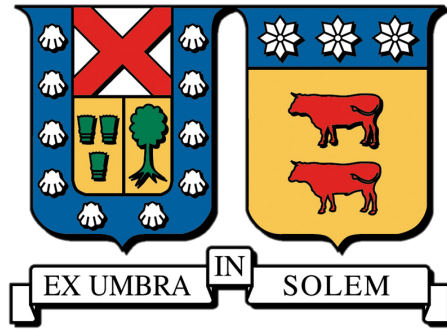




UNIVERSIDAD TÉCNICA FEDERICO SANTA MARÍA
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Spin wave spectra for ferromagnetic film

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A thesis submitted in partial fulfillment of the requirements for the degree of
Master in science, mention physics

August, 2024

Abstract

The fundamental magnetic excitations, spin waves, are capable of transporting energy and angular momentum without the movement of electric charge, implying the reduction of heat energy losses. Besides, the wavelengths of the spin waves are much shorter than the electromagnetic waves at the same frequency, which brings new perspectives to the miniaturization of magnonic devices. Thus, the study of the dynamic of the spin in nanostructured magnetic materials is a topic with great relevance in the area of nanomagnetism. This project studies different types of multilayered ferromagnetic systems, with emphasis in the nonreciprocal spin-wave phenomena. The results obtain in this thesis show that contribution of noncollinear magnetic moments may mimic the role of the biquadratic interaction in the macrospin model, implying that such a biquadratic term may be overestimated in coupled ferromagnetic films with thicknesses exceeding the material's intrinsic exchange length. By the other hand, in multilayered synthetic antiferromagnetics, an asymmetric spin-wave dispersion occurs in structures with an even number of layers, while a reciprocal dispersion of two counterpropagating waves is observed for systems with an odd number of layers. As both the number of layers and their thickness increase, dynamic dipolar interactions give rise to distinct surface modes, which arises between the bulk modes. The behavior of these surface modes suggests certain topological features of the modes.

Resumen

Las excitaciones magnéticas fundamentales, las ondas de espín, son capaces de transportar energía y momento angular sin el movimiento de carga eléctrica, lo que implica la reducción de pérdidas de energía por calor. Además, las longitudes de onda de las ondas de espín son mucho más cortas que las de las ondas electromagnéticas a la misma frecuencia, lo que aporta nuevas perspectivas para la miniaturización de dispositivos magnónicos. Por lo tanto, el estudio de la dinámica del espín en materiales magnéticos nanoestructurados es un tema de gran relevancia en el área de nanomagnetismo. Este proyecto, a través del método de matriz dinámica, estudia diferentes tipos de sistemas ferromagnéticos, con énfasis en la comprensión desde las propiedades estáticas hasta los fenómenos no recíprocos de las ondas de espín. Los resultados obtenidos en esta tesis muestran que la contribución de momentos magnéticos no colineales puede imitar el papel de la interacción biquadrática en el modelo de macrorspin, lo que implica que dicho término biquadrático puede estar sobreestimado en películas ferromagnéticas acopladas con espesores que superan la longitud de intercambio intrínseca del material. Por otro lado, una dispersión asimétrica de ondas de espín ocurre en estructuras con un número par de capas, mientras que una dispersión recíproca de dos ondas contrapropagantes se observa en sistemas con un número impar de capas. A medida que aumentan tanto el número de capas como su espesor, las interacciones dipolares dinámicas dan lugar a modos de superficie distintos.

Acknowledgements

A mi familia por su amor y apoyo en todo momento durante estos años en la universidad, que han habido altos y bajos. A mi profesor guía Rodolfo Gallardo por su apoyo no-acotado, su paciencia y dedicación en estos tres años de trabajo. A mis amigos por su cariño y la vida compartida, haciendo que la vida sea mucho más amena. A Kitty, compañera fiel de noches de estudio, siestas por la tarde y del día a día. Te amé toda tu vida y seguiré amándote por el resto de la mía. Finalmente, agradecimiento al proyecto FONDECYT Regular N°1210607 que colaboró en este trabajo.

*"Veja o mar que faz com areia,
Passa por cima não machuca nem juiz."*
Mestre Chinaxé

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Chapter 1

Introduction

1.0.1 Magnetism and nanostructures

Magnetism has been present and it has been useful to the development of human civilization. The research and fascination for this area started in ancient world with the discovery of lodestones, which attracts some pieces of iron [1]. In ancient Greece, Aristotle and Thales of Miletus could have the first scientific discussion about magnetism [2]. In India, it was found the medical text *Sushruta Samhita*, an elderly book which described the use of magnetite to remove arrows embedded in person's body [3]. The invention of the magnetic compass enabled the great voyages and thereby the age of discovery. The invention of the dynamo eventually led to the large-scale production of electricity and all the benefits electricity brought with it. High-density magnetic data storage development paved the way for today's society with its abundance of information. Other applications like magnetocaloric refrigeration or high-power electromotors have also evolved from research in magnetism. Today, magnetism has penetrated practically all branches of science and technology.

Nanomagnetism [4] is the sub-branch of magnetism that concerns the study of the properties of magnetic materials at ever increasingly smaller dimensions. Research in nanomagnetism aims to manipulate, control and exploit magnetic phenomena on the nanometer scale. Questions of interest in this scientific domain are how the magnetization adapts to finite-size effects arising in low-dimensional systems, such as, e.g. thin films, nanowires, or nanodots. Over the past decades, extensive studies have investigated ways to use external stimuli to control the magnetization in such systems. Methods to manipulate the magnetization on the nanoscale include the application of magnetic fields, spin-polarized currents, or approaches exploiting other coupling mechanisms like the magnetoelectric effect.

Along with scientific curiosity, nanomagnetism research has traditionally been motivated by the rapid progress in information technology and its demand for low-power, high-density, and non-volatile storage media [5]. This demand skyrocketed with the discovery of the giant magnetoresistance effect, which gave the Nobel Prize in Physics to Grünberg and Fert [6, 7]. This response, along with the tunnel magnetoresistance effect, describes a significant change of the electric resistance in a magnetic multilayer system depending on the relative orientation of the magnetization direction of the layers [8, 9]. These discoveries started the field of spintronics [10–12], in which the interaction of the electron spin and the magnetization is of central importance. These studies led to a trend in nanomagnetism where much of the research was focused on thin films and interface effects. In addition to its relevance for spintronic applications, the scientific interest in nanomagnetism extends far beyond the desire to improve device miniaturization and other technological aspects.

Most of these studies on the shape and size dependence in nanomagnetism concerned patterned thin film elements of varying thickness [4, 14]. However, it can be extended into third dimension structures which gives type of magnetization structures such as multilayers, nanotubes, nanorings, 3D vortex domain walls, Bloch point structures, skyrmions, among others (see Fig. 1.1).

At nanometer scales, the magnetization is prone to be uniform. Nevertheless, under specific conditions, the magnetization can present a texture, at small fields, in such a way that it can depend on the space. Such magnetic textures can be influenced by non-uniform anisotropies or interfacial interaction, like the Dzyaloshinskii-Moriya interaction, which can induce the formation of helicoidal and skyrmionic structures [15–17]. Also, in some materials, stripe domains are formed

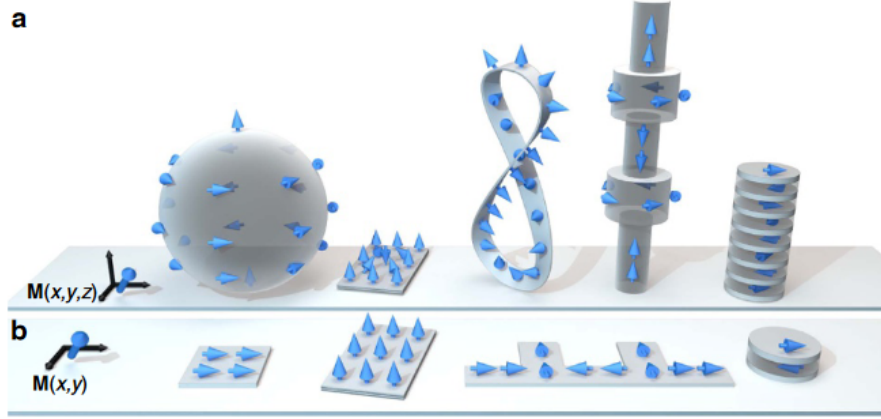


Figure 1.1: Figure from [13]. Schematic view comparing some examples of geometries and magnetic configurations (indicated by blue arrows) for (a) 3D and (b) 2D nanomagnetism. The dependence of the magnetization $\mathbf{M}$ on spatial coordinates (black arrows) is indicated for both cases. New synthesis, characterization and computational methods have the potential to make the leap to 3D. The combination of more complex magnetic states and additional degrees of freedom in 3D nanomagnets leads to the emergence of new physical phenomena, which may find applications in multiple areas. (a) Examples of 3D nanomagnets, from left to right: magnetic sphere with vortex configuration. Magnetic thin film element with a skyrmion. Symmetry breaking is caused by bulk or interfacial Dzyaloshinskii-Moriya interaction. Möbius strip with perpendicular magnetization; a DW is present in the ground state due to the object's topology. Cylindrical NW with modulated diameter, with different magnetic configurations depending on the diameter. Antiferromagnetic (AF) superlattice (interlayers not shown for clarity) with a wide soliton in the middle. (b) Examples of 2D nanomagnets, from left to right: Single-domain magnet. Magnetic multi-layered element with perpendicular anisotropy. Nanostrip with protrusions for DW trapping. Bi-layered magnet with AF coupling due to indirect exchange via an interlayer (not shown for clarity).

to reduce the stray field [18]. Other examples are the domain walls, where the magnetization rotates between two magnetic domains [19].

Spin waves are the fundamental low-energy excitations of ferromagnets or antiferromagnets. These excitations manifest as oscillations in the direction of the magnetization vector throughout the material. Because the dynamic properties of the spin waves strongly depend on the equilibrium ground state, it is very useful to study the behavior of such waves over a magnetic ground state with a particular magnetic texture.

In this thesis, the spin-wave dynamic is studied different ferromagnetic multilayered structures. In these systems, a non-uniform equilibrium magnetization along the thickness is considered. The non-uniform ground state is induced by changing the magnetic properties along the normal of the film. The calculations are based on the Landau Lifshitz equation of motion, where the spin-wave modes, evaluated at zero and finite wave vectors, are obtained. To consider the variation of the magnetization along the thickness, the theoretical calculations also consider applying the dynamic matrix method, where the system is subdivided into ultrathin sublayers. Thus, after a convergence test, the dynamic characteristics of the thick nanostructure are obtained.

1.0.2 Thesis structure

This thesis studies three dimensional magnetism (ref) and the main interest of this thesis is an overview about static and dynamics properties of ferromagnetic films. First, the ferromagnetic resonance for a film with noncollinear magnetization, where the thickness has an important role, is studied. Finally, spin waves are propagate in a thick synthetic antiferromagnet observing particular behavior in its band structure. To drive this thesis, it will be used the semi-classical approximation introduced by Brown in 1963 in his book *Micromagnetics* [20] to obtain analytical relations for each system. The thesis has the following structure:

- Chapter 2 are focused to introduce the necessary theoretical foundations. The interactions which are included into the system and the method used.

- Chapter 3 will describe the development of the theoretical method used in this thesis. Solving the Landau-Lifshitz equation for the pertinent system and describing the static and dynamic properties for an arbitrary k .
- Chapter 4 shows the first results of this thesis, the static and dynamic properties for two cases, the first one is a thick ferromagnetic film with noncollinear magnetization and the second one, a synthetic antiferromagnet varying its thickness.
- Chapter 5 showed the last results of this thesis, similar to chapter 4, here spin waves are propagate in a synthetic antiferromagnet and different parameters are manipulate to see changes in the dispersion relation.
- Chapter 6 summarizes this thesis, giving final remarks and future view to research.

Chapter 2

Theoretical foundations

This chapter describes the theoretical foundations of nanomagnetism. The different interactions associated with the magnetic nanostructures are illustrated. The equation of motion of the magnetization is described, and some basic aspects of the dynamic matrix model are shown.

2.1 Properties of the magnetic materials

Broadly speaking, all materials can be divided into two classes with regard to their magnetic properties: those that contain atoms or ions possessing permanent magnetic moments and those that do not. Within the group of materials containing permanent magnetic moments, it is possible to go further to distinguish between those that manifest long-range order among the magnetic moments (below a critical temperature) and those that do not. Finally, it is possible to classify those with magnetic order according to the particular alignment pattern that the moments exhibit. The major classifications of media according to magnetic properties are illustrated in Figure 2.2 and discussed later.

The magnetic properties of linear materials can be conveniently discussed with reference to the magnetic susceptibility χ defined as [21]

$$\mathbf{M} = \mathbf{M}_0 + \chi \cdot \mathbf{H}, \quad (2.1)$$

where $\mathbf{M}$ is the net macroscopic magnetic moment per unit volume (usually called magnetization), $\mathbf{H}$ is the applied field, and $\mathbf{M}_0$ is the spontaneous magnetization in the absence of an applied field. In general, the susceptibility χ is represented by a 3×3 matrix. For isotropic materials, however, the induced magnetization is either parallel or antiparallel to the applied field, and the susceptibility is a scalar.

2.1.1 Ferromagnetism

Ferromagnets are materials in which the elementary permanent moments spontaneously align (below a critical temperature [22]). Although these moments do interact via their dipolar magnetic fields, the interaction giving rise to the spontaneous alignment is orders of magnitude stronger and of quantum mechanical origin [23]. This is called exchange interaction.

In the absence of external fields, the magnetic order of ferromagnet generally breaks up into complex patterns of magnetic domains [22] (see Figure 2.1). The moments are all aligned within a given domain but change direction rapidly at the boundaries between domains. Thus, each domain acts like a tiny magnet that is usually small in volume compared with the size of the material sample but still contains a large number of elementary magnetic moments. In equilibrium, these domains orient themselves so as to minimize the net magnetic moment of the macroscopic sample (Figure 2.1). This minimizes the magnetic fringing field external to the sample and thus minimizes the stored magnetostatic energy. When an external field is applied, the domains begin to align with the magnetic field, giving rise to a net magnetization. Thus, an isotropic ferromagnetic also has a initial susceptibility [24]

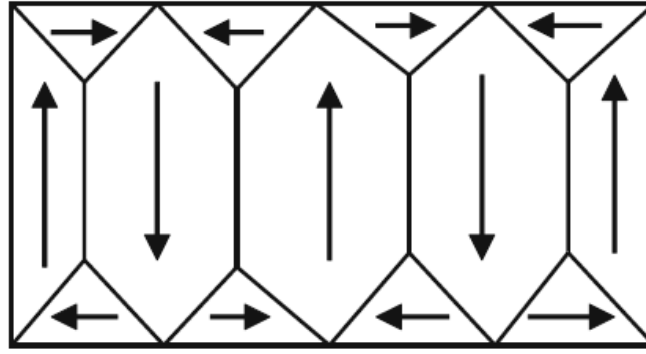


Figure 2.1: Schematic representation of a magnetic domain pattern in a ferro- or ferrimagnet. Each domain contains a large number of microscopic magnetic moments

2.1.2 Paramagnetism

Materials that contain permanent magnetic moments but not spontaneous long-range order are called paramagnetic. In thermal equilibrium, without an applied magnetic field, the moments are randomly oriented so that no net magnetic moment is exhibited. Application of an external field then causes a partial alignment of the moments, generating a net magnetic moment. Since the moments tend to align parallel to the applied field, isotropic paramagnets exhibit a positive scalar susceptibility [24].

