter, which are needed to compute direct detection and relic density, we now face the task of implementing this model to obtain results. This can be done using *micrOMEGAs* [51], a program built around *CalcHEP* [52], which is capable of determining the odd particles of the model—namely, the DM candidates—to compute relic density, direct detection, and other quantities. The issue here is that *CalcHEP* can only consider processes at the tree level. Therefore, without external help, it cannot compute the relevant quantities needed for our model.

The model, without the loop-induced processes, was implemented in a *LanHEP* file [53], which allows us to convert it into *CalcHEP* format files. With this, *micrOMEGAs* can only calculate direct detection processes involving the dark-charged $V^\pm$ bosons. We then modified the *LanHEP* file to include the functions $C_0^{(dd)}$ and $C_1^{(dd)}$, as well as the Wilson coefficients of the effective Lagrangian, which implements the effective vertex $\psi^a \bar{\psi}^a \tilde{\phi}$. Now, *micrOMEGAs* can compute the processes of elastic scattering between dark fermions and nucleons.

As mentioned in the previous chapter, we cannot obtain an effective operator to include the box diagrams due to the Lorentz structure of the amplitude. Luckily for us, *micrOMEGAs* has an external routine called Improved Cross Section, which allows us to implement loop-induced processes numerically. It works as follows: with the aid of *LanHEP*, we tell *micrOMEGAs* that the annihilation processes $\bar{\psi}^a \psi^a \rightarrow XX$ exist, with $X = \phi, h, W^\pm, Z, t$, and that they are proportional to some dummy constant g_{dummy} , which we set to zero. Then, we provide tables with different values of the cross-section as a function of the incoming 3-momenta p_1 for each process. These values are calculated numerically using an external C code that employs *LoopTools* [54] to calculate the scalar Passarino-Veltman functions. The Improved Cross Section routine assigns each table to its corresponding process, replacing the zero value of the cross-section with the value we have calculated.

6.2 Analysis

In this section, we are going to analyze the behavior of the cross section for different processes as a function of the free parameters of the model, which, just to remind you, are the masses M_F , M_V , and M_ϕ , the dark coupling constant g_D , and the mixing angle of the scalars θ_D , specifically $\sin(\theta_D)$.

6.2.1 Trivial dependence on coupling and scalars mixing angle

We begin our analysis by checking the dependence of the cross sections calculated with *micrOMEGAs* and our external code. From Figures 6.1 and 6.2, we can see that the cross sections scale trivially when we increase the coupling constant, as expected. These graphs are in logarithmic scale, so the slope of the curve tells us the exponent of the g_D dependence. We also plotted this for different values of the sine of the scalar mixing angle θ_D in the case of Figure 6.1 and the left panel of Figure 6.2. In these figures, we can see that, because the dark fermions remain degenerate, the graphs for their elastic scattering with nucleons are identical and vary

very little with the increase of $\sin(\theta_D)$. This contrasts with the scattering of V bosons with nucleons, where the curves are more separated. All three of these graphs were obtained with *micrOMEGAs*, and their cross section values increase with the mixing angle, as expected.

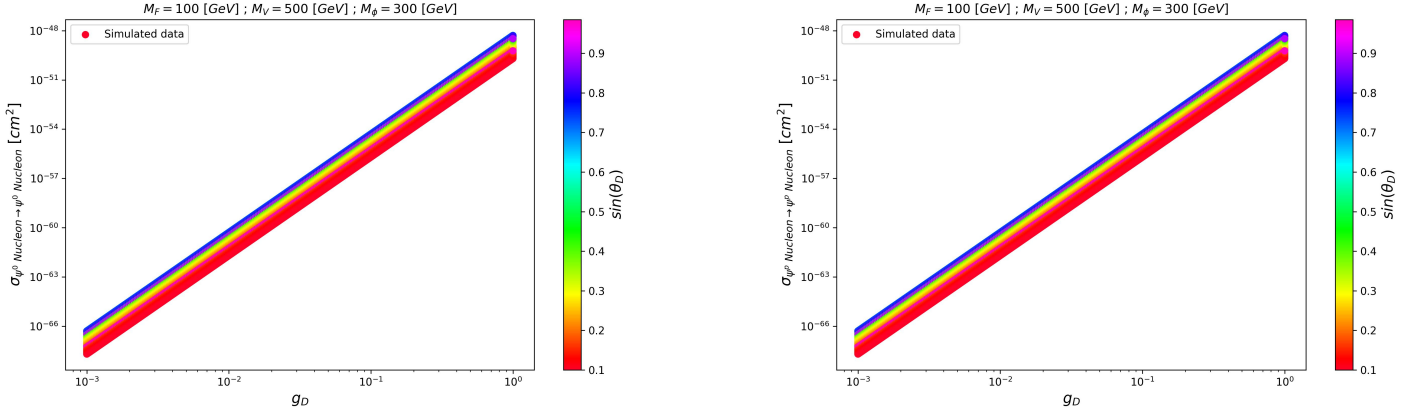


FIGURE 6.1: (a) Cross section of elastic scattering $\psi^0 \text{Nucleon}$ as a function of the dark coupling and the sine of the mixing angle. (b) Cross section of elastic scattering $\psi^+ \text{Nucleon}$ as a function of the dark coupling and the sine of the mixing angle.

In Figures 6.3, 6.4, and 6.5, we have plotted the $\sin(\theta_D)$ dependence of the annihilation cross sections as a function of p_1^1 , which were calculated using the external LoopTools C-code. As you can see, the plots in the right panel of Figure 6.4 and the two graphs in Figure 6.5 are identical in form, varying only in their cross section values. This is due to the fact that their diagram structure, from the side of the dark sector, is exactly the same: the loop-triangle. The only difference is the coupling to the scalar, which depends on the SM particle. Apart from that, they all scale with the increasing value of $\sin(\theta_D)$ as well.

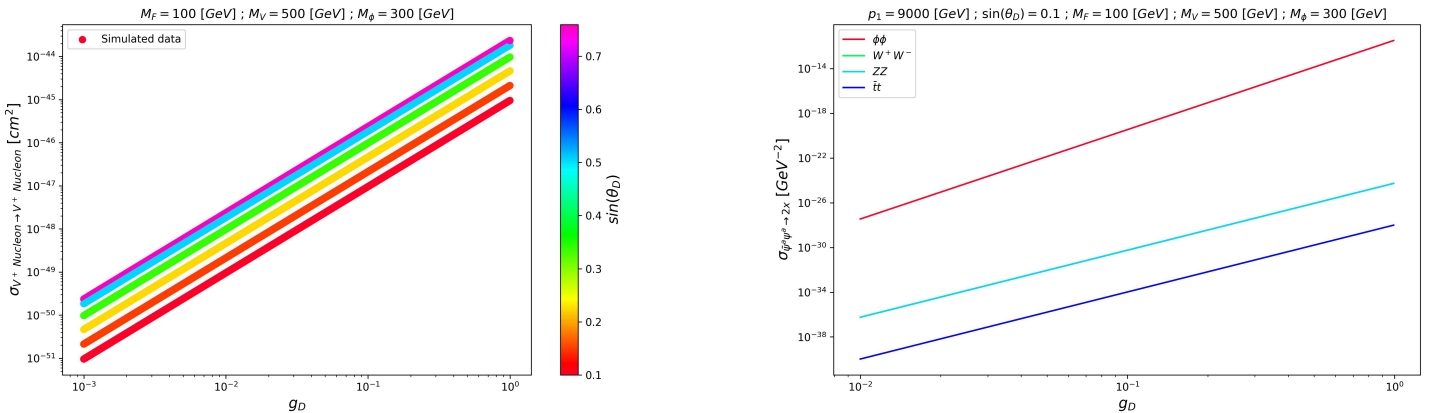


FIGURE 6.2: (a) Cross section of elastic scattering $V^+ \text{Nucleon}$ as a function of the dark coupling and the sine of the mixing angle. (b) Cross section of different dark fermion annihilations into scalar and SM final states, as a function of g_D .

¹The magnitude of the 3-momenta belonging to one of the incoming dark fermions.

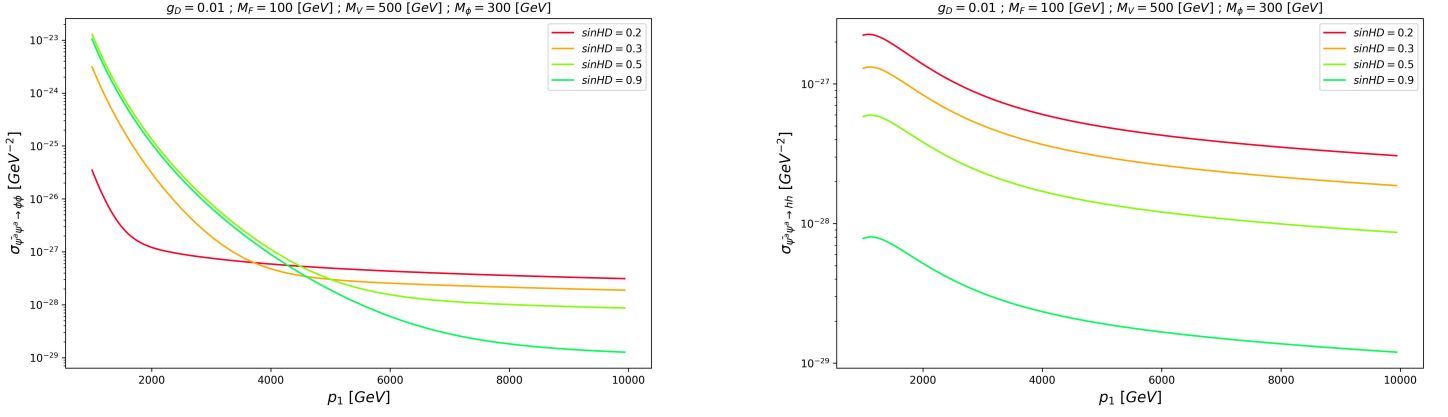


FIGURE 6.3: Cross section of dark fermion annihilation into two dark higgs (a) and SM higgs (b), as a function of p_1 for different values of the sine of their mixing angle.

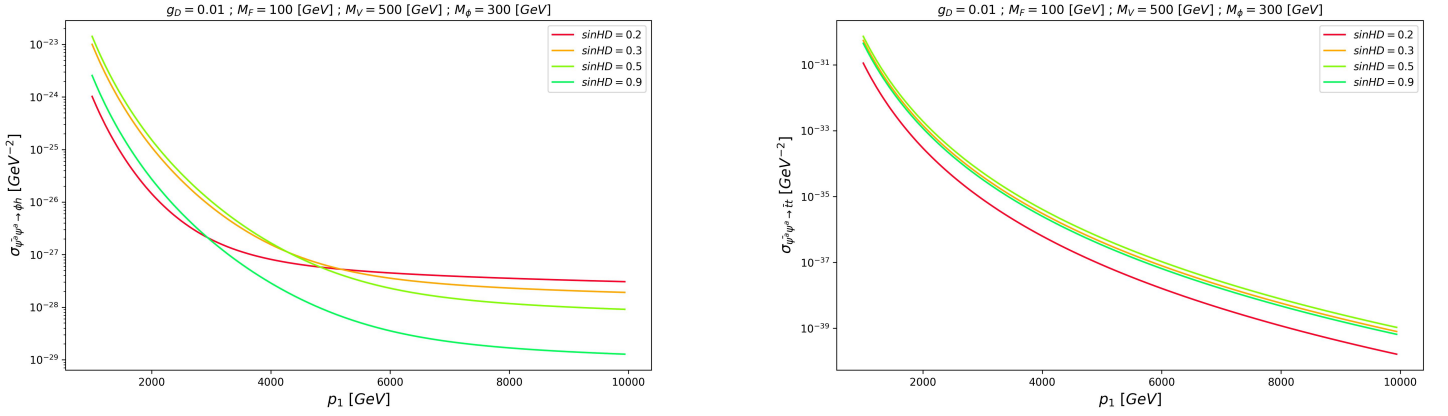


FIGURE 6.4: Cross section of dark fermion annihilation into each one of the scalars (a) and a pair top antitop quark (b), as a function of p_1 for different values of the sine of their mixing angle.