2.1.3 Diamagnetism

Materials that do not contain atoms or ions with permanent magnetic moments respond to an applied field with an induced magnetization that is opposed to the applied field and are called diamagnets. The response of a diamagnetic material to an externally applied magnetic field can be described in terms of a microscopic application of Lenz's law. As a magnetic field is applied to such a material, electronic orbital motions are modified so as to generate an opposing magnetic field [24]. Diamagnetic contributions in electrical insulators come from bound electrons circulating in atomic orbitals. Classically, the diamagnetic contribution from conduction electrons in metals and semiconductors can be shown to vanish in thermal equilibrium. There is, however, a small non-vanishing diamagnetic effect from conduction electrons that arises from the quantization of angular momentum. Isotropic diamagnets are characterized by a negative scalar susceptibility since the induced moments oppose the applied field. Virtually all materials have a diamagnetic contribution to their total response to a magnetic field. In materials containing permanent magnetic moments, however, the diamagnetic contribution is usually overshadowed by the response of those moments [24].

2.1.4 Ferrimagnetism and Antiferromagnetism

In some materials, the quantum mechanical coupling between moments is such that adjacent moments tend to line up in opposite directions. The long-range order can be described in terms of two opposing ferromagnetic sublattices. If the net magnetizations are equal, the material is called an antiferromagnet. If the net magnetizations are unequal, the material is a ferrimagnet [24]. In general, ferrimagnets are not limited to two sublattices; the distinguishing characteristic is that the equilibrium magnetization of at least one of the sublattices must be opposite to the other. For microwave frequencies and below, ferrimagnets can usually be modeled simply as ferromagnets with a total magnetization determined by the net magnetization of the sublattices.

Antiferromagnets, on the other hand, behave like anisotropic paramagnets. In the absence of an external field, the magnetization of the sublattices cancels, yielding no net magnetic moment. The susceptibility along the direction parallel (or antiparallel) to the moments is very small since the application of a field parallel (or antiparallel) to a moment yields no net torque [23]. At finite temperatures, however, thermal agitations prevent the moments from being perfectly aligned so that the parallel susceptibility vanishes rigorously only at 0 K. In contrast, the susceptibility

perpendicular to the moments is much larger since the moments on both sublattices will tend to rotate toward the applied field.

At nonzero temperatures, thermal fluctuations prevent perfect alignment in any material exhibiting long-range magnetic order. As the temperature increases, the fluctuations become larger and larger until the magnetic order is destroyed [22]. The transition temperature above which magnetic order is destroyed is called Curie temperature for ferromagnets and the Néel temperature for ferri- and antiferromagnets. Above this transition temperature, ferromagnets, ferrimagnets, and antiferromagnets exhibit a paramagnetic susceptibility [22].

Finally, when a ferromagnet or ferrimagnet is placed in a sufficiently strong static magnetic field, all of the domains become aligned with the applied field, and the material is said to be saturated; strengthening the field will not result in an increased magnetic moment. However, there will still be a susceptibility for small perturbations perpendicular to the static field. Thus, the small signal susceptibility can be seen to be anisotropic in a manner similar to an antiferromagnet. If the perturbations are rapidly varying in time, off-diagonal elements of the susceptibility tensor begin to be important, and the response of the medium becomes considerably more involved. These are precisely the conditions under which dipolar spin waves propagate.

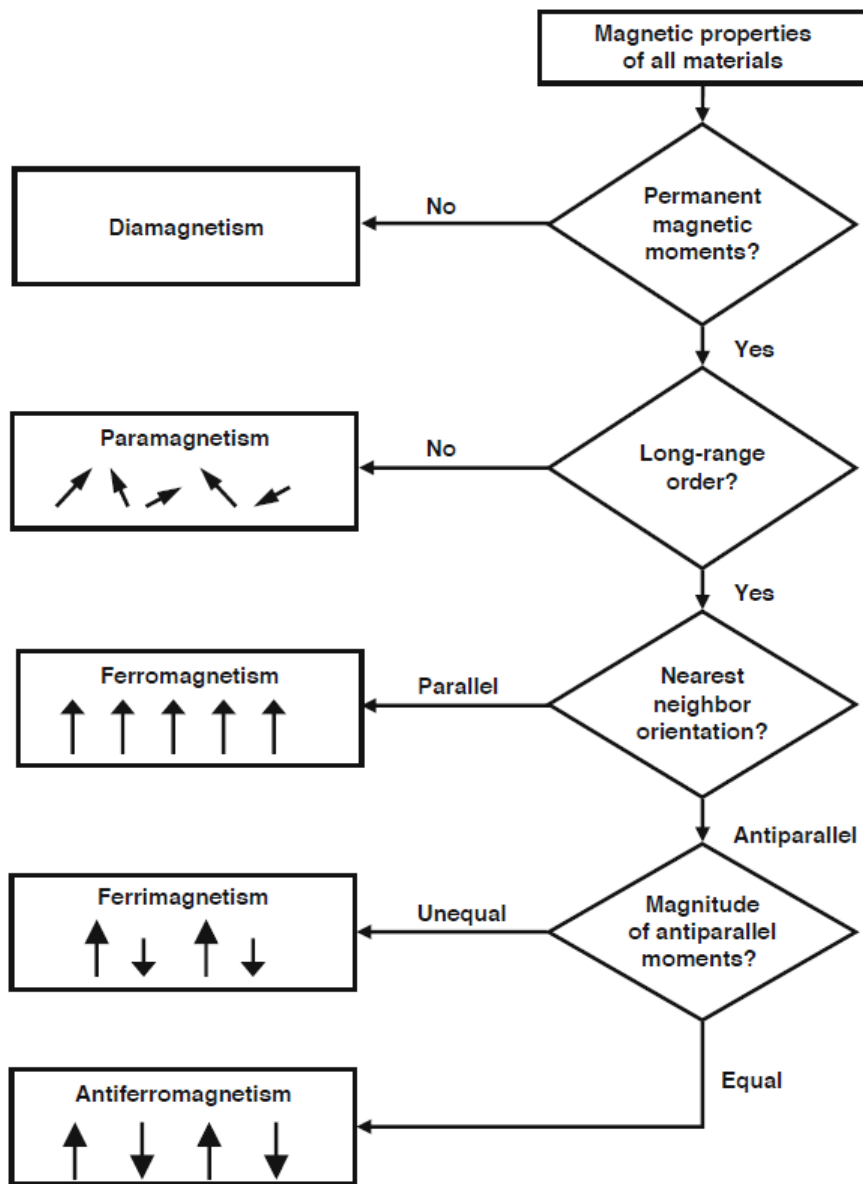


Figure 2.2: Figure from [24]. The major classifications of magnetic properties of media. Antiferromagnetism can be viewed as a special case of ferrimagnetism.

It is worth mentioning that there are other kinds of materials that present particular magnetic configurations, for example, helicoidal structures, which is a form of magnetic ordering where spins of neighboring magnetic moments arrange themselves in a spiral or helical pattern, with a characteristic turn angle of somewhere between 0 and 180 degrees [25, 26]. Nevertheless, they are not relevant in this thesis and, hence, are not discussed in detail.

2.2 Magnetic Energies

2.2.1 Zeeman energy

Suppose the magnetized sample is in the presence of an external magnetic field $\mathbf{H}_0$. In that case, the magnetic moments of the sample will interact with the external field, and the parallel alignment of the magnetization $\mathbf{M}$ and the field $\mathbf{H}_0$ is the low energy state. This interaction is called Zeeman energy, and the corresponding energy density is given by [22, 24]:

$$\epsilon_0 = -\mu_0 \mathbf{M} \cdot \mathbf{H}_0. \quad (2.2)$$

2.2.2 Magnetocrystalline Anisotropy

Magnetocrystalline anisotropy means that a ferromagnetic sample prefers energetically specific directions of magnetization [23]. The preferred directions are determined by the symmetries and structures of the crystal lattice [22]. One of the origins of the anisotropy in magnetically ordered crystals is the magnetic interaction of elementary magnetic moments. The most common anisotropy effect is connected to the existence of one only easy direction, and in literature, it is referred to as uniaxial anisotropy. Thus, the anisotropy free energy density $\epsilon_{\text{an}}(\mathbf{M})$ will be rotationally-symmetric with respect to the easy axis and will depend only on the relative orientation of $\mathbf{M}$ with respect to this axis. It is supposed, for the sake of simplicity, that the easy direction coincides with the cartesian axis z . Therefore, the expression of $\epsilon_{\text{u}}(\mathbf{M})$ can be written as an even function of $m_z = \cos \theta$, or equivalently using as independent variable $m_x^2 + m_y^2 = 1 - m_z^2 = \sin^2 \theta$. This expression, developed in power series, assumes the following form [22, 23]:

$$\epsilon_{\text{u}}(\mathbf{M}) = K_0 + K_1 \sin^2 \theta + K_2 \sin^4 \theta + K_3 \sin^6 \theta + \dots, \quad (2.3)$$

where $K_1, K_2, K_3, \dots$, are the anisotropy constants having the dimensions of energy per unit volume [J/m^3].

2.2.3 Demagnetizing field

An essential contribution of energy in this system is the demagnetization energy. This energy's origin is due to Maxwell's equations in a spontaneously magnetized body. As seen in electrodynamics courses, the magnetic field in vacuum or magnetic induction $\mathbf{B}$ can be described by two quantities: the magnetic field in matter $\mathbf{H}$ and the magnetization $\mathbf{M}$. The relationship among these expressions is [21]:

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}). \quad (2.4)$$

Furthermore, the second law of Maxwell expressed $\mathbf{B}$ as:

$$\nabla \cdot \mathbf{B} = 0. \quad (2.5)$$

The magnetic field $\mathbf{H}$ is, unlike the magnetic field $\mathbf{B}$, non-source free. Substituting equation (2.4) into equation (2.5) it is obtained a relationship between $\mathbf{H}$ and $\mathbf{M}$, namely

$$\nabla \cdot \mathbf{H} = -\nabla \cdot \mathbf{M}. \quad (2.6)$$

The $\mathbf{H}$ -field can be decomposed into two fields [22, 27].

$$\mathbf{H} = \mathbf{H}_0 + \mathbf{H}_{\text{dem}}, \quad (2.7)$$

where $\mathbf{H}_0$ is the external field caused by currents or external fields/magnets meanwhile, the second field $\mathbf{H}_{\text{dem}}$ is the demagnetizing field inside the sample. This decomposition is done for a particular reason, and it is possible to set the demagnetizing field as a conservative field, which means it is

possible to define this field as a gradient of potential generated by magnetic charges. Using the first law of Maxwell's equations, but now the magnetic charges are given by [21]

$$\rho = -\nabla \cdot \mathbf{M}. \quad (2.8)$$

In homogeneous magnetized samples, the magnetic charges are located on the interface because the gradient of the magnetization is not zero. Using Lenz's law, the demagnetizing field must act opposite to the magnetization. For ellipsoidal homogeneously magnetized samples, the demagnetizing field can be calculated precisely. It turns out that the demagnetizing field is linearly related to the magnetization and, therefore, also homogeneous [23, 27].

$$\mathbf{H}_{\text{dem}} = -\hat{\mathcal{N}}\mathbf{M}, \quad (2.9)$$

where $\hat{\mathcal{N}}$ is the demagnetization tensor and becomes diagonal in the axes coinciding with the axes of the ellipsoid

$$\hat{\mathcal{N}} = \begin{pmatrix} \mathcal{N}_{xx} & 0 & 0 \\ 0 & \mathcal{N}_{yy} & 0 \\ 0 & 0 & \mathcal{N}_{zz} \end{pmatrix}. \quad (2.10)$$

Here, $\mathcal{N}_{xx}$, $\mathcal{N}_{yy}$ and $\mathcal{N}_{zz}$ are the demagnetizing factors that depend on the ratio between the semi-axes of the ellipsoid and the trace of this tensor is equal to one or 4π depending if is not normalized. As mentioned before, the demagnetizing factors depend on the ratio between the semi-axis, for example, for a magnetic sphere, due to the symmetry of the shape, the demagnetizing factors will be $\mathcal{N}_{xx} = \mathcal{N}_{yy} = \mathcal{N}_{zz} = 1/3$. Another case is for an infinitely long cylinder with its long axis along the y -axis (see Figure 1.9 from Ref [23]), the demagnetizing factors are $\mathcal{N}_{xx} = \mathcal{N}_{zz} = 1/2$. Finally, for a thin, widely extended film, the length, and width are much larger than the thickness. Therefore, the components of the demagnetizing field parallel to the surface (x and y directions) can be neglected since the magnetic charges are very far apart [23]. This corresponds to $\mathcal{N}_{xx} = 0$ and $\mathcal{N}_{yy} = 0$. Since the trace of the demagnetization tensor is one, $\mathcal{N}_{zz} = 1$ must apply. Then, it is easy to show that

$$\epsilon_{\text{dem}} = -\frac{1}{2}\mu_0\mathbf{M} \cdot \mathbf{H}_{\text{dem}}. \quad (2.11)$$

2.2.4 Exchange energy

It is known from quantum mechanics that the form of a wave function and, consequently, the mean energy of the Coulomb interaction of two electrons depend on the mutual orientation of their spins. The part of the Coulomb energy that depends on this orientation is called exchange energy. This energy is the cause of the magnetic ordering, mentioned in subsection 2.1.1. The simplest way to express the exchange energy is through Heisenberg Hamiltonian [24, 28]:

$$\mathcal{H} = -2\frac{\mathcal{J}}{\hbar^2}\mathbf{S}_1 \cdot \mathbf{S}_2 \quad (2.12)$$

Here, $\mathbf{S}_1$ and $\mathbf{S}_2$ are angular momentum operators and $\mathcal{J}$ is the exchange constant. This interaction is described in discrete form, nevertheless, in the continuous case, it is easy to show that such exchange energy becomes [22]

$$\epsilon_{\text{exc}} = \frac{A_{\text{ex}}}{M_s^2} [(\nabla M_x)^2 + (\nabla M_y)^2 + (\nabla M_z)^2] \quad (2.13)$$

where A_{ex} is the exchange constant, and M_s is the saturation magnetization.

2.3 Effective fields

For further theoretical considerations, the effective magnetic field $\mathbf{H}^e$ is of great importance. It is calculated by forming the gradient of the total energy density with respect to the coordinates of the magnetization $\mathbf{M}$ [22, 23], namely

$$\mu_0\mathbf{H}^e = -\frac{1}{M_s}\nabla_{\mathbf{M}}\epsilon \quad (2.14)$$

where ϵ is the sum of the energy densities previously mentioned.

2.4 Landau-Lifshitz equation

In this section, some aspects of the magnetization motion are commented on. Under the presence of an effective field, the magnetization will be deflected from its equilibrium position. As a result, the magnetization will precess around such a field. During this process, the magnetization will return to the equilibrium position through a spiral movement, as shown in Figure 2.3. Note that after a time, the magnetization reaches the direction of the effective field. This effect is due to the intrinsic damping of the system. The time scale for this phenomenon is in the nanosecond range [23]. This behavior was studied and described theoretically by Landau and Lifshitz in 1935 [23]. The Landau-Lifshitz (LL) equation describes the precession (without damping) of the magnetization around the effective field [22, 24]. This equation is

$$\frac{d\mathbf{M}}{dt} = -\gamma\mu_0\mathbf{M} \times \mathbf{H}^e, \quad (2.15)$$

here, γ is the gyromagnetic ratio, representing the relation between the magnetic moment and angular momentum.

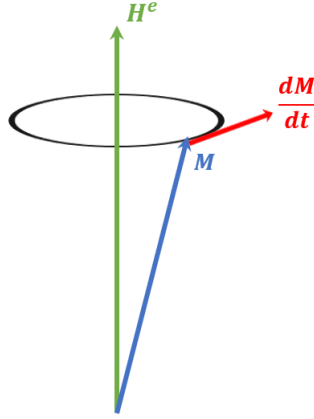


Figure 2.3: Representation of the trajectory of the magnetization motion.

An essential feature of the LL equation is that it ensures the conservation of the vector $\mathbf{M}$ length. Indeed, multiplying both sides of equation (2.15) by $\mathbf{M}$ it is found:

$$\frac{\partial}{\partial t}\mathbf{M}^2 = 0. \quad (2.16)$$

It is worth mentioning that from the LL equation of motion, the equilibrium states are also obtained. These states are calculated from the equilibrium condition $\mathbf{M} \times \mathbf{H}^e = 0$.

2.4.1 Spin waves

Spin waves were introduced by Felix Bloch in 1930 in his work *Theory of ferromagnetism* [29] to explain the reduction of the spontaneous magnetization of a ferromagnet close to its Curie temperature, over the years, this theory was developed by a huge quantity of scientist, being the most prominent Holstein and Primakoff [30] and Dyson [31]. The experimental detection of the spin waves was led by different people. Brockhouse [32] could detected by using neutron scattering or Grünberg [33] using inelastic light scattering. The study of spin waves has been advancing to be a very active research field, with notorious progress in either experimental or theoretical studies [34, 35].

As an introductory example, a linear chain of classical spins is illustrated in Fig. 2.4. Here, a chain of N spins with magnitude S , coupled with nearest-neighbor exchange interaction, uniformly spaced, by a distance a is shown. By considering only the Heisenberg interaction [36], the energy of the system is given by

$$U_{\text{exc}} = -2J \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+1}. \quad (2.17)$$

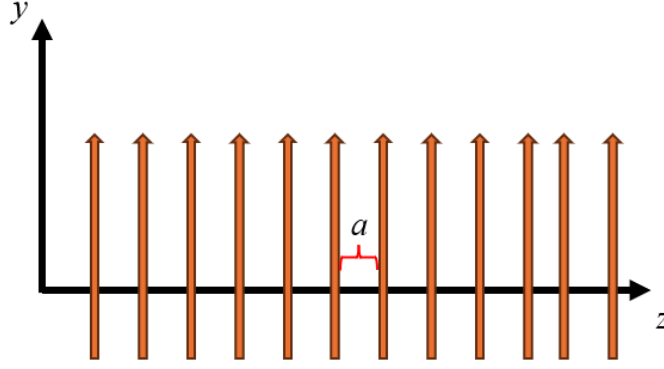


Figure 2.4: Linear chain of classical spins in the ground state

Where J is the nearest neighbor exchange parameter and $\mathbf{S}_i$ denotes the classical spin at the coordinate $z_i = ia$. In the ground state, all spins are parallel, as in the Fig. 2.4, so that with $\mathbf{S}_i \cdot \mathbf{S}_{i+1} = S^2$ the exchange energy of the system is $U_0 = -2JNS^2$ [37].

First, the spin waves are treated as a linear chain of spins that have motion governed by the classical mechanical equation for the torque. This torque is acting on the spin $\mathbf{S}_i$ can be found by considering that it has an associated magnetic moment $\boldsymbol{\mu}_i = -g\mu_B \mathbf{S}_i$, and that torque acting on the moment has the form $\boldsymbol{\tau} = \boldsymbol{\mu}_i \times \mathbf{B}_{\text{eff}}$, where $\mathbf{B}_{\text{eff}}$ is an effective field representing all interactions on the spin $\mathbf{S}_i$. This field can be found considering that the energy of the moment has the form $U_i = -\boldsymbol{\mu}_i \cdot \mathbf{B}_{\text{eff}}$. From Eq. (2.17), the effective field of a linear spin chain is given by

$$\mathbf{H}_{\text{eff}}^{\text{exc}} = -\frac{2J}{g\mu_B\mu_0}(\mathbf{S}_{i-1} + \mathbf{S}_{i+1}). \quad (2.18)$$

Besides of considering the exchange interactions with their neighbors, the spins are subject to an applied static magnetic field $\mathbf{H}$, so that the total field acting on the spins is $\mathbf{B}_{\text{eff}} = \mu_0(\mathbf{H} + \mathbf{H}_{\text{eff}}^{\text{exc}})$. Using the torque equation $d\hbar\mathbf{S}/dt = \boldsymbol{\tau}$ is obtained the equation of motion for the spin $\mathbf{S}_i$

$$\frac{d\mathbf{S}_i}{dt} = -\gamma\mu_0\mathbf{S}_i \times (\mathbf{H} + \mathbf{H}_{\text{eff}}^{\text{exc}}), \quad (2.19)$$

where $\gamma = g\mu_B/\hbar$ is the gyromagnetic ratio. Since the magnetic moment has a direction opposite to the spin, to be consistent with the ground state depicted in Fig. 2.4, the external field is applied in z -direction, so that $\mathbf{H} = -H\hat{z}$. Then the equation for the spin component S_i^z becomes

$$\frac{dS_i^z}{dt} = \gamma\mu_0 S_i^x \left[H + \frac{2J}{g\mu_B\mu_0}(S_{i-1}^y + S_{i+1}^y) \right] - \gamma\mu_0 \frac{2J}{g\mu_B\mu_0}(S_{i-1}^x + S_{i+1}^x)S_i^y.$$

Considering that the amplitude of the spin excitation is small, it is linearized this equation using $S_i^z S_i^x \ll S_i^y \approx S$. Then equations for the two transverse spin components become

$$\frac{dS_i^y}{dt} = \gamma\mu_0 H S_i^x + \frac{2JS}{\hbar}(2S_i^x - S_{i-1}^x - S_{i+1}^x), \quad (2.20)$$

$$\frac{dS_i^x}{dt} = -\gamma\mu_0 H S_i^z - \frac{2JS}{\hbar}(2S_i^z - S_{i-1}^z - S_{i+1}^z). \quad (2.21)$$

The Eqs. (2.20) and (2.21) show that the motion of the spin in any site is coupled to the motions of the neighboring spins, indicating that their solutions must be collective excitations. Consider for possible solutions excitations in the form of harmonic travelling waves

$$S_i^z = A_z e^{i(kz_i - \omega t)}, \quad S_i^x = A_x e^{i(kz_i - \omega t)}, \quad (2.22)$$

where ω is the angular frequency and k is the wave number. It is introduced these expression into Eq. (2.20) leads to

$$-i\omega A_z = A_x \left[\gamma\mu_0 H + \frac{2JS}{\hbar}(2 - e^{-ika} - e^{ika}) \right], \quad (2.23)$$

where the term $i(kz_i - \omega t)$ were cancelled on both sides. Then, the previous equation can be written as

$$-i\omega A_z = A_x \left[\gamma\mu_0 H + \frac{4JS}{\hbar}(1 - \cos ka) \right]. \quad (2.24)$$

Similarly, its reproduce the procedure to Eq. (2.21) leading to

$$-i\omega A_x = -A_z \left[\gamma\mu_0 H + \frac{4JS}{\hbar}(1 - \cos ka) \right] \quad (2.25)$$

Writing the Eqs. (2.24) and (2.25) in matrix form

$$\begin{pmatrix} -[\gamma\mu_0 H + \frac{4JS}{\hbar}(1 - \cos ka)] & [\gamma\mu_0 H + \frac{4JS}{\hbar}(1 - \cos ka)] \\ [\gamma\mu_0 H + \frac{4JS}{\hbar}(1 - \cos ka)] & -i\omega \end{pmatrix} \begin{pmatrix} A_z \\ A_x \end{pmatrix} = 0 \quad (2.26)$$

Solving this matrix for the frequency is obtained

$$\omega_k = \gamma\mu_0 H + \frac{4JS}{\hbar}(1 - \cos ka) \quad (2.27)$$

This equation represents the dependence of the spin wave on the wave number and is commonly called **dispersion relation** for a linear ferromagnetic chain (See Appendix A for general cases of dispersion relation). Substitution of Eqs (2.27) in either (2.24) or (2.25) gives the relation between the amplitudes of the spin components $A_x = -iA_z \equiv -iA_0$. Thus, the real parts of the transverse spin components become

$$S_i^z = A_0 \cos(kz_i - \omega_k t), \quad S_i^x = A_0 \sin(kz_i - \omega_k t), \quad (2.28)$$

while the longitudinal component is $S_i^y \approx S$. The equations illustrate the classical depiction of a one-dimensional spin wave, where in spins precess circularly around the equilibrium direction, as illustrated in Fig. 2.5 at a certain instant of time. In a traveling wave propagating in the $+z$ direction, the spin precession maintains consistent amplitude along the chain while exhibiting a position-dependent phase variation with $\varphi_i = kz_i$.

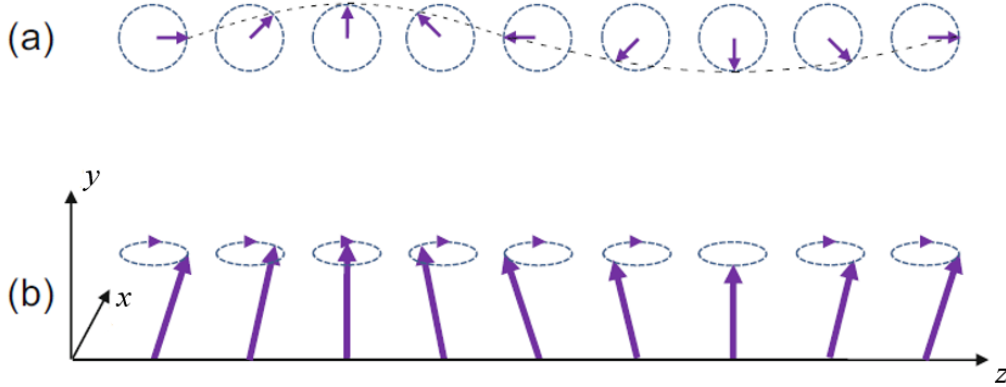


Figure 2.5: Visualisation of a 1D spin wave from [36].

Illustration of a spin wave in a lineal chain of classical spins propagating in the $+z$ direction (a)
Top view of the spins (b) Side view

2.4.2 Ferromagnetic resonance

In typical ferromagnetic resonance experiments, the magnetization is excited by a high-frequency magnetic field in the microwave range. At the same time, the sample is exposed to the presence of an external time-independent magnetic field $\mathbf{H}_0$. At a given field $\mathbf{H}_0 = \mathbf{H}_{\text{FMR}}$ and microwave frequency ω , the sample absorbs the energy of the microwave irradiation. This phenomenon is called ferromagnetic resonance (FMR), and it is very useful to characterize the sample and understand the fundamental physics behind the system [38].

From a calculations point of view, the frequency that matches the one obtained from ferromagnetic resonance experiments is simply the frequency of the spin waves evaluated at zero wave vector ($k = 0$). Such spin waves are calculated for slight deviations around the equilibrium, which is consistent with the ferromagnetic resonance setup since such an experimental technique is a perturbative method in the sense that it measures magnetic excitations with small magnetization oscillation amplitudes.

Retaking the ellipsoid, if this one is saturated by a homogeneous static external field $\mathbf{H}_0$ in the z -axis (see Figure 1.9 from Ref. [23]), the frequency of the ferromagnetic resonance is given by

$$\omega_{\text{FMR}} = \omega_M \sqrt{[H_0 + (\mathcal{N}_{xx} - \mathcal{N}_{zz})][H_0 + (\mathcal{N}_{xx} - \mathcal{N}_{zz})]}, \quad (2.29)$$

which is known as the Kittel formula [39].

2.4.3 Dynamic matrix method

In this section, the idea is to give a general notion of the Dynamic matrix method, which will be used in this thesis to study thick ferromagnetic systems. Let's consider a thick film of thickness D , which is subdivided into N sublayers. Then, the coordinate system, (u, v, w) is associated with the orthonormal basis $\{\hat{e}_u, \hat{e}_v, \hat{e}_w\}$, such that axis u is parallel to the direction of propagation of the spin waves and v is normal to the film. The magnetic cells used to discretize films will be the sublayers of thickness $b = D/N$, parallel to the (u, w) plane [see Fig.2.6(a)]. To implement the constraint, it is helpful to transform the eigenvalue problem from the Landau-Lifshitz equation to a new basis $\{\hat{e}_1, \hat{e}_2\}$ locally orthogonal to $M_S \hat{e}_3$, where $\hat{e}_3$ is the direction of the equilibrium magnetization [40, 41]. Then, the result from Bruckner [42] is used for the transformation matrix $\hat{\mathbf{R}}$. Namely

$$\hat{\mathbf{R}} = \begin{pmatrix} \hat{e}_1 \cdot \hat{e}_u & \hat{e}_2 \cdot \hat{e}_u \\ \hat{e}_1 \cdot \hat{e}_w & \hat{e}_2 \cdot \hat{e}_w \\ \hat{e}_1 \cdot \hat{e}_v & \hat{e}_2 \cdot \hat{e}_v \end{pmatrix} \quad \hat{e}_3 = M_S \hat{e}_t \quad \hat{e}_1 = \hat{e}_3 \times \hat{e}_{\text{ref}} \quad \hat{e}_2 = \hat{e}_1 \times \hat{e}_3. \quad (2.30)$$

This transformation is unitary for vectors orthogonal to $M_S \hat{e}_3$. Choosing a convenient direction

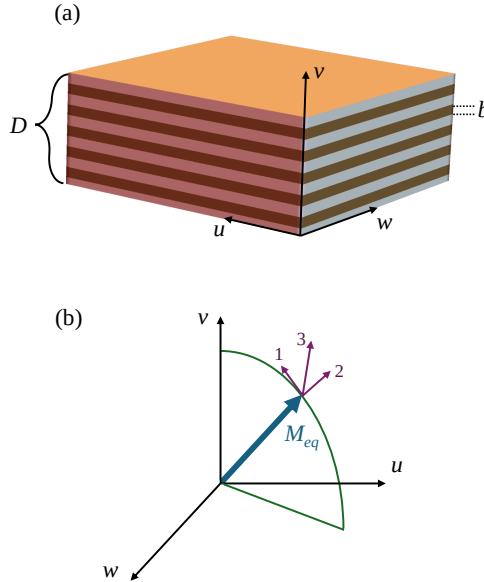


Figure 2.6: Adaption from Figure from [42].(a) Geometry of magnetic array. Extended films are subdivided vertically into infinite horizontal sublayers.(b) (u, v, w) and $(1, 2, 3)$ coordinate systems attached to the magnetic medium and equilibrium magnetization, respectively

for the magnetization (like $\pm \hat{e}_z$). Now, multiplying for $\hat{\mathbf{R}}$ both sides of Eq. (2.15) and using the unitary property for matrices $\hat{\mathbf{R}}^\dagger \hat{\mathbf{R}} = \hat{\mathbf{I}}$, obtaining the rotated eigenvalue problem

$$\omega \mathbf{m} = \tilde{\mathbf{A}} \mathbf{m} \quad (2.31)$$

or, in extended form

$$\omega \begin{pmatrix} m_1^{(1)} \\ \vdots \\ m_1^{(N)} \\ m_2^{(1)} \\ \vdots \\ m_2^{(N)} \end{pmatrix} = \begin{pmatrix} \cdot & \cdots & \cdot & \cdot & \cdots & \cdot \\ \vdots & A_{11} & \vdots & \vdots & A_{12} & \vdots \\ \cdot & \cdots & \cdot & \cdot & \cdots & \cdot \\ \vdots & A_{21} & \vdots & \vdots & A_{22} & \vdots \\ \cdot & \cdots & \cdot & \cdot & \cdots & \cdot \end{pmatrix} \begin{pmatrix} m_1^{(1)} \\ \vdots \\ m_1^{(N)} \\ m_2^{(1)} \\ \vdots \\ m_2^{(N)} \end{pmatrix},$$

where $\tilde{\mathbf{A}}$ is the dynamic matrix whose dimensions are $2N \times 2N$. If short-range exchange and long-range dipole-dipole interactions are not included in the model, the $\tilde{\mathbf{A}}$ is block-diagonal [43]. If, on the contrary, those interactions are taken into account, none of the off-diagonal elements are zero, and the matrix $\tilde{\mathbf{A}}$ has no special properties.