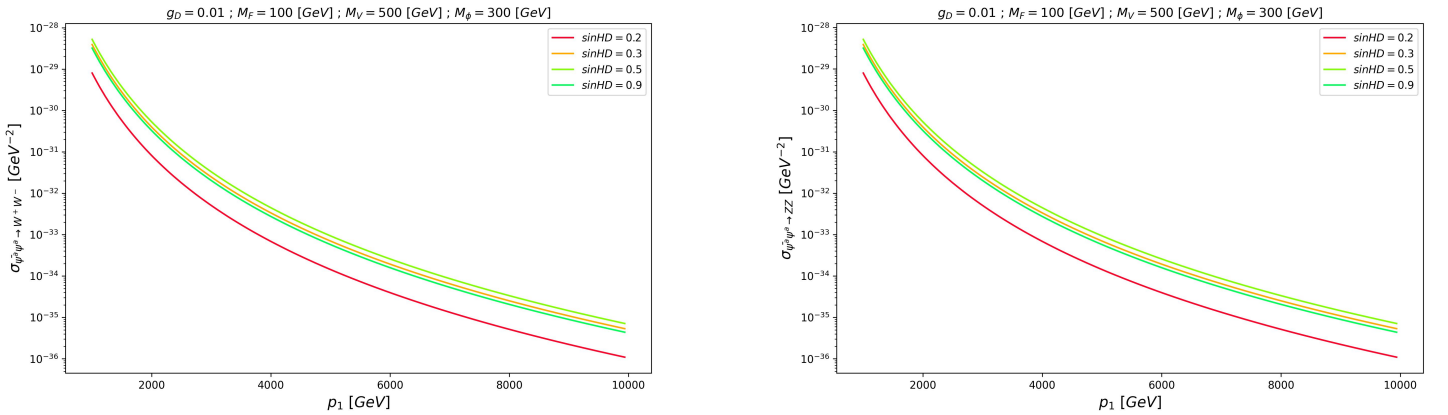


FIGURE 6.5: Cross section of dark fermion annihilation into W (a) and Z bosons (b), as a function of p_1 for different values of the sine of their mixing angle.

In the case of annihilation into scalars, the dependence is somewhat less trivial. For $\phi\phi$ and ϕh (left panels of Figures 6.3 and 6.4, respectively), there is a change in the behavior of the cross section when the incoming

3-momentum reaches values around 5000 [GeV], at which point the cross section decreases for higher values of $\sin(\theta_D)$. For hh production, the cross section decreases as the mixing angle increases, as seen in the right panel of Figure 6.3. These behaviors may seem strange when compared to the production of SM particles. However, the mixing angle mediates which scalar dominates the interactions, so the higher the value of $\sin(\theta_D)$, the greater the predominance of the corresponding scalar. From a physical perspective, we expect that the WIMPs interact very weakly with the rest of the visible sector, so the value of the mixing angle must be such that the standard Higgs boson dominates the interactions. Therefore, we set $\sin(\theta_D) = 0.1$ and $g_D = 0.01$ for the rest of this analysis.

6.2.2 Direct Detection

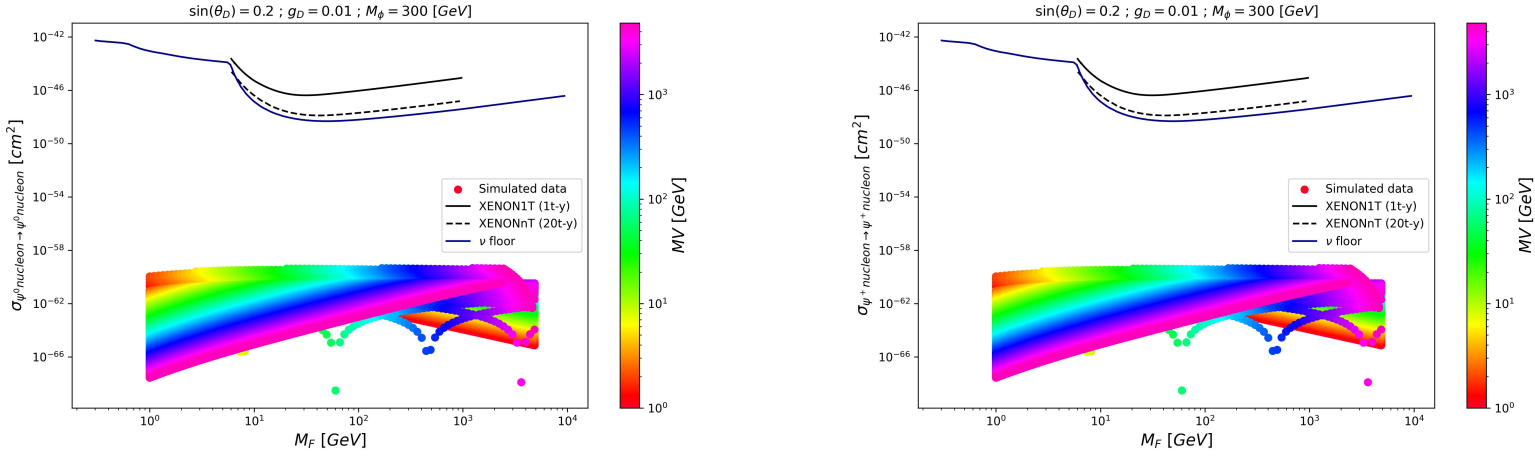


FIGURE 6.6: Elastic scattering cross section of dark fermions ψ^0 (a) and ψ^+ (b) with nucleons, as a function of the dark fermion mass M_F , for different values of the vector mass M_V , plotted against the exclusion curves of XENON1T and XENONnT.