Chapter 3

Theory for multilayered thick films

This chapter reviews the theory related to the thesis work. The LL equation is solved with the pertinent reference system, where the static and dynamic properties are analyzed for an arbitrary wave vector $\mathbf{k}$. Finally, the dynamical matrix method is explained, which allows for obtaining the frequencies (eigenvalues) and dynamic magnetizations (eigenvectors) of the system.

A scheme of the ferromagnetic film is shown in Fig. 3.1. A system of local coordinates (X_n, Y_n, Z_n) is defined, where the equilibrium orientation of the magnetization is pointing along Z_n . It will be assumed that the spin waves are propagating along the z -axis. The angle φ_n corresponds to the equilibrium angle of the n -th layer measured from $\hat{z}$. The temporal evolution of the system is described by the Landau-Lifshitz equation, namely

$$\dot{\mathbf{M}}_n(\mathbf{r}, t) = -\gamma \mathbf{M}_n(\mathbf{r}, t) \times \mathbf{H}_n^e(\mathbf{r}, t). \quad (3.1)$$

Here, the dot denotes the time derivative, and γ is the absolute value of the gyromagnetic ratio. Also, $\mathbf{H}_n^e(\mathbf{r}, t)$ is the effective field acting on layer n . In the linear regime, both magnetization and effective field can be written as $\mathbf{M}_n(\mathbf{r}, t) = M_s \hat{Z}_n + \mathbf{m}_n(\mathbf{r}, t)$ and $\mathbf{H}_n^e(\mathbf{r}, t) = \mathbf{H}_n^{e0} + \mathbf{h}_n^e(\mathbf{r}, t)$ (see Fig. 3.1). Thus, the linearized equations of motion (3.1) become

$$i\Omega m_{X_n}(\mathbf{r}) = -m_{Y_n}(\mathbf{r}) H_{Z_n}^{e0} + M_s h_{Y_n}^e(\mathbf{r}) \quad (3.2a)$$

$$i\Omega m_{Y_n}(\mathbf{r}) = m_{X_n}(\mathbf{r}) H_{Z_n}^{e0} - M_s h_{X_n}^e(\mathbf{r}), \quad (3.2b)$$

where $\Omega = \omega/\gamma$ and it has been assumed a harmonic dynamic response $\mathbf{m}(\mathbf{r}, t) = \mathbf{m}(\mathbf{r}) e^{i\omega t}$. Furthermore, for the in-plane case $H_{Y_n}^{e0} = 0$ and according to the equilibrium condition $M_s H_{X_n}^{e0} = 0$. On the other side, the effective field is the sum of the energetic contributions of the system. Thus, this field can be stated as $\mathbf{H}^e = \mathbf{H}_0 + \mathbf{H}_{\text{dip}} + \mathbf{H}_u + \mathbf{H}_{\text{ex}}$.

The monochromatic spin waves are propagating along z so that the dynamic magnetization components can be written as $\mathbf{m}(\mathbf{r}) = \mathbf{m}(\mathbf{k}) e^{ikz}$, where k is the magnitude of the wave vector. Generalizing the dynamic effective fields $h_{Y_n, X_n}^e(\mathbf{r})$, the terms $M_s h_{X_n}^e(\mathbf{r})$ and $M_s h_{Y_n}^e(\mathbf{r})$ in Eq (3.2) can be expressed as

$$M_s h_{X_n}^e(\mathbf{r}) = \sum_{\nu=1}^N m_{X_\nu}(\mathbf{r}) T_{X_n}^{X_\nu} + \sum_{\nu=1}^N m_{Y_\nu}(\mathbf{r}) T_{X_n}^{Y_\nu}, \quad (3.3)$$

and

$$M_s h_{Y_n}^e(\mathbf{r}) = \sum_{\nu=1}^N m_{X_\nu}(\mathbf{r}) T_{Y_n}^{X_\nu} + \sum_{\nu=1}^N m_{Y_\nu}(\mathbf{r}) T_{Y_n}^{Y_\nu}, \quad (3.4)$$

Then,

$$i\Omega m_{X_n}(\mathbf{r}) = -m_{Y_n}(\mathbf{r})H_{Z_n}^{e0} + \sum_{\nu=1}^N m_{X_\nu}(\mathbf{r})T_{Y_n}^{X_\nu} + \sum_{\nu=1}^N m_{Y_\nu}(\mathbf{r})T_{Y_n}^{Y_\nu} \quad (3.5a)$$

$$i\Omega m_{Y_n}(\mathbf{r}) = m_{X_n}(\mathbf{r})H_{Z_n}^{e0} - \sum_{\nu=1}^N m_{X_\nu}(\mathbf{r})T_{X_n}^{X_\nu} - \sum_{\nu=1}^N m_{Y_\nu}(\mathbf{r})T_{X_n}^{Y_\nu} \quad (3.5b)$$

Thus, Eq (3.2) can be expressed in matrix form

$$i\Omega \mathbf{m}(\mathbf{r}) = \tilde{\mathbf{A}} \mathbf{m}(\mathbf{r}), \quad (3.6)$$

where the transpose of matrix $\mathbf{m}(\mathbf{r})$ is given by $\mathbf{m}^T(\mathbf{r}) = [m_{X_1}, \dots, m_{X_N}, m_{Y_1}, \dots, m_{Y_N}]$ and $\tilde{\mathbf{A}}$ has block form, this is

$$\tilde{\mathbf{A}} = \begin{pmatrix} A_{X_n}^{X_\nu} & A_{X_n}^{Y_\nu} \\ A_{Y_n}^{X_\nu} & A_{Y_n}^{Y_\nu} \end{pmatrix}.$$

The sub-matrix elements, for $n = \nu$, are given by

$$A_{X_n}^{X_n} = M_s T_{Y_n}^{X_n} ; \quad A_{X_n}^{Y_n} = M_s T_{Y_n}^{Y_n} - H_{Z_n}^{e0} \quad (3.7)$$

$$A_{Y_n}^{X_n} = H_{Z_n}^{e0} - M_s T_{X_n}^{X_n} ; \quad A_{Y_n}^{Y_n} = -M_s T_{X_n}^{Y_n} \quad (3.8)$$

and, for $n \neq \nu$, they are

$$A_{X_n}^{X_\nu} = M_s T_{Y_n}^{X_\nu} ; \quad A_{X_n}^{Y_\nu} = M_s T_{Y_n}^{Y_\nu} \quad (3.9)$$

$$A_{Y_n}^{X_\nu} = -M_s T_{X_n}^{X_\nu} ; \quad A_{Y_n}^{Y_\nu} = -M_s T_{X_n}^{Y_\nu} \quad (3.10)$$

The first four terms of the matrix contain the interaction of the intralayer nature, so they correspond to the same layer n , and the remaining terms are of interlayer nature, referring to the interaction between layers n and ν . These interlayer elements are of exchange and dipolar nature and are fundamental for describing the dynamics of the system. Finally, the elements $T_{\kappa_n}^{\xi'_\nu}$ are obtained from the calculation of the dynamic effective fields (see next section).

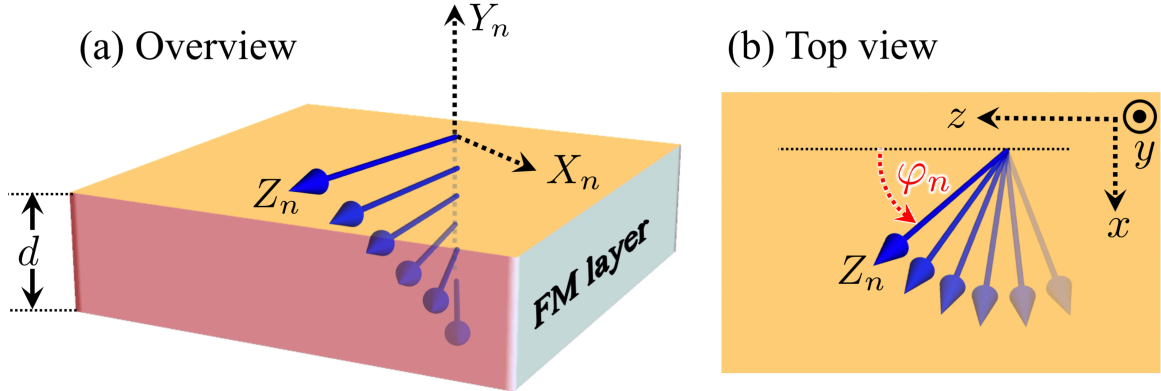


Figure 3.1: (a) Overview of the magnetic multilayer system. The coordinates (x, y, z) are fixed. Overall, every film has different magnetic properties. Local coordinates (X_n, Y_n, Z_n) are defined according to the equilibrium orientation of the n . (b) Correspondence between angle φ_n with the coordinate Z_n

Effective magnetic fields

3.0.1 Zeeman field

According to the in-plane geometry shown in Fig. 3.1, the contribution of the external field to the effective field acting over the sublayer n is

$$H_{Z_n}^0 = H \cos(\varphi_H - \varphi_n) \quad (3.11)$$

and

$$H_{X_n}^0 = H \sin(\varphi_H - \varphi_n). \quad (3.12)$$

The last contribution ($H_{X_\eta}^0$) is crucial because it will be part of the equilibrium condition that allows calculating the equilibrium angles.

3.0.2 Uniaxial anisotropy

For simplicity, it is assumed that the system has an easy axis along x for all layers. Thus, the energy density, in this case, is given by

$$\epsilon_u = -\frac{H_u}{2M_s} [\mathbf{M}(\mathbf{r}) \cdot \hat{x}]^2, \quad (3.13)$$

where $H_u^{(\eta)} = 2K_u^{(\eta)}/M_s^{(\eta)}$. Then, $\hat{x} = \cos \varphi_n \hat{X}_n + \sin \varphi_n \hat{Z}_n$, and therefore

$$\epsilon_u = -\frac{H_u}{2M_s} (m_{X_n}(\mathbf{r}) \cos \varphi_n + M_{Z_n} \sin \varphi_n)^2. \quad (3.14)$$

By considering up to second order in the spin deviation, the contribution of the uniaxial anisotropy field to the effective field at zero order acting over the sublayer n is ($H_\eta = -\delta\epsilon/\delta m_\eta$)

$$H_{X_n}^{u0} = H_u \cos \varphi_n \sin \varphi_n, \quad (3.15)$$

$$H_{Z_n}^{u0} = H_u \sin^2 \varphi_n, \quad (3.16)$$

and

$$h_{X_n}^u(\mathbf{r}) = \frac{H_u}{M_s} m_{X_n}(\mathbf{r}) \cos^2 \varphi_n. \quad (3.17)$$

This is the contribution of the uniaxial anisotropy field to the effective field of linear order in dynamic magnetization $\mathbf{m}$

3.0.3 Exchange field

The exchange field is given by

$$\mathbf{H}^{\text{ex}(n)}(\mathbf{r}) = (\nabla \cdot [\lambda_{\text{ex}}(\mathbf{r})^2 \nabla] \mathbf{M}(\mathbf{r})). \quad (3.18)$$

Here $\lambda_{\text{ex}}(\mathbf{r}) = \sqrt{2A/\mu_0(M_s)^2}$ is the exchange length of the layer n and A is the exchange constant. Writing the magnetization in the linear regime and taking into account that the exchange length is constant, the exchange field components are

$$H_{Z_n}^{\text{ex}0(n)} = 0, \quad (3.19)$$

and

$$h_{X,Y}^{\text{ex}(n)}(\mathbf{r}) = -[\lambda_{\text{ex}}]^2 k^2 m_{X,Y}(\mathbf{r}). \quad (3.20)$$

3.0.4 Dipolar fields

Dynamic dipolar fields due to volumetric magnetic charges

The dynamic dipolar field due to volumetric magnetic charges can be obtained as follows: First, as mentioned before, dynamic components of the n -th magnetization can be written as $m_{X_n, Y_n}(\mathbf{r}) = m_{X_n, Y_n}(\mathbf{k}) e^{i\mathbf{k} \cdot \mathbf{r}}$. Then, the volumetric magnetic charge density $\rho(\mathbf{r}) = -\nabla_{\mathbf{r}} \cdot \mathbf{M}(\mathbf{r})$ is

$$\rho(\mathbf{r}) = -\frac{dM_x(\mathbf{r})}{dx} - \frac{dM_y(\mathbf{r})}{dy} - \frac{dM_z(\mathbf{r})}{dz} \quad (3.21)$$

$$= -\frac{dM_z(\mathbf{r})}{dz}, \quad (3.22)$$

where $M_z(\mathbf{r}) = M_s \cos \varphi_n - m_{X_n}(\mathbf{r}) \sin \varphi_n$. Therefore

$$\rho(\mathbf{r}) = im_{X_n}(\mathbf{k})k \sin \varphi_n e^{i\mathbf{k} \cdot \mathbf{r}}. \quad (3.23)$$

Here, it is assumed a uniform distribution of the dynamic magnetization along the thickness, therefore the results presented here should apply to thin films. Then, the magnetostatic potential can be calculated from

$$\phi_V(\mathbf{r}) = \frac{1}{4\pi} \int \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^3\mathbf{r}', \quad (3.24)$$

which gives

$$\phi_V(\mathbf{r}) = im_{X_n}(\mathbf{k})k \sin \varphi_n \int \frac{e^{i\mathbf{k} \cdot \mathbf{r}'}}{|\mathbf{r} - \mathbf{r}'|} d^3\mathbf{r}'. \quad (3.25)$$

The integral in Eq. (3.25) can be analytically evaluated inside the film n , this is

$$\int \frac{e^{i\mathbf{k} \cdot \mathbf{r}'}}{|\mathbf{r} - \mathbf{r}'|} d^3\mathbf{r}' = \int \frac{dx' dy' dz' e^{i\mathbf{k} \cdot \mathbf{r}'}}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}}. \quad (3.26)$$