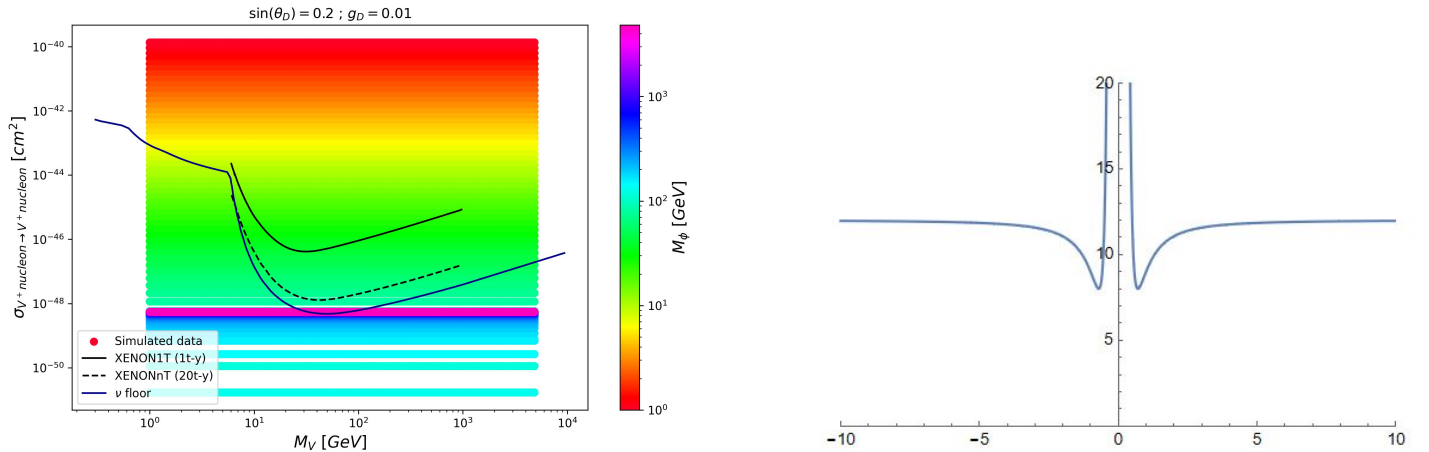


FIGURE 6.7: (a) Elastic scattering of V^+ with nucleons as a function of their mass, for different values of the mass of the dark scalar M_ϕ , plotted against the exclusion curves of XENON1T [55] and XENONnT [56]. (b) Behavior of squared amplitude as a function M_V , for $t = 1$.

Next in our analysis, we compute the elastic scattering processes between the three species of dark matter and nucleons, and contrast them with data from the direct detection experiments XENON1T [55] and XENONnT

[56]. In Figure 6.6, we present the dark fermion elastic scattering with nucleons as a function of their mass and for different values of M_V , which is identical for both ψ^0 and ψ^+ . The simulated data is well below the neutrino floor², at least seventeen orders of magnitude lower. These small values are what we should expect from loop-induced processes and allow us to adjust the masses of these particles more freely. However, it will be a difficult task for our experimental colleagues to detect these particles.

In the left panel of Figure 6.7, we can see the plot for the scattering of V^+ with nucleons as a function of the vector mass, for different values of the dark Higgs mass. Here, the simulated data falls within the same region as the exclusion curves, so values of M_ϕ below ~ 20 [GeV] are excluded by XENON1T, and values below ~ 70 [GeV] are excluded by XENONnT. Interestingly, the dependence on the vector mass remains constant. If we examine the squared amplitude for the elastic scattering of $V^\pm$ with any quark q^3 , we observe that the dependence on M_V behaves as:

$$|\mathcal{M}|^2 \sim \frac{12M_V^4 - 4M_V^2 t + t^2}{M_V^4} \quad (6.1)$$

This graph can be found in the right panel of Figure 6.7. We see that it becomes constant for the first values of M_V and remains constant as $M_V \rightarrow \infty$. We conclude that direct detection only provides a lower limit for the value of M_ϕ .

6.2.3 Mass dependence in annihilation processes

Now, we analyze the plots obtained with our external C-code using *LoopTools* for the annihilation of a dark fermion-antifermion pair into scalars and SM particles. These plots are valid for any of the three fermions, labeled simply as ψ^a . In each figure, we present the two kinematic regimes: where $M_V > M_F$ and $M_V < M_F$, corresponding to the left and right panels, respectively. We vary the order of magnitude of both masses, from 10^{-3} to 10^3 [GeV].

For the scalar final state (Figures 6.8, 6.9, and 6.10), the plots appear almost identical between the cases $\phi\phi$, hh , and ϕh . We observe that as the order of magnitude of the masses increases, the value of the cross section decreases. Only in the cases where the masses are on the order of $\sim 10^2$ and $\sim 10^3$ [GeV], for the regime $M_V > M_F$, and where the masses are on the order of $\sim 10^3$ [GeV], for the regime $M_V < M_F$, does the cross section become higher than the others below a certain value of p_1 , depending on the final state. There is also a strange peak in the case of $\sim 10^3$ [GeV], for $M_V < M_F$, around $p_1 \sim 5000$ [GeV], which corresponds to the mass of the V boson in this case. This could be some form of resonance, possibly arising from the contribution of the box loop diagrams.

²This is where coherent neutrino scattering of nuclei becomes a relevant background

³*micrOMEGAs* deals with the effective part of the nuclear structure.

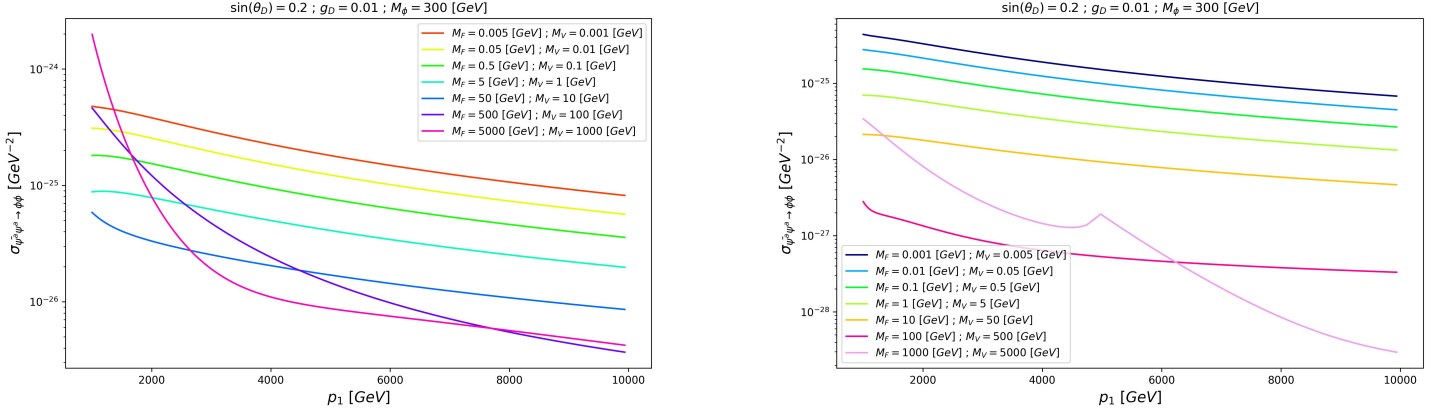


FIGURE 6.8: Cross section for annihilation of dark fermions into dark scalars as a function of p_1 , for $M_V > M_F$ (a) and $M_V < M_F$ (b).