Then, using $x - x' = x''$, $z - z' = z''$, $(r'')^2 = (x'')^2 + (z'')^2$ and using polar coordinates

$$\int \frac{dr'' d\theta dy' r'' e^{i|\mathbf{k}| \cdot (r'' + y') \cos \theta}}{\sqrt{(r'')^2 + (y-y')^2}},$$

integrating directly

$$\int \frac{e^{i\mathbf{k} \cdot \mathbf{r}'}}{|\mathbf{r} - \mathbf{r}'|} d^3\mathbf{r}' = 2\pi e^{i\mathbf{k} \cdot \mathbf{r}} \int_{\kappa_n}^{\kappa_n+d} \frac{e^{-|\mathbf{k}| |y-y'|}}{|\mathbf{k}|} dy',$$

so

$$\begin{aligned} \int \frac{e^{i\mathbf{k} \cdot \mathbf{r}'}}{|\mathbf{r} - \mathbf{r}'|} d^3\mathbf{r}' &= 2\pi e^{i\mathbf{k} \cdot \mathbf{r}} \int_{\kappa_n}^{\kappa_n+d} \frac{e^{-|\mathbf{k}| |y-y'|}}{|\mathbf{k}|} dy' \\ &= 2\pi e^{i\mathbf{k} \cdot \mathbf{r}} \left[\int_{\kappa_n}^y \frac{e^{-|\mathbf{k}| (y-y')}}{|\mathbf{k}|} dy' + \int_y^{d+\kappa_n} \frac{e^{-|\mathbf{k}| (y'-y)}}{|\mathbf{k}|} dy' \right] \\ &= 4\pi e^{i\mathbf{k} \cdot \mathbf{r}} \frac{1 - \cosh[|\mathbf{k}| (d/2 + \kappa_n - y)] e^{-|\mathbf{k}| d/2}}{|\mathbf{k}|^2}, \end{aligned} \quad (3.27)$$

where, according to system shown in Fig. 3.1, $\kappa_n = (n-1)d$. Thus, κ_n is defined in such a way that $\kappa_1 = 0$ and $\kappa_N = 0$ [Note that the coordinate y is measured from the bottom of the layer $(n+1)$]. Then, the potential $\phi_V(\mathbf{r})$ becomes

$$\phi_V(\mathbf{r}) = i4\pi m_{X_n}(\mathbf{k})k \sin \varphi_n e^{i\mathbf{k} \cdot \mathbf{r}} \frac{1 - \cosh[|\mathbf{k}| (d/2 + \kappa_n - y)] e^{-|\mathbf{k}| d/2}}{|\mathbf{k}|^2}. \quad (3.28)$$

The dynamic dipolar field components can be obtained from $h_n^{V(n)}(\mathbf{r}) = -\partial_n \phi_V(\mathbf{r})$. i.e.

$$h_z^{V(n)}(z) = 4\pi m_{X_n}(k) \sin \varphi_n \left\{ 1 - \cosh[|k| (d/2 + \kappa_n - y)] e^{-|k| d/2} \right\} e^{ikz}$$

$$h_y^{V(n)}(z) = -i4\pi m_{X_n}(k)k \sin \varphi_n \frac{\sinh[|k| (d/2 + \kappa_n - y)] e^{-|k| d/2}}{|\mathbf{k}|} e^{ikz},$$

where we have used $\mathbf{k} \cdot \mathbf{r} = kz$. Now, the fields are averaged along the n -th layer of thickness d as $h_y^{V(n)}(z) = 0$ and

$$h_z^{V(n)}(z) = 4\pi m_{X_n}(k) [1 - \zeta(k, d)] \sin \varphi_n e^{ikz}, \quad (3.29)$$

where

$$\zeta(k, d) = \frac{2 \sinh[|k| d/2] e^{-|k| d/2}}{d |k|}. \quad (3.30)$$

Thus, $h_X^{V(n)}(z) = -h_z^V(z) \sin \varphi_n$.

Now, the stray field generated by the n -th layer is evaluated at the upper layers. Therefore, $y - y' > 0$, then

$$\int \frac{e^{i\mathbf{k} \cdot \mathbf{r}'}}{|\mathbf{r} - \mathbf{r}'|} d^3 \mathbf{r}' = 2\pi e^{i\mathbf{k} \cdot \mathbf{r}'} \left[\int_{\kappa_n}^{\kappa_n+d} \frac{e^{-|\mathbf{k}|(y-y')}}{|\mathbf{k}|} dy' \right] \quad (3.31)$$

$$= 2\pi e^{i\mathbf{k} \cdot \mathbf{r}'} \left[\frac{-e^{(-y+\kappa_n)|\mathbf{k}|} + e^{(-y+\kappa_n+d)|\mathbf{k}|}}{|\mathbf{k}|^2} \right] \quad (3.32)$$

$$= 2\pi e^{i\mathbf{k} \cdot \mathbf{r}'} \frac{(-1 + e^{d|\mathbf{k}|})e^{(-y+\kappa_n)|\mathbf{k}|}}{|\mathbf{k}|^2}. \quad (3.33)$$

Then, the magnetic potential $\phi_V(\mathbf{r})$ is

$$\phi_V(\mathbf{r}) = i2\pi m_{X_n}(\mathbf{k})k \sin \varphi_n e^{i\mathbf{k} \cdot \mathbf{r}} \frac{(-1 + e^{d|\mathbf{k}|})e^{(-y+\kappa_n)|\mathbf{k}|}}{|\mathbf{k}|^2}. \quad (3.34)$$

Analogously to the previous case, to obtain the dynamic dipolar field components should be obtained using $h_{z,y}^{V(n)} = -\partial_n \phi_V(\mathbf{r})$

$$h_z^{V(n)}(z) = 2\pi m_{X_n}(\mathbf{k})k^2 \sin \varphi_n e^{i\mathbf{k} \cdot \mathbf{r}} \frac{(e^{d|\mathbf{k}|} - 1)e^{|\mathbf{k}|(-y+\kappa_n)}}{|\mathbf{k}|}$$

$$h_y^{V(n)}(z) = i2\pi m_{X_n}(\mathbf{k})k \sin \varphi_n e^{i\mathbf{k} \cdot \mathbf{r}} \frac{(e^{d|\mathbf{k}|} - 1)e^{|\mathbf{k}|(-y+\kappa_n)}}{|\mathbf{k}|^2}.$$

Averaging the fields along the thickness of layer ν , the dynamic fields are given by

$$h_X^{V(\nu)} = -2\pi m_{X_n}(\mathbf{k})k^2 \sin \varphi_n \sin \varphi_\nu e^{i\mathbf{k} \cdot \mathbf{r}} \frac{(e^{d|\mathbf{k}|} - 1)^2 e^{|\mathbf{k}|(\kappa_n - \kappa_\nu - d)}}{d|\mathbf{k}|^3}$$

$$h_Y^{V(\nu)} = i2\pi m_{X_n}(\mathbf{k})k \sin \varphi_n e^{i\mathbf{k} \cdot \mathbf{r}} \frac{(e^{d|\mathbf{k}|} - 1)^2 e^{|\mathbf{k}|(\kappa_n - \kappa_\nu - d)}}{d|\mathbf{k}|^2}.$$

Now, the dipolar interaction between the n -th layer with those below it, is calculated. Here, the condition $y - y' < 0$ implicates that

$$\int \frac{e^{i\mathbf{k} \cdot \mathbf{r}'}}{|\mathbf{r} - \mathbf{r}'|} d^3 \mathbf{r}' = 2\pi e^{i\mathbf{k} \cdot \mathbf{r}'} \left[\int_{\kappa_n}^{\kappa_n+d} \frac{e^{-|\mathbf{k}|(y'-y)}}{|\mathbf{k}|} dy' \right] \quad (3.35)$$

$$= 2\pi e^{i\mathbf{k} \cdot \mathbf{r}'} \frac{(e^{d|\mathbf{k}|} - 1)e^{(-y+\kappa_n+d)|\mathbf{k}|}}{|\mathbf{k}|^2}. \quad (3.36)$$

Analogously to $y - y' > 0$ case, the magnetic potential is given by

$$\phi_V(\mathbf{r}) = i2\pi m_{X_n}(\mathbf{k})k \sin \varphi_n e^{i\mathbf{k} \cdot \mathbf{r}} \frac{(e^{d|\mathbf{k}|} - 1)e^{(-y+\kappa_n+d)|\mathbf{k}|}}{|\mathbf{k}|^2}, \quad (3.37)$$

obtaining the components of the dipolar field as

$$h_z^{V(n)}(z) = 2\pi m_{X_n}(\mathbf{k})k^2 \sin \varphi_n e^{i\mathbf{k} \cdot \mathbf{r}} \frac{(e^{d|\mathbf{k}|} - 1)e^{-|\mathbf{k}|(-y+\kappa_n+d)}}{|\mathbf{k}|^2}$$

and

$$h_y^{V(n)}(z) = -i2\pi m_{X_n}(\mathbf{k})k \sin \varphi_n e^{i\mathbf{k} \cdot \mathbf{r}} \frac{(e^{d|\mathbf{k}|} - 1)e^{-|\mathbf{k}|(-y+\kappa_n+d)}}{|\mathbf{k}|}.$$

Then, averaging the fields along the thickness of the layer ν , it is obtained

$$h_X^{V(\nu)} = -2\pi m_{X_n}(\mathbf{k})k^2 \sin \varphi_n \sin \varphi_\nu e^{i\mathbf{k} \cdot \mathbf{r}} \frac{(e^{d|\mathbf{k}|} - 1)^2 e^{|\mathbf{k}|(\kappa_\nu - \kappa_n - d)}}{d|\mathbf{k}|^3}$$

and

$$h_Y^{V(\nu)} = -i2\pi m_{X_n}(\mathbf{k})k \sin \varphi_n e^{i\mathbf{k}\cdot\mathbf{r}} \frac{(e^{d|\mathbf{k}|} - 1)^2 e^{|\mathbf{k}|(\kappa_\nu - \kappa_n - d)}}{d|\mathbf{k}|^2}.$$

For both the upper and lower layers, a similar behavior is observed, with a difference in sign between them. Due to the nature of the dipolar interaction, the field lines converge to create magnets. In the case of volumetric charges, the normal components of the field change sign, while the tangential components retain their orientation. Calculated lines of the volumetric charges (with typical parameters of Permalloy) are shown in Fig. 3.2, where the stray fields inside and outside a magnetic layer are shown.

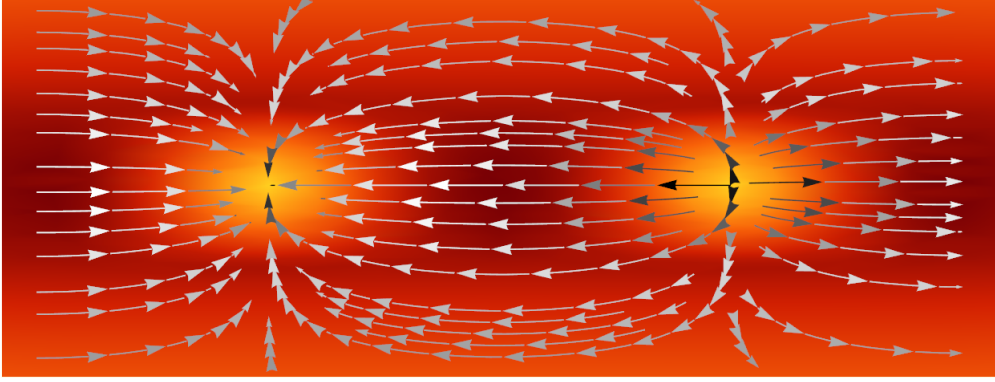


Figure 3.2: Stray field of volumetric charges.

Dynamic dipolar fields due to superficial magnetic charges

The magnetic potential generated by surface magnetic charges is given by

$$\phi_S(\mathbf{r}) = \frac{1}{4\pi} \int \frac{M_n(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^2 S', \quad (3.38)$$

where $M_n(\mathbf{r}')$ is the normal component of the magnetization. Then,

$$\phi_S(\mathbf{r}) = \int \frac{m_{Y_n}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}'}}{|\mathbf{r} - \mathbf{r}'|} d^2 S'_{y'=d+\kappa_n} - \int \frac{m_{Y_n}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}'}}{|\mathbf{r} - \mathbf{r}'|} d^2 S'_{y'=\kappa_n}. \quad (3.39)$$

Then, following the same idea of Eq. (3.26), the surface magnetic potential is

$$\int \frac{e^{i\mathbf{k}\cdot\mathbf{r}'}}{|\mathbf{r} - \mathbf{r}'|} d^2 S' = 2\pi \frac{e^{-|\mathbf{k}||y-y'|}}{|\mathbf{k}|} e^{i\mathbf{k}\cdot\mathbf{r}}. \quad (3.40)$$

Therefore,

$$\begin{aligned} \phi_S(\mathbf{r}) &= 2\pi m_{Y_n}(\mathbf{k}) \left(\frac{e^{|\mathbf{k}|(y-d/2-\kappa_n)}}{|\mathbf{k}|} - \frac{e^{-|\mathbf{k}|(y-d/2-\kappa_n)}}{|\mathbf{k}|} \right) e^{-|\mathbf{k}|d/2} e^{i\mathbf{k}\cdot\mathbf{r}} \\ &= 4\pi m_{Y_n}(\mathbf{k}) \sinh[|\mathbf{k}|(y-d/2-\kappa_n)] \frac{e^{-|\mathbf{k}|d/2}}{|\mathbf{k}|} e^{i\mathbf{k}\cdot\mathbf{r}}. \end{aligned} \quad (3.41)$$

Then, the components of the dynamic dipolar fields are

$$h_y^{S(n)}(z) = -4\pi m_{Y_n}(k) \cosh[|k|(y-d/2-\kappa_n)] e^{-|k|d/2} e^{ikz}, \quad (3.42)$$

and

$$h_z^{S(n)}(z) = -4\pi i m_{Y_n}(k) k \frac{\sinh[|k|(y-d/2-\kappa_n)] e^{-|k|d/2}}{|k|} e^{ikz}. \quad (3.43)$$

Then, by taking the average along the thickness, it is obtained $h_z^{S(\nu)}(z) = 0$ and

$$h_y^{S(n)}(z) = -4\pi m_{Y_n}(k) \zeta(k, d) e^{ikz}. \quad (3.44)$$

Now, the dynamic dipolar field is evaluated in the upper layers. Therefore, $y - y' > 0$ and

$$\phi_S(\mathbf{r}) = 2\pi m_{Y_n}(\mathbf{k}) \left(\frac{e^{-|\mathbf{k}|(y-\kappa_n-d)}}{|\mathbf{k}|} - \frac{e^{-|\mathbf{k}|(y-\kappa_n)}}{|\mathbf{k}|} \right) e^{i\mathbf{k}\cdot\mathbf{r}}. \quad (3.45)$$

Following the steps of the previous case (calculation for volumetric charges), the components of the dynamic dipolar fields are

$$h_z^{S(n)}(z) = -i2\pi m_{Y_n}(\mathbf{k}) k e^{i\mathbf{k}\cdot\mathbf{r}} \left(\frac{e^{-|\mathbf{k}|(y-\kappa_n-d)}}{|\mathbf{k}|} - \frac{e^{-|\mathbf{k}|(y-\kappa_n)}}{|\mathbf{k}|} \right)$$

and

$$h_y^{S(n)}(z) = -2\pi m_{Y_n}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}} \left(e^{-|\mathbf{k}|(y-\kappa_n)} - e^{-|\mathbf{k}|(y-\kappa_n-d)} \right).$$

Averaging along the thickness of the ν -th layer, it is obtained

$$h_X^{S(\nu)}(z) = i2\pi m_{Y_n} k \sin \varphi_\nu e^{i\mathbf{k}\cdot\mathbf{r}} \frac{(e^{d|\mathbf{k}|} - 1)^2 e^{(\kappa_n - \kappa_\nu - d)|\mathbf{k}|}}{d|\mathbf{k}|^2} \quad (3.46)$$

and

$$h_Y^{S(\nu)}(z) = 2\pi m_{Y_n} e^{i\mathbf{k}\cdot\mathbf{r}} \frac{(e^{d|\mathbf{k}|} - 1)^2 e^{(\kappa_n - \kappa_\nu - d)|\mathbf{k}|}}{d|\mathbf{k}|}. \quad (3.47)$$

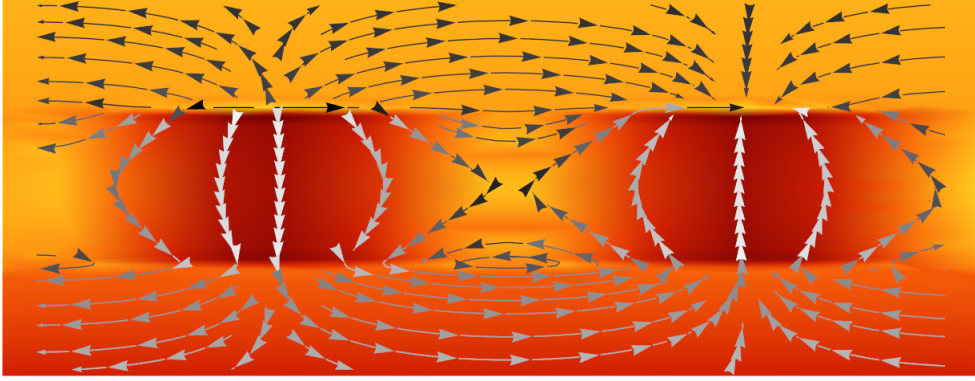


Figure 3.3: Stray field of superficial charges

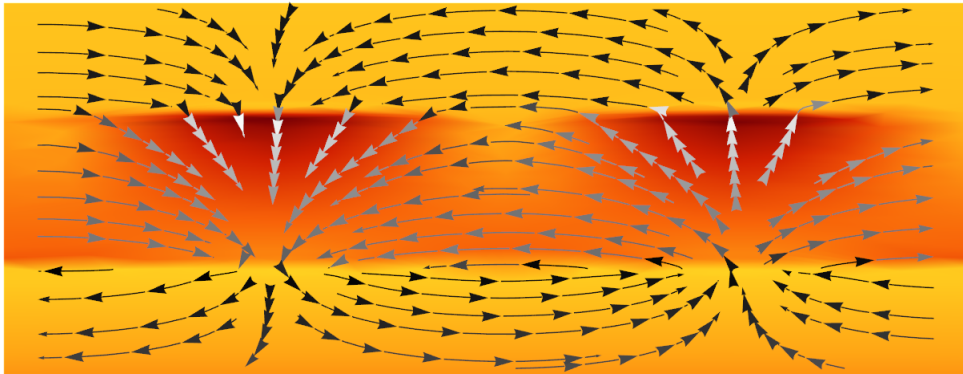


Figure 3.4: Total stray field of a film, as a result of the superposition of volumetric and surface charges.

Now, it follows to analyze the case for the lower layers, where $y - y' < 0$. Thus,

$$\phi_S(\mathbf{r}) = 2\pi m_{Y_n}(\mathbf{k}) \left(\frac{e^{-|\mathbf{k}|(-y+\kappa_n+d)}}{|\mathbf{k}|} - \frac{e^{-|\mathbf{k}|(-y+\kappa_n)}}{|\mathbf{k}|} \right) e^{i\mathbf{k}\cdot\mathbf{r}}. \quad (3.48)$$

The dynamic components of the dipolar field are

$$h_z^{S(n)}(z) = -i2\pi m_{Y_n}(\mathbf{k}) k e^{i\mathbf{k}\cdot\mathbf{r}} \left(\frac{e^{-|\mathbf{k}|(-y+\kappa_n+d)}}{|\mathbf{k}|} - \frac{e^{-|\mathbf{k}|(-y+\kappa_n)}}{|\mathbf{k}|} \right)$$

and

$$h_y^{S(n)}(z) = -2\pi m_{Y_n}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}} \left(e^{-|\mathbf{k}|(-y+\kappa_n+d)} - e^{-|\mathbf{k}|(-y+\kappa_n)} \right).$$

Averaging along the thickness of the ν layer,

$$h_X^{S(\nu)}(z) = -i2\pi m_{Y_n} k \sin \varphi_\nu e^{i\mathbf{k}\cdot\mathbf{r}} \frac{(e^{d|\mathbf{k}|} - 1)^2 e^{(\kappa_\nu - \kappa_n - d)|\mathbf{k}|}}{d|\mathbf{k}|^2} \quad (3.49)$$

and

$$h_Y^{S(\nu)}(z) = 2\pi m_{Y_n} e^{i\mathbf{k}\cdot\mathbf{r}} \frac{(e^{d|\mathbf{k}|} - 1)^2 e^{(\kappa_\nu - \kappa_n - d)|\mathbf{k}|}}{d|\mathbf{k}|} \quad (3.50)$$

Similar to the case of volumetric charges, the behavior is comparable because the dipolar interaction aims to prevent the formation of magnets. However, for surface charges, the normal component maintains its orientation, while the direction of the tangential component depends on whether the upper or lower layers are considered. The stray fields due to surface magnetic charges are shown in Fig. 3.3, while the total stray field, which includes both volumetric and surface magnetic charges is depicted in Fig. 3.4.

3.0.5 Interlayer exchange field

The interlayer exchange, energy per unit area, between two neighboring layers is

$$\epsilon_{\text{bl}}^{(\eta\nu)} = -J_{\text{bl}} \frac{\mathbf{M}^{(\eta)} \cdot \mathbf{M}^{(\nu)}}{M_s^{(\eta)} M_s^{(\nu)}}. \quad (3.51)$$

Then, to obtain the components for the static and dynamics exchange fields, it is necessary to project the basis of layer ν into layer η . Using Fig. 3.1 as a reference and with some trigonometry, it is obtained that

$$\begin{aligned} \hat{Z}_\nu &= \cos(\varphi_\nu - \varphi_\eta) \hat{Z}_\eta - \sin(\varphi_\nu - \varphi_\eta) \hat{X}_\eta, \\ \hat{X}_\nu &= \sin(\varphi_\nu - \varphi_\eta) \hat{Z}_\eta + \cos(\varphi_\nu - \varphi_\eta) \hat{X}_\eta. \end{aligned} \quad (3.52)$$

Therefore, the magnetization $\mathbf{M}^{(\nu)}$ can be written in terms of the coordinates (X_η, Y_η, Z_η) . Thus, the energy density becomes

$$\begin{aligned} \epsilon_{\text{bl}}^{(\eta\nu)} &= -\frac{J_{\text{bl}}}{M_s^{(\eta)} M_s^{(\nu)}} (M_s^{(\eta)} M_s^{(\nu)} \cos(\varphi_\nu - \varphi_\eta) + M_s^{(\eta)} m_{X_\nu} \sin(\varphi_\nu - \varphi_\eta) \\ &\quad - m_{X_\eta} M_s^{(\nu)} \sin(\varphi_\nu - \varphi_\eta) + m_{X_\eta} m_{X_\nu} \cos(\varphi_\nu - \varphi_\eta) + m_{Y_\eta} m_{Y_\nu}) \end{aligned}$$

Similar to the previous cases, this energy density allows for obtaining effective fields. In this case, the effective exchange field components are

$$\begin{aligned} h_{Y_\eta}^{\text{bl}(\eta)}(\mathbf{r}, t) &= \sum_\nu (\delta_{\eta+1}^\nu + \delta_{\eta-1}^\nu) \frac{J_{\text{bl}}}{d\mu_0 M_s^{(\eta)} M_s^{(\nu)}} m_{Y_\nu}^{(\nu)}(\mathbf{r}, t) \\ h_{X_\eta}^{\text{bl}(\eta)}(\mathbf{r}, t) &= \sum_\nu (\delta_{\eta+1}^\nu + \delta_{\eta-1}^\nu) \frac{J_{\text{bl}}}{d\mu_0 M_s^{(\eta)} M_s^{(\nu)}} \cos(\varphi_\eta - \varphi_\nu) m_{X_\nu}^{(\nu)}(\mathbf{r}, t) \end{aligned}$$

and

$$H_{Z_\eta}^{\text{bl}0(\eta)}(\mathbf{r}) = \sum_\nu (\delta_{\eta+1}^\nu + \delta_{\eta-1}^\nu) \frac{J_{\text{bl}}}{d\mu_0 M_s^{(\eta)}} \cos(\varphi_\eta - \varphi_\nu) \quad (3.53)$$

$$H_{X_\eta}^{\text{bl}0(\eta)}(\mathbf{r}) = \sum_\nu (\delta_{\eta+1}^\nu + \delta_{\eta-1}^\nu) \frac{J_{\text{bl}}}{d\mu_0 M_s^{(\nu)}} \sin(\varphi_\eta - \varphi_\nu) \quad (3.54)$$

3.0.6 Equilibrium condition

From LL equation, it is obtained the equilibrium conditions of the system, this is $H_{Y_n}^{\text{e0}} = H_{X_n}^{\text{e0}} = 0$. The first condition $H_{Y_n}^{\text{e0}} = 0$ implies that the equilibrium magnetization is in the plane. The in-plane static component of the effective field $H_{X_n}^{\text{e0}}$ contains the angle φ_n , which is necessary to find for every sublayer. Such a component is given by

$$H_{X_n}^{\text{e0}} = H \sin(\varphi_H - \varphi_n) + H_u \cos \varphi_n \sin \varphi_n + \sum_{\nu} \frac{J_{n\nu} \sin(\varphi_n - \varphi_{\nu})}{d_n \mu_0 M_{S_n}}, \quad (3.55)$$

where H is the magnitude of the external field, φ_H is the angle that H makes with the z axis and H_u is the uniaxial anisotropy field of sublayer n , with x being the easy axis. The term $J_{n,\nu}$ corresponds to the exchange constant strength, and d_n is the thickness of n -th sublayer. It is worth mentioning that the exchange interaction acts over nearest neighbors, which can be written as

$$J_{n,\nu} = J_{n,\nu+1} \delta_n^{\nu+1} + J_{n,\nu-1} \delta_n^{\nu-1}, \quad (3.56)$$

where δ_i^j is the Kronecker delta function ($\delta_i^j = 0$ for $i \neq j$ and $\delta_i^j = 1$ for $i = j$). If n and ν correspond to the same FM layer, the exchange term becomes $J_{n,\nu} = J_{\text{intra}}$. In the case that n and ν correspond to the sublayers separated by the spacer s , such as coupling of the synthetic antiferromagnet, then $J_{n,\nu} = J_{\text{inter}}$, where J_{inter} is the interlayer exchange constant

3.1 Matrix elements

By comparing the Eqs.(3.7)-(3.10), with the dynamic and static fields, we obtain $T_{Y_n}^{X_n} = 0$ and $T_{X_n}^{Y_n} = 0$. Besides, the nonzero terms are

$$T_{X_n}^{X_n} = -4\pi\zeta(k, d) - 4\pi[\lambda_{\text{ex}}]^2 k^2 \quad (3.57)$$

$$T_{Y_n}^{Y_n} = \frac{H_u}{M_s} \cos^2 \varphi_n - 4\pi[\lambda_{\text{ex}}]^2 k^2 - 4\pi[1 - \zeta(k, d)] \sin^2 \varphi_n \quad (3.58)$$

$$T_{Y_n}^{X_{\nu}} = \text{sgn}(\kappa_{\nu} - \kappa_n) i 2\pi k \sin \varphi_n \frac{(e^{|\mathbf{k}|d} - 1)^2 e^{(-d - |\kappa_n - \kappa_{\nu}|)|\mathbf{k}|}}{|\mathbf{k}|^2 d} \quad (3.59)$$

$$T_{X_n}^{Y_{\nu}} = \text{sgn}(\kappa_{\nu} - \kappa_n) i 2\pi k \sin \varphi_{\nu} \frac{(e^{|\mathbf{k}|d} - 1)^2 e^{(-d - |\kappa_n - \kappa_{\nu}|)|\mathbf{k}|}}{|\mathbf{k}|^2 d} \quad (3.60)$$

$$T_{X_n}^{X_{\nu}} = \frac{J_{n,\nu} \cos(\varphi_n - \varphi_{\nu})}{d\mu_0 M_s^{(n)} M_s^{(\nu)}} + 2\pi k^2 \sin \varphi_n \sin \varphi_{\nu} \frac{(e^{|\mathbf{k}|d} - 1)^2 e^{(-d - |\kappa_n - \kappa_{\nu}|)|\mathbf{k}|}}{|\mathbf{k}|^3 d} \quad (3.61)$$

$$T_{Y_n}^{Y_{\nu}} = \frac{J_{n,\nu}}{d\mu_0 M_s^{(n)} M_s^{(\nu)}} + 2\pi \frac{(e^{|\mathbf{k}|d} - 1)^2 e^{(-d - |\kappa_n - \kappa_{\nu}|)|\mathbf{k}|}}{|\mathbf{k}| d}. \quad (3.62)$$

Chapter 4

Results: Static and dynamic properties of noncollinear magnetized ferromagnetic films

This chapter comprises the first results of the thesis. It is studied the static and dynamic properties for thick ferromagnetic films which allow the formation of texture along the thickness. Two critical cases are studied and compared with macrospin model. The results showed that the contribution of noncollinear magnetic moments may mimic the role of the biquadratic interaction of in the macrospin model.

The static and dynamic properties of nanostructured magnetic systems have been widely studied due to their fundamental physics and technological application perspectives [34, 44–46]. These structures can be classified into categories based mainly on their shape, composition and magnetic properties. According to the shape, the magnetic nanostructures can be classified into magnetic nanoparticles [47–49], magnetic nanowires and nanotubes [50–54], ultrathin magnetic films [55, 56], magnetic quantum dots [57, 58], magnonic crystals [59–64], and magnetic superlattices [65–67], among others. Due to the complexity of the magnetization distribution and geometry of the nanomagnetics, their physical properties have been studied mainly using computational tools [68–70]. Nonetheless, analytical or semianalytical theories are feasible in cases where the geometry of the nanostructure has some symmetries. For instance, magnetic nanotubes, magnetic stripes and multilayers can be analytically modeled [51, 71–77]. Similarly, spin textures such as domain walls and magnetic skyrmions can also be comprehensively investigated using analytical methods [78–82], enabling a better understating of the underlying physics governing these magnetic systems.

In this chapter its studied the statics and dynamics properties of ferromagnetic films with noncollinear states of the magnetization and wave vector $k = 0$. The analysis is done through the dynamic matrix method, which subdivides the film into multiple sublayers to account for variations in the magnetization across the thickness and are compare with the macrospin model [22], where the magnetic moments of the structure are considered as a single macroscopic entity with a fixed magnitude and orientation, which is only affected by an effective field, Nevertheless, this model cannot considered the thermal impact of fluctuations on the particle's magnetic moment, which can modify some magnetic characteristics [83]. Besides, the macrospin model does not consider the effects of surface interactions, which can be significant in certain type of nanostructures [84, 85]. As the thickness of the nanostructure increases, for instance, there is a notable difference in the magnetic behavior of the moments at the interfaces compared to that of the bulk region due to variations in exchange coupling. Additionally, the system tries to avoid the formation of surface magnetic charges. As a result, a change in the equilibrium magnetization across the thickness is anticipated, rendering the macrospin model is invalid.