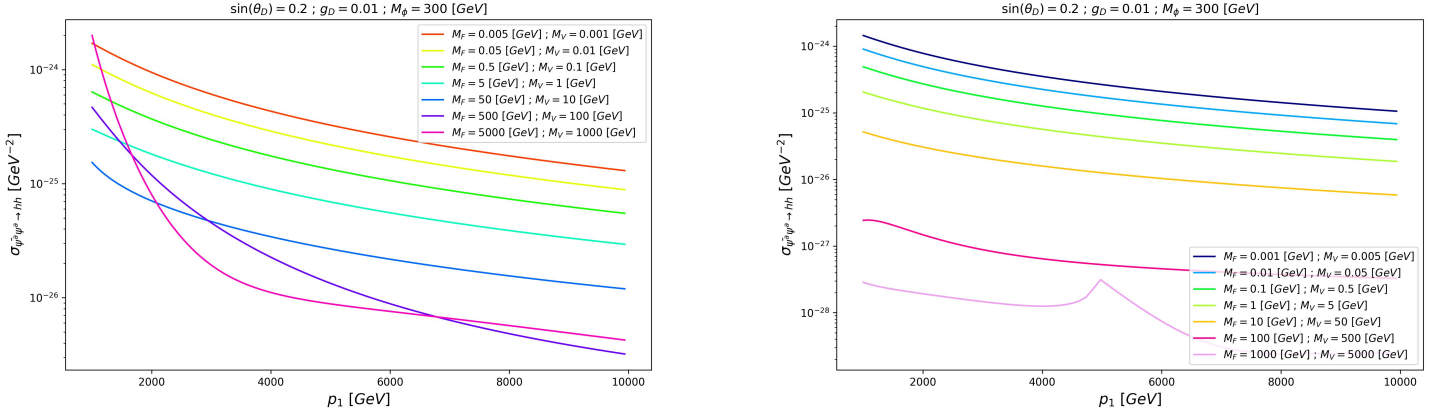


FIGURE 6.9: Cross section for annihilation of dark fermions into SM scalars as a function of p_1 , for $M_V > M_F$ (a) and $M_V < M_F$ (b).

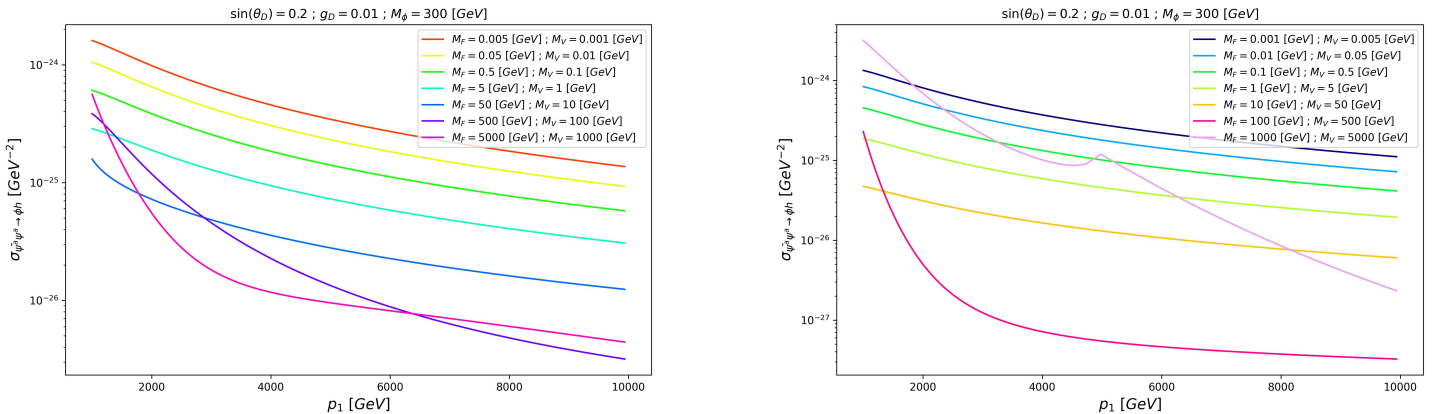


FIGURE 6.10: Cross section for annihilation of dark fermions into a SM and a dark scalars as a function of p_1 , for $M_V > M_F$ (a) and $M_V < M_F$ (b).

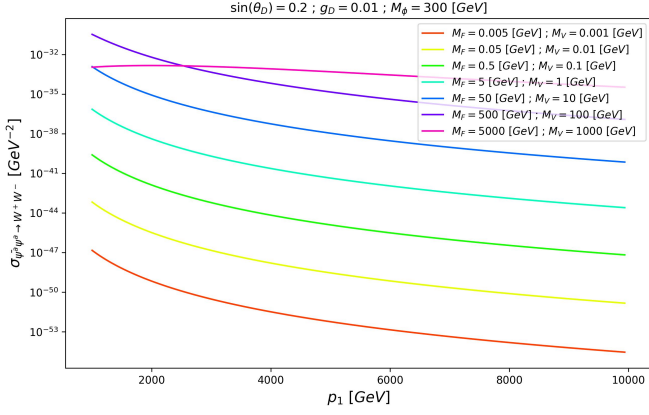
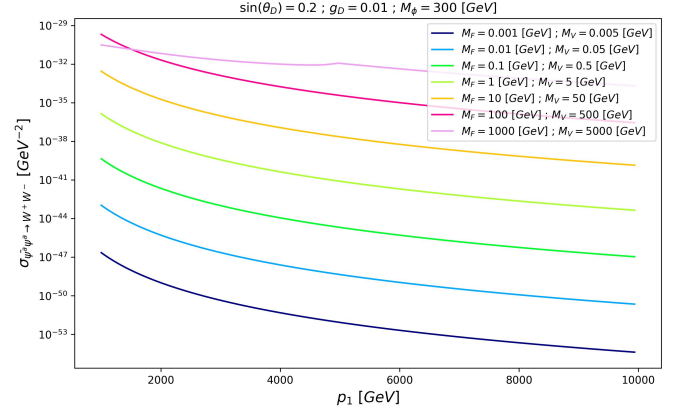


FIGURE 6.11: Cross section for annihilation of dark fermions into W bosons as a function of p_1 , for $M_V > M_F$ (a) and $M_V < M_F$ (b).