This study investigates the equilibrium magnetization states and the ferromagnetic resonance response in a film with a nonuniform anisotropy profile across the thickness and a synthetic antiferromagnet. In the second system, the interfacial interlayer exchange interaction result in a

noncollinear magnetization rotation at low fields, creating a twisted magnetization states that mimics a similar role to the well-know biquadratic exchange interaction [86].

The idea of this work was to compare the differences of the dynamic matrix model(DMM) and the macrospin model(MCS) for two cases with presence of texture and $k = 0$. The first corresponds a single layer with nonhomogeneous uniaxial anisotropy, where different anisotropy profiles along the thickness are considered. Second case is a synthetic antiferromagnetic structure, where the antiparallel alignment of the magnetization is promoted by a interlayer exchange interaction with $J < 0$. It has to be mentioned that for both system are consistent with a noncollinear magnetization rotation when the external magnetic field is applied along the film plane. The system under study is shown in Fig. 3.1. Here, a ferromagnetic structure with a noncollinear in-plane magnetization along the film's normal is considered, where such a rotated magnetization state can be induced by gradients of some magnetic parameter of interfacial effects that break the symmetry along the film thickness.

For the analysis, the reference material is Permalloy(Py). Hence, the magnetic parameters considered in the discussion are $M_s = 1200$ kA/m for the saturation magnetization, and $A_{ex} = 18$ pJ/m for the exchange constant. The exchange parameter J , associated with the interlayer exchange coupling, is written in terms of the constant A_{ex} . This relation can be obtained by analyzing the transition from the discrete to the continuous case since if the number of sublayers increases, the dynamic properties of the continuous case are reached. Thus, if the number of layers is considerable, it can be demonstrated that $J = 2A_{ex}/b$ [87].

For the FM single layer are considered different anisotropy profiles along the thickness. To make the comparison reasonable, the profiles keep the same average of anisotropy value ($\langle \mu_0 H_u \rangle = 100$ mT). The easy axis is given by x ($\varphi_H = 0$), while the external field is applied along z ($\varphi_H = 0$). Fig.4.1(a)-(c) depicts the case of a uniform anisotropy profile (Case 1) which reproduces results from macrospin model observed by Farle [88]. Fig.4.1(b) shows a uniform magnetization reorientation as an external field along the hard axis is increased. Fig.4.1(c) illustrates the two ferromagnetic low-frequency FMR modes within the frequency range of 0-30 GHz, with higher-order modes being excited at frequencies beyond this range, although they are not presented in the figure. The solid line represents the uniform in-phase mode, while the dash line corresponds to the first excited out-of-phase mode. Here, it is observed that the frequency becomes zero at the nucleation field ($\mu_0 H_N$), defined as the field in which the first deviation from a uniformly magnetized state appears. Case 2 shows an example of a non-uniform anisotropy, as illustrated in Fig.4.1(d)-(f), where the anisotropy parameter has a gradual variation across the film thickness. The magnetization equilibrium angle is shown in Fig.4.1(e), where a clear twisted state of the magnetization is observed before the saturation [$\mu_0 H_N \approx 122$ mT, as shown in Fig.4.1(f)]. The magnetic moments with low-local anisotropy [darker points in Fig. 4.1(e)] follow the external field first and then follow the rest of the magnetization. The frequency-field dependence of the modes does not change notably [see Figs.4.1(c) and 4.1(f)], but the nucleation field increases due to the zones with high anisotropy. Finally, an interfacial like anisotropy profiles is considered in Case 3, where the effective anisotropy field is localized at the surface of the FM layer, as shown in Fig.4.1(g).

Here, the noncollinear magnetization state is enhanced due to the strong nonhomogeneity of the anisotropy, as shown in Fig.4.1(h). The magnitude of the nucleation field significantly exceeds that of the previous cases, being more than three times larger than in the uniform scenario. As the thickness decreases, the difference between the nucleation fields for uniform and nonuniform anisotropies is reduced because the exchange interaction dominates, forcing the magnetic moments to rotate coherently. This behavior is evidenced in the inset of Fig.4.1(g), where the nucleation field (calculated for an anisotropy profile of Case 3) is shown as a function of the thickness. On the other side, the frequency of the low-frequency mode is reduced at small fields [see solid line in Fig. 4.1(i)], reaching a maximum at around $\mu_0 H = 200$ mT. This dynamic behavior can be explained through the calculations of the magnetization profiles that are represented by the in-plane dynamic magnetization component m_{X_n} , as shown in the insets of Fig. 4.1. At small fields, the magnetization oscillates with a large amplitude on the zones with reduced anisotropy; therefore, it implies that the frequency of such modes is low due to the reduction of the local internal field. As the external field increases, the oscillation amplitude gradually increases in the high anisotropy region [lighter lines in the insets of Fig. 4.1(i)], increasing thus the frequency. However, as the external field increases further, the frequency reduces again because the amplitude of the oscillations becomes significant in the zones with high anisotropy, where, in turn, the magnetic moments are not fully oriented along the external field direction [89].

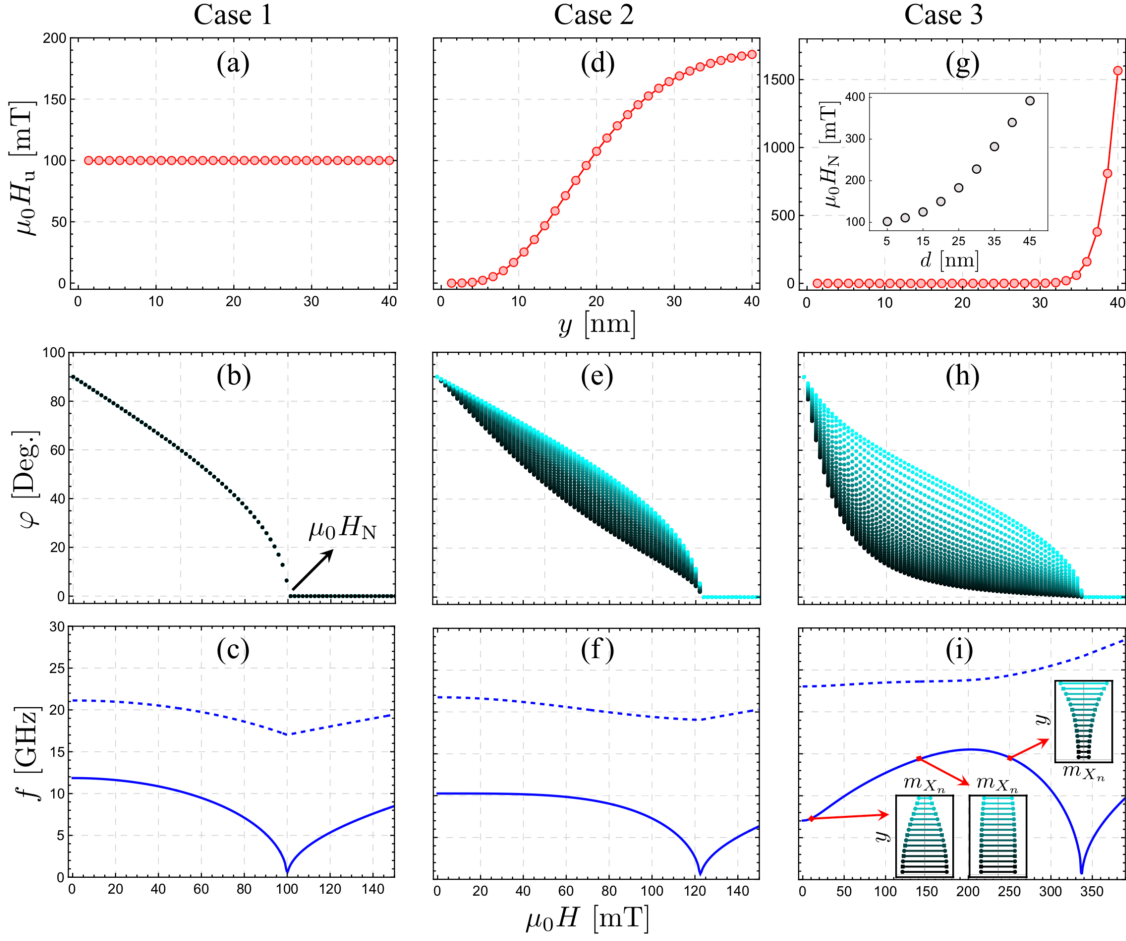


Figure 4.1: Static and dynamic properties of a ferromagnetic film with a nonhomogeneous anisotropy profile evaluated at $d = 40$ nm. In (a)–(c), a constant anisotropy is considered, while in (d)–(f), a modulation in the whole bulk is assumed. An interfacial-like anisotropy profile is used in (g)–(i). The top panels show the anisotropy profile, the central panels depict the magnetization’s in-plane equilibrium angle, and the lower panels show the ferromagnetic resonance frequency f as a function of the external field. The darker (lighter) points in (e) and (h) correspond to the magnetization located at the zones with low (high) anisotropy. The inset in (g) illustrates the nucleation field $\mu_0 H_N$ as a function of the film thickness. The insets in (i) correspond to the in-plane dynamic magnetization components evaluated at $\mu_0 H = 10$ mT, 140 mT, and 250 mT. In plots (c), (f), and (i), the solid line represents the in-phase uniform mode, while the dashed line corresponds to the first excited out-of-phase mode.

Now, the case of a synthetic antiferromagnet is addressed. Here, the interfacial nature of the interlayer exchange interaction is responsible for the noncoherent rotation of the magnetization under the application of an external field. Fig. 4.2 shows three cases where the thicknesses of the individual layers have been varied for a symmetric antiferromagnet ($d_1 = d_2$), while an interlayer interaction strength (bilinear exchange term) of $J_{bl} = -1$ mJ/m² is used to remain the antiferromagnetic state stable at a zero applied field, $\mu_0 H = 0$. An in-plane uniaxial anisotropy of $\mu_0 H_u = 100$ mT is implemented to the bottom layer, resulting in the magnetization of this layer remaining parallel to the external field at small fields [see inset in Fig. 4.2(a)]. Similarly to the previous case, Fig. 4.2(a)–(c) are the results known from [90,91], getting a convergence with the FMR and hysteresis loop of those papers and this is going to be the base to compare and drive the discussion to the other cases. As expected, three distinctive regions are obtained by applying an external field along the easy axis ($\varphi_H = 90^\circ$). In small fields, the magnetization remains antiparallel. Then, at a critical field $\mu_0 H_c$, the antiferromagnetic state becomes unstable, and the magnetization begins to rotate. This intermediate state is called a spin-flop regime [92,93], which is

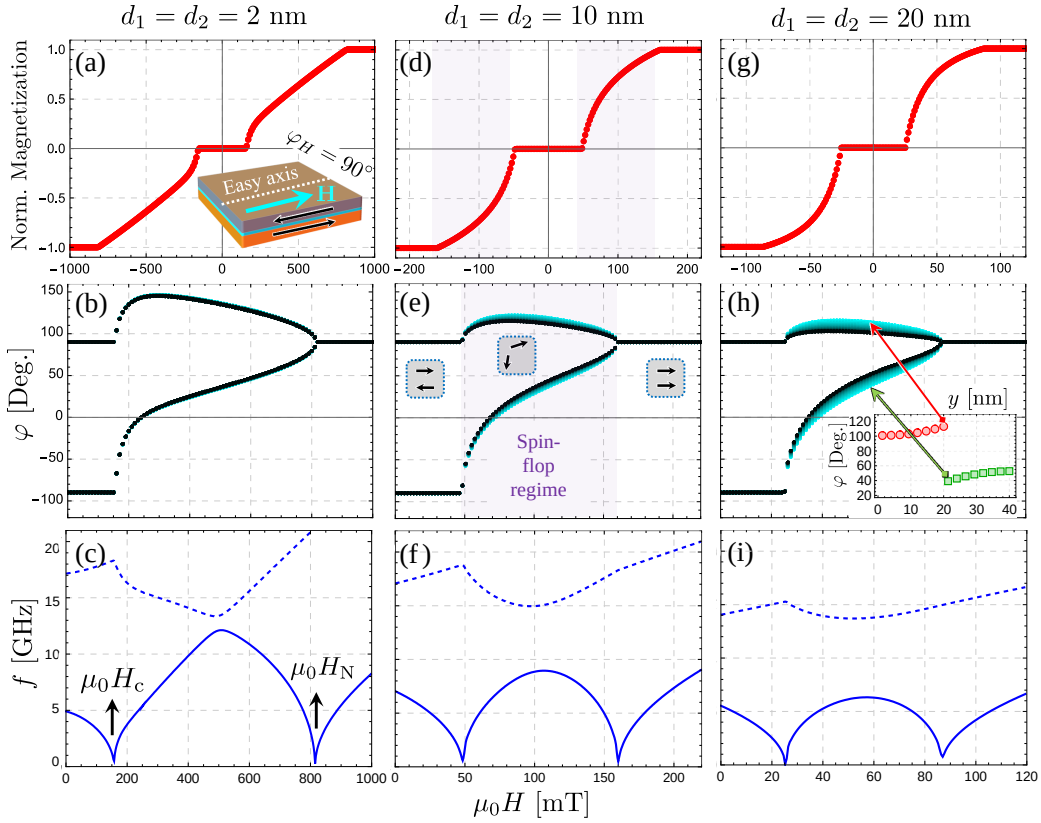


Figure 4.2: Hysteresis loop, equilibrium angles, and ferromagnetic resonance frequency of synthetic antiferromagnetic nanostructures. In all cases, the external field is applied along the easy axis x ($\varpi_H = 90^\circ$). (a)–(c), (d)–(f), and (g)–(i) correspond to the cases $d_1 = d_2 = 2$ nm, $d_1 = d_2 = 10$ nm, and $d_1 = d_2 = 20$ nm, respectively. In (b), (e), and (h), the lighter color in the curves represents the magnetic moments at the nonmagnetic/ferromagnetic interfaces, while the darker points correspond to the magnetization at the top/bottom of the films. The inset in (h) shows the twisting equilibrium angles of both layers as a function of the normal coordinate y evaluated at $\mu_0 H = 60$ mT. Circles correspond to the bottom layer, while squares represent the equilibrium angle of the top layer. The spin-flop regime is highlighted in (e). In plots (c), (f), and (i), the two low-frequency modes, also referred to as optic and acoustic modes, are illustrated as a function of the external field.

an equilibrium configuration where the magnetization are neither antiparallel nor parallel [see Fig. 4.2(e)]. As the external field increases further, the saturated state is reached at $\mu_0 H = \mu_0 H_N$, where $\mu_0 H_N$ is the nucleation field defined above. According to the model presented in this paper, the magnetic moments do not evolve coherently during the rotation in the spin-flop zone; instead, they describe a noncoherent rotation caused by the interfacial interlayer exchange interaction that couples the magnetization more strongly in the nonmagnetic/ferromagnetic interfaces. The magnetization angles at the bottom and top of the film ($y = 0$ and $y = 40$ nm, respectively) are closer to the external field angle $\varphi_H = 90^\circ$. This behavior is observed in the inset of Fig. 4.2(h) for the case $d_1 = d_2 = 20$ nm, where the equilibrium angles are evaluated at $\mu_0 H = 60$ mT. The equilibrium angles of the magnetic moments at the interfaces are described by the lighter curves in Fig. 4.2(b),(e),(h), while the darker points correspond to the magnetization angles at the top and bottom of the film. As the thicknesses of the ferromagnetic layers decrease, the magnetization rotation becomes more coherent, which is caused by the intralayer exchange energy, as mentioned above [see Figs. 4.2(b) and (e)]. In Figs. 4.2(c), (f) and (i), two low-frequency modes, known as the optic and acoustic modes, are depicted as a function of the external magnetic field. The assignment of optic and acoustic characteristics is based on the phase of magnetization oscillations, which undergoes a change within the spin-flop zone, as detailed in Ref. [93].