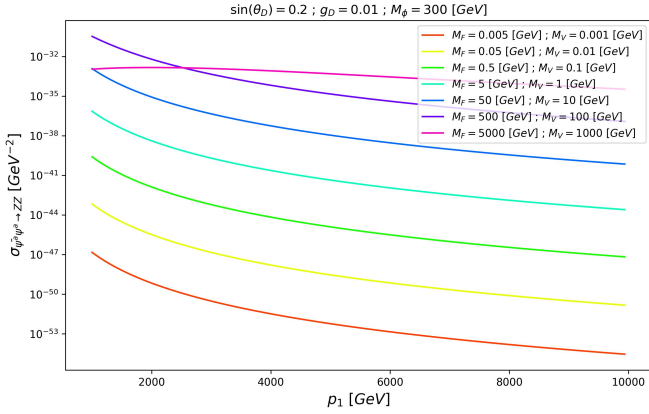
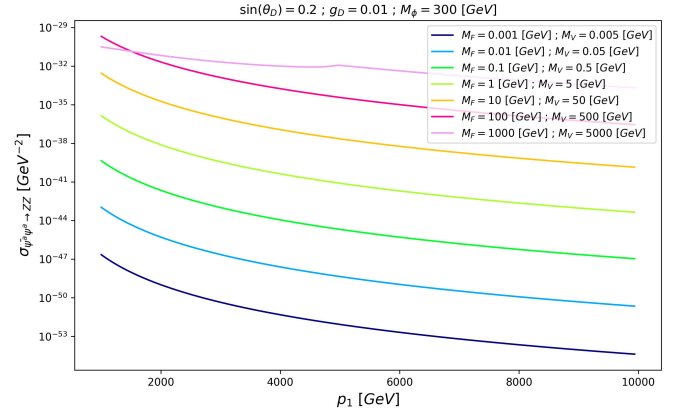


FIGURE 6.12: Cross section for annihilation of dark fermions into Z bosons as a function of p_1 , for $M_V > M_F$ (a) and $M_V < M_F$ (b).

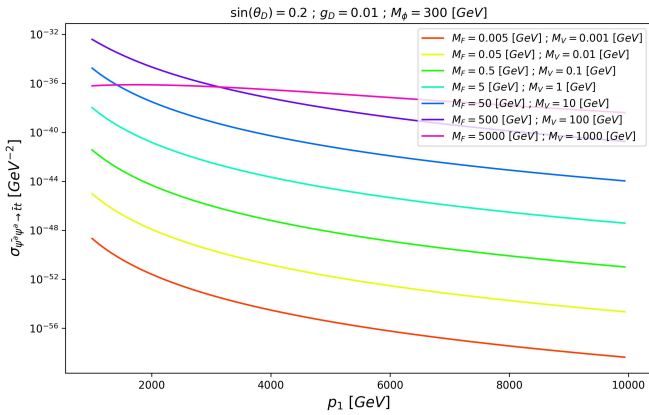
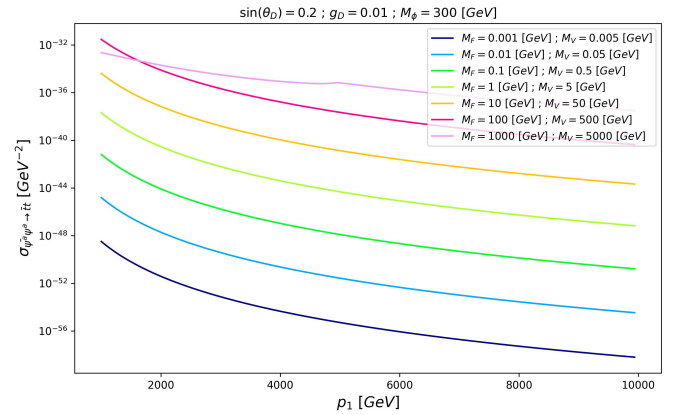


FIGURE 6.13: Cross section for annihilation of dark fermions into a pair $t \bar{t}$ as a function of p_1 , for $M_V > M_F$ (a) and $M_V < M_F$ (b).

The plots in Figures 6.11, 6.12, and 6.13 share almost the same form, particularly those corresponding to the weak boson final states. Unlike the scalars, the cross section increases as the order of magnitude of the

masses increases. Additionally, all the curves behave similarly, except for those with masses on the order of $\sim 10^3$ [GeV], in both regimes.

It is important to remind that, as we saw in Chapter 2, the higher the value of the cross section, the lower the value of the relic density. Finally, it is important to mention that all cross sections, in both regimes, are suppressed by large values of p_1 .

6.3 Future work and projections

Despite the fact that we have set the first limits on the non-trivial part of the parameter space, setting a lower value for the mass of the dark Higgs boson, depending on the direct detection experiment, and we have verified that the cross sections for loop-induced annihilations behave correctly, we still cannot compute the relic density calculations. This is due to the fact that we have a process left to implement into the programs, which is $\bar{\psi}^0 \psi^0 \rightarrow V^0 V^0$. This process cannot occur at tree-level because there is no vertex that includes $\bar{\psi}^0 \psi^0$. Remember that these fermions belong to a triplet in the adjoint representation of $SU(2)_D$, which changes the form of the covariant derivative, adding a commutator that mixes the different components of the triplet. Thus, this process can only occur from next-to-leading order diagrams. This adds seven new diagrams, some of them simpler and similar in structure to the ones we treated in this thesis, and others with a new level of complexity, which add Passarino-Veltman functions of three and four propagators of rank 2 or even 3. This is still a work in progress.

On the other hand, collider physics can be analyzed in the future because the neutral vector V^0 is not charged under the dark Z_2 group, so it can decay via three loops (see Figures 6.14 and 6.15 below) and can be a long-lived particle, which can be detected as missing energy in the detectors.

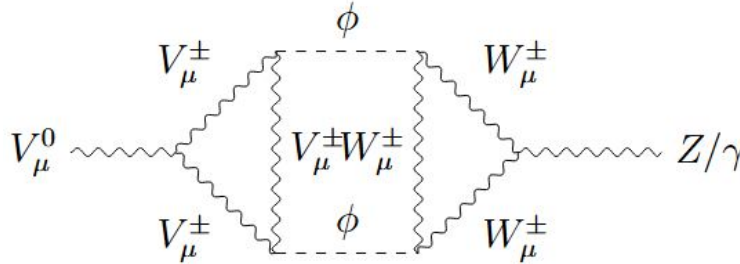


FIGURE 6.14: Decay of V^0 with loop of $W^\pm$ bosons.

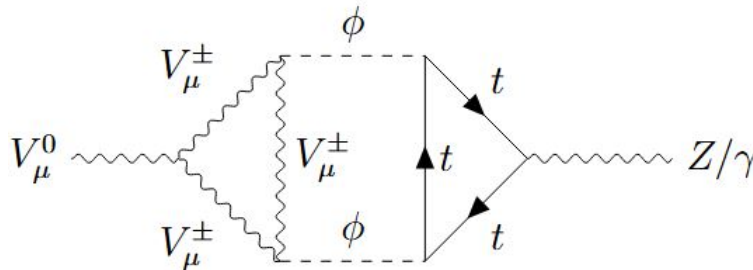


FIGURE 6.15: Decay of V^0 with loop of t bosons.

Principle remarks and conclusions

In this thesis, we successfully introduced a dark matter model that extends the Standard Model of particle physics by including three new fields and three new potential dark matter species. We managed to decompose the diagrams of loop-induced processes using the mathematical techniques of the Passarino-Veltman reduction algorithm.

On the one hand, the algorithm sometimes produces enormous expressions with many terms, which we had to treat carefully in order to perform the algebra. However, the positive side is that it allows us to fully program the mathematics behind the Feynman diagrams in an efficient way, enabling the computation of different cross sections. As a result, we were able to implement the model almost completely, introducing an effective Lagrangian that allowed us to incorporate analytically the elastic scattering of dark fermions into scalars and SM particles in *LanHEP*. Furthermore, we managed to numerically include the dark fermion annihilation processes in *micrOMEGAs*, with the creation of a C-code that incorporates *LoopTools* to compute numerically the Passarino-Veltman scalar integrals. The external routine was also used to improve the cross section calculations.

As a result, we performed the first analysis of the model and examined the behavior of the different processes and their dependence on the free parameters of the parameter space. We found that the cross sections scale trivially with the dark coupling constant g_D and the sine of the scalar mixing angle θ_D , as expected. On the other hand, we took the first steps toward obtaining the relic density predicted for this model by plotting and analyzing the loop-induced annihilation cross sections, all of which behaved as expected. Finally, using the effective Lagrangian approach, we were able to compute the direct detection processes for our dark matter species with the aid of *micrOMEGAs*, concluding that we have ample freedom to set masses for the fermions and vectors, and that the mass of the dark scalar is bounded from below at ~ 20 [GeV] according to XENON1T and at ~ 70 [GeV] for XENONnT.

There is still much work to be done. In the future, we will include the diagrams for the process $\bar{\psi}^0 \psi^0 \rightarrow V^0 V^0$, which will introduce additional algebraic complexities, but will allow us to finally compute the relic density for this model. Other future work includes finding other experiments and processes to better constrain the parameter space and impose limits on the values of the masses. Without further ado, I thank all of you for having reached this point. Until next time.

Feynman Rules of the model

Here we present the Feynman rules of the model, disposed with their corresponding vertex. For the case of scalar, we only include the three-legged vertices.

A.1 Fermions

$$\begin{aligned} \psi^\pm \text{ --- } \swarrow \quad \nwarrow \text{ --- } \bar{\psi}^0 \quad \text{---} \quad \text{wavy line } V_\mu^\mp &= \pm \frac{1}{2} g_D \gamma^\mu \\ \psi^\pm \text{ --- } \swarrow \quad \nwarrow \text{ --- } \bar{\psi}^\pm \quad \text{---} \quad \text{wavy line } V_\mu^0 &= \mp \frac{1}{2} g_D \gamma^\mu \end{aligned}$$

FIGURE A.1: *Dark fermions vertices.*

A.2 Bosons

$$V_\mu^+ \text{ --- } \swarrow \quad \nwarrow \text{ --- } V_\nu^- \quad \text{---} \quad \text{wavy line } V_\rho^0 = g_D [g^{\nu\rho}(p_3 - p_2)^\mu + g^{\mu\rho}(p_1 - p_3)^\nu + g^{\mu\nu}(p_2 - p_1)^\rho]$$

FIGURE A.2: *Three-legged boson vertex.*

$$\begin{aligned}
 & \text{Four-legged gauge vertex: } V_\mu^+, V_\rho^{0,-}, V_\nu^-, V_\sigma^{0,+} \text{ connected by wavy lines} \\
 & \quad = -g_D^2 (2g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}) \\
 & \text{Three-legged Goldstone boson vertex: } V_f^{+,0,0}, V_f^{-,-,+}, V_\mu^{0,+,-} \text{ connected by dashed lines} \\
 & \quad = -\frac{1}{2}g_D(p_3 - p_2)^\mu \\
 & \quad = \frac{1}{2}g_D(p_3 - p_1)^\mu \\
 & \quad = \frac{1}{2}g_D(p_1 - p_2)^\mu
 \end{aligned}$$

FIGURE A.3: Four-legged gauge and three-legged Goldstone boson vertex.

$$\begin{aligned}
 & \text{Left vertex: } V_C^{0,\pm}, V_c^{\pm,\mp}, V_\mu^{\mp,0} \text{ connected by dotted lines} \\
 & \quad = \mp g_D(p_2 + p_3)^\mu \\
 & \text{Right vertex: } V_C^{0,\pm}, V_c^{\pm,\mp}, V_f^{\mp,0} \text{ connected by dotted lines} \\
 & \quad = \pm \frac{i}{2}g_D M_V
 \end{aligned}$$

FIGURE A.4: Ghost particles vertices

$$\begin{aligned}
 & \text{Left vertex: } V_\mu^{0,+,0,+}, V_\nu^{0,-,0,-}, V_f^{0,-,-,0}, V_f^{0,+,+,0} \text{ connected by wavy lines} \\
 & \quad = \frac{1}{2}g_D^2 g^{\mu\nu} \\
 & \text{Right vertex: } V_\mu^{0,+,0}, V_\nu^{0,-,0}, V_f^{0,-,-}, V_f^{0,+,+} \text{ connected by dashed lines} \\
 & \quad = -6\lambda_D \\
 & \quad = -4\lambda_D \\
 & \quad = -2\lambda_D
 \end{aligned}$$

FIGURE A.5: Four-legged vertices with gauge and Goldstone bosons.

A.3 Bosons and scalars

$$\begin{array}{ccc}
 \begin{array}{c} V_{\mu}^{0,+} \\ \diagdown \\ \text{---} \\ \diagup \\ V_{\nu}^{0,-} \end{array} & \begin{array}{c} \text{---} \phi, h \\ = \\ M_V g_D \cos(\theta_D) g^{\mu\nu} \\ - M_V g_D \sin(\theta_D) g^{\mu\nu} \end{array} & \\
 \begin{array}{c} V_{\mu}^{0,+} \\ \diagdown \\ \text{---} \\ \diagup \\ V_f^{0,-} \end{array} & \begin{array}{c} \text{---} \phi, h \\ = \\ -\frac{i}{2} g_D \cos(\theta_D) (p_2 - p_3)^{\mu} \\ -\frac{i}{2} g_D \sin(\theta_D) (p_2 - p_3)^{\mu} \end{array} &
 \end{array}$$

FIGURE A.6: Three-legged vertices involving gauge and Goldstones bosons, with scalars.

$$\begin{array}{ccc}
 \begin{array}{c} V_f^{0,+} \\ \diagdown \\ \text{---} \\ \diagup \\ V_f^{0,-} \end{array} & \begin{array}{c} \text{---} \phi, h \\ = \\ 2 \sin(\theta_D) \lambda_D v_D - \cos(\theta_D) \lambda_{h\phi} v_S \\ -2 \cos(\theta_D) \lambda_D v_D - \sin(\theta_D) \lambda_{h\phi} v_S \end{array} & \\
 \begin{array}{c} V_C^{0,+} \\ \diagdown \\ \text{---} \\ \diagup \\ V_c^{0,-} \end{array} & \begin{array}{c} \text{---} \phi, h \\ = \\ -\frac{1}{2} M_V g_D \cos(\theta_D) \\ \frac{1}{2} M_V g_D \sin(\theta_D) \end{array} &
 \end{array}$$

FIGURE A.7: Three-legged vertices involving ghost particles and Goldstones bosons, with scalars.

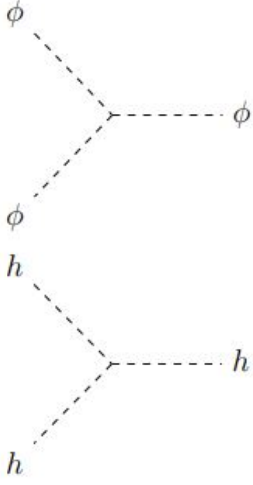
$$\begin{array}{ccc}
 \begin{array}{c} V_{\mu}^{0,+} \\ \diagdown \\ \text{---} \\ \diagup \\ V_{\nu}^{0,-} \end{array} & \begin{array}{c} \text{---} \phi, h \\ = \\ \frac{1}{2} g_D^2 \cos^2(\theta_D) g^{\mu\nu} \\ \frac{1}{2} g_D^2 \sin^2(\theta_D) g^{\mu\nu} \end{array} & \\
 \begin{array}{c} V_{\mu}^{0,+} \\ \diagdown \\ \text{---} \\ \diagup \\ V_{\nu}^{0,-} \end{array} & \begin{array}{c} \text{---} \phi \\ = \\ -\frac{1}{2} g_D^2 \cos(\theta_D) \sin(\theta_D) g^{\mu\nu} \end{array} &
 \end{array}$$

FIGURE A.8: Four-legged vertices involving gauge bosons, with scalars.

$$\begin{array}{ccc}
 \begin{array}{c} V_f^{0,+} \\ \diagdown \\ \text{---} \\ \diagup \\ V_f^{0,-} \end{array} & \begin{array}{c} \text{---} \phi, h \\ = \\ -2\lambda_D \cos^2(\theta_D) - \lambda_{\phi h} \sin^2(\theta_D) \\ -2\lambda_D \sin^2(\theta_D) - \lambda_{\phi h} \cos^2(\theta_D) \end{array} & \\
 \begin{array}{c} V_f^{0,+} \\ \diagdown \\ \text{---} \\ \diagup \\ V_f^{0,-} \end{array} & \begin{array}{c} \text{---} h \\ = \\ (2\lambda_D - \lambda_{\phi h}) \cos(\theta_D) \sin(\theta_D) \end{array} &
 \end{array}$$

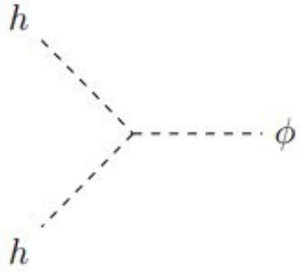
FIGURE A.9: Four-legged vertices involving Goldstones bosons, with scalars.

A.4 Scalars only

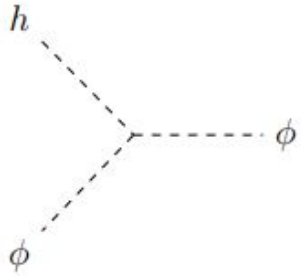


$$\begin{aligned}
 \phi \quad \phi &\quad \phi = -3[2\lambda_S v_S \sin^3(\theta_D) + 2\lambda_D v_D \cos^3(\theta_D) + \lambda_{h\phi} \sin(\theta_D) \cos(\theta_D)(v_S \cos(\theta_D) + v_D \sin(\theta_D))] \\
 h \quad h &\quad h = -3[2\lambda_s v_s \cos^3(\theta_D) - 2\lambda_D v_D \sin^3(\theta_D) + \lambda_{h\phi} \sin(\theta_D) \cos(\theta_D)(v_S \sin(\theta_D) - v_D \cos(\theta_D))]
 \end{aligned}$$

FIGURE A.10: Vertices with only one type of scalar.



$$\begin{aligned}
 h \quad h &\quad \phi = \\
 &\quad v_D \lambda_{h\phi} \cos(\theta_D)(3 \sin^2(\theta_D) - 1) + v_S \lambda_{h\phi} \sin(\theta_D)(2 - 3 \sin^2(\theta_D)) \\
 &\quad - 6v_D \lambda_D \cos(\theta_D) \sin^2(\theta_D) - 6v_S \lambda_S \cos^2(\theta_D) \sin(\theta_D)
 \end{aligned}$$



$$\begin{aligned}
 h \quad \phi &\quad h = \\
 &\quad v_s \lambda_{h\phi} \cos(\theta_D)(3 \sin^2(\theta_D) - 1) + v_D \lambda_{h\phi} \sin(\theta_D)(2 - 3 \sin^2(\theta_D)) \\
 &\quad + 6v_D \lambda_D \cos^2(\theta_D) \sin(\theta_D) - 6v_s \lambda_S \cos(\theta_D) \sin^2(\theta_D)
 \end{aligned}$$

FIGURE A.11: Vertices mixing the scalars.

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