Now, the model based on the dynamic matrix method is compared with the macrospin model,

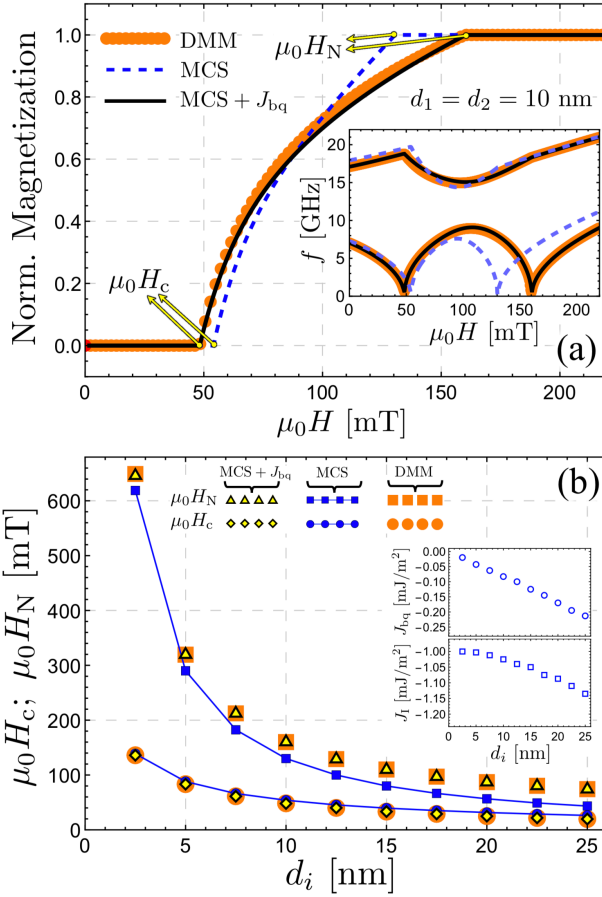
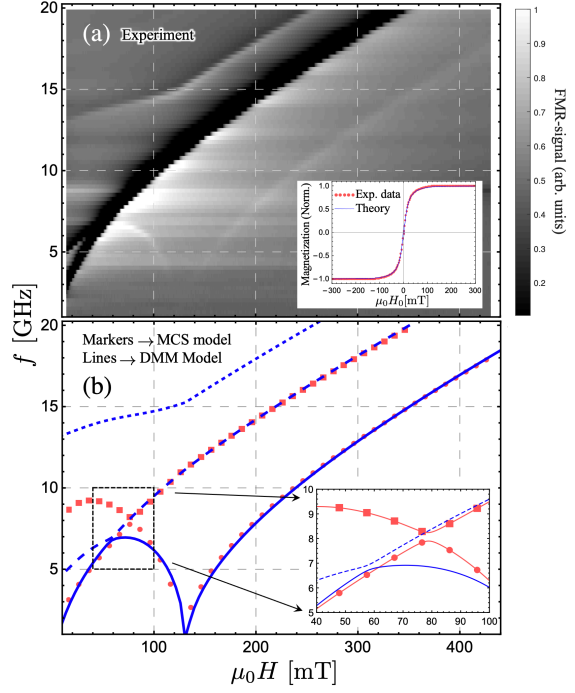


Figure 4.3: (a) Hysteresis loop for the case $d_1 = d_2 = 10$ nm. Orange circular dots show the dynamic matrix method (DMM) results, while the dashed (solid) line corresponds to the macrospin (macrospin plus J_{bq}) model. Inset in (a) depicts the FMR frequency as a function of the external field. (b) Critical and nucleation fields as a function of the thickness d_i . The DMM, MCS, and MCS + J_{bq} models are compared. The matching between DMM and MCS + J_{bq} is reached by modifying the interlayer exchange constant J_I and including J_{bq} as shown in the inset in (b). For the DMM case, the interlayer exchange constant is fixed as $J_I = -1$ mJ/m².

which assumes that the magnetization is uniform in the whole thickness of the magnetic material. The case $d_1 = d_2 = 10$ nm is taken as an example to analyze both models in Fig. 4.2. The first quadrant of the hysteresis loop at positive fields is shown in Fig. 4.3(a), where the DMM that considers noncoherent magnetization (solid orange circles) is compared with the macrospin approach (blue dashed lines). As observed, both models do not match perfectly, which is seen by the mismatch between the field $\mu_0 H_c$ and $\mu_0 H_N$. Nevertheless, including the well-known biquadratic exchange term, described by the constant J_{bq} , into the MCS model, it is possible to reach the results based on the noncoherent approach described in this chapter. The comparison between MCS + J_{bq} (solid black line) and DMM is depicted in Fig. 4.3(a), where the inset shows that the dynamic response (frequency vs. external field) also matches under the inclusion of J_{bq} . Of course, the requirement of a good agreement between the different models implicates that in the MCS + J_{bq} approach, the bilinear exchange constant J_{bl} must be modified, while in the DMM, it is fixed to be $J_{bl} = -1$ mJ/m². Fig. 4.3(b) shows the critical and nucleation field ($\mu_0 H_c$ and $\mu_0 H_N$, respectively) as a function of the layer thickness d_i , where $d_i = d_1 = d_2$. Here, the discrepancies between the MCS and DMM models are observed for different thicknesses, while the case MCS + J_{bq} matches perfectly under a modification of the interlayer exchange constant and the inclusion of J_{bq} , as shown in the inset of Fig. 4.3(b). An essential remark regarding these results is that the effects induced by the noncoherent rotation of the magnetization can be captured into the MCS model by including a biquadratic interaction. This implies that the contribution of the biquadratic term, typically associated with interface morphology characteristics, might be overestimated in the case of thick ferromagnetic films, where magnetization twisted states are feasible. In the context of thin or ultrathin film bilayers, the emergence of distinctive twisted states within the equilibrium magnetization is precluded by the intralayer exchange coupling. Consequently, the macrospin model effectively captures both the static and dynamic properties of these synthetic antiferromagnets [94–100]. In addition, a rotated state is also dependent on the strength of the interlayer exchange constant J_{bl} a parameter intricately linked to spacer thickness. Namely, if J_{bl} is small, the rotated states are absent even for thick films, making the macrospin model again appropriate for describing the system's behavior.

Figure 4.4: (a) Frequency as a function of the external field. Color code represents the FMR signal in arbitrary units. The hysteresis loop is shown in the inset, where dots represent the measurements, and the lines are obtained from the DMM model. (b) Markers (dots and squares) and lines correspond to the calculated results using the MCS and DMM models, respectively. The inset in (b) illustrates the frequency range close to the avoided-crossing gap, which appears due to the thickness asymmetry of the stack. The parameters utilized for the calculations are described in the text.



Finally, a comparison is realized between the theoretical approach and the static and dynamic measurements conducted on a synthetic antiferromagnet consisting of two ferromagnetic layers made of permalloy (Py) separated by an iridium (Ir) spacer. The thicknesses of the Py films are 40 and 30 nm, while the Ir thickness is 0.4 nm [Py(40)/Ir(0.4)/Py(30)], respectively. Due to the nature of the Ir spacer, and antiferromagnetic state is induced between the FM layers ($J_{bl} < 0$) Ref. [101]. The gray color code in Fig 4.4(a) illustrates the experimental results for the frequency as a function of the external field, $f(H)$, while the inset depicts the hysteresis loop. Both measurements [$f(H)$ and the hysteresis loop] are compared with the model presented in this paper [lines in Fig. 4.4b], where one can observe that the model reproduces the data perfectly using the following parameters: $M_S^{\text{Py}} = 791$ kA/m, $A_{\text{ex}}^{\text{Py}} = 9$ pJ/m, $J_{bl} = -0.6$ mJ/m², and $J_{bq} = -0.06$ mJ/m². In contrast, the macrospin model is not able to reproduce the data quantitatively in the range of small fields ($H < H_N$). Indeed, the frequency of the high-frequency mode is inconsistent with the experiments (see squares at $\mu_0 H < 80$ mT). Only a qualitative agreement is reached in this case, where the fitted parameters are $J_{bl} = -0.7$ mJ/m² and $J_{bq} = -0.5$ mJ/m² (M_S^{Py} and $A_{\text{ex}}^{\text{Py}}$ are the same used previously). The inset in Fig. 4.4(a) illustrates the frequency gap resulting from avoided crossing, an effect attributed to the thickness asymmetry of the synthetic antiferromagnet Ref. [99, 100, 102], while both models exhibit this mode coupling, the observation occurs at distinct frequency and wave-vector ranges for each. Comparing the two theoretical models makes it apparent that MCS model necessitates a substantial value of the biquadratic exchange to replicate the data qualitatively. Indeed, the MCS approach employs a value of J_{bq} that is approximately nine times greater than used in the DMM. Therefore, the correct calculations of the rotated ground state significantly change the magnitude of the biquadratic exchange constant. This observation carries significant implications, suggesting that the biquadratic term could have been inaccurately treated and derived in prior studies of thick films where the equilibrium magnetization configuration displays a degree of torsion along the thickness. Incorporating the rotated magnetization configuration into the model substantially reduces the biquadratic interaction's strength, aligning with its interfacial nature. A considerable magnitude of the biquadratic exchange constant is found in the literature when the coupled layers have a thickness exceeding the material's intrinsic exchange length ($\ell_{\text{ex}} = \sqrt{2A_{\text{ex}}/\mu_0 M_s^2}$) Ref. [103–108], where the macrospin model has been used to reproduce the measurements. In Ref [105], for instance, the $f(H)$ curve is similar to the one illustrated in Fig. 4.4 for a sample [Py(30)/Ru(0.49)/Py(30)] [see Fig.1(a) of Ref. [105]], where the MCS model requires a biquadratic constant of $J_{bq} = -0.514$ mJ/m². A further noteworthy observation depicted in Fig. 4.4 is that both models (MCS and DMM) exhibit exceptional agreement with the measurements in the saturated region ($\mu_0 H > \mu_0 H_N$) and succeeding the point of satu-

ration. This analysis may not be crucial in thin films with minimal magnetic texture throughout their thickness. However, the analysis of the rotated states is fundamental for thick ferromagnetic films that exhibit interfacial or even graded magnetic properties. For more information about the derivation and how is introduced the biquadratic exchange interaction into the dynamic matrix method in Appendix B

Chapter 5

Results: Spin-wave modes in multilayered synthetic antiferromagnets

This chapter comprises the second results of this thesis. The spin wave spectra of multilayered synthetic antiferromagnet is studied. This study reveals that depending on whether the number of layers is odd or even, the spin wave band structure becomes symmetric or asymmetric under wave vector reversion. Besides, under the increase in the number of layers, the bands will segregate into groups associated with bulk excitations. However, for an even number of layers, a chiral surface mode emerges between the bulk modes, representing a mode excited at the top or bottom of the multilayered structure.

As seen in chapter 4, SAFs are engineered magnetic materials designed to possess distinctive and highly adjustable static and dynamics properties [109,110]. In contrast to natural antiferromagnets materials, where magnetic moments inherently align in opposing directions, SAFs are artificially constructed by sandwiching two or more FM layers with a thin nonmagnetic spacer layer. By precisely tuning the properties of the spacer, adjacent ferromagnetic layers can align their magnetic moments antiparallel to each other, leading to a reduction in overall magnetic stay fields [109]. This antiparallel alignment is mediated by an interlayer exchange interaction, whose strength is influenced by the thickness and composition of the spacer layer [111–114], which can comprise metallic [115,116], semiconducting [117], and insulating [118] materials. The controllable properties of SAFs can also be exploited to stabilize spin textures [110,119] .

The studied system in this part is shown in Fig 5.1. Here, multiple ferromagnetic layers are coupled and separated among them for a spacer of thickness s , which promotes the antiferromagnet coupling. Every FM layer has the same magnetic properties and thickness d_l in the sample. The dynamic matrix method is used to obtain the spin wave spectra in the synthetic antiferromagnet, where the idea is to divide the film into different sublayers or slabs connected through dipolar and exchange interactions [87,120]. How the system is treated and described is similar to the first results showed in chapter 4, but now, the collective spin waves are set to propagate along z axis, and hence, the wave vector is given by $\mathbf{k} = k\hat{z}$. This consideration must be done because the backward volume mode will be analyzed, such modes with no presence of external magnetic field ($H = 0$), and the evolution of these modes when the external field increases.

The parameters used for the calculations are the following: saturation magnetization $M_s = 800$ kA/m, exchange constant $A = 10$ pJ/m(exchange length $l_{\text{ex}} = 4.46$ nm), and the spacer thickness $s = 0.4$ nm. These magnetic parameters are based on Permalloy, and the spacer thickness is based on samples described in chapter 4, where a 0.4 nm Ir spacer causes antiparallel coupling between FM layers. The interlayer exchange constant is $J_{\text{inter}} = -1$ mJ/m².

First, to study the magnetic properties of this system is interesting to explore any dependence or behavior in the dispersion relation in the quantity of FM layers which are coupled with a fixed thickness $d_l = 5$ nm, Fig 5.2 shows the SW dispersion for different number of layer (N_L). The case

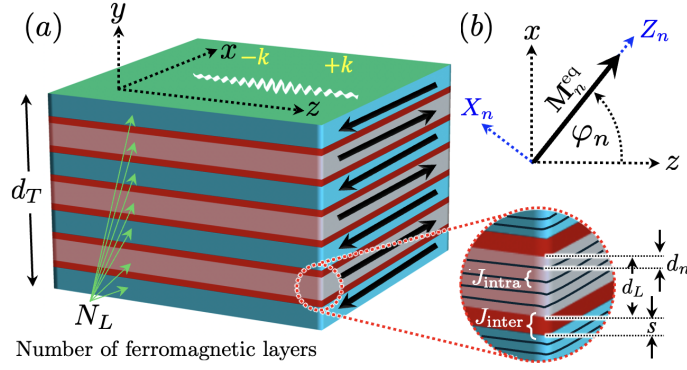


Figure 5.1: (a) Schematic representation of the multilayered synthetic antiferromagnet. This structure consists of N_L ferromagnetic layers with antiparallel in-plane magnetizations. Each ferromagnetic layer has a thickness denoted as d_L and is separated by a spacer layer of thickness s . According to the theoretical model, the ferromagnetic film is divided into many sublayers of thickness d_n , and the sublayers are exchange-coupled by means J_{inter} if the sublayers are at the interfaces in contact with the spacer, and J_{intra} if the sublayers belong to the same ferromagnetic film. The total thickness of the structure is given by $d_T = N_L d_L + (N_L - 1)s$. The spin waves are assumed to propagate along the in-plane z -axis, with y and x lying, respectively, in the normal and plane of the films. (b) Definition of the local reference system (X_n, Y_n, Z_n) , where $Y_n = y$ is normal to the multilayer and X_n lies in the film's plane. Coordinate Z_n points along the equilibrium magnetization $\mathbf{M}_n^{\text{eq}}$ of the n -th ferromagnetic layer

depicted in Fig 5.2(a) ($N_L = 2$) corresponds to the typical dispersion of synthetic antiferromagnet composed of two FM layers. The nonreciprocity obtained in the SW dispersion is consistent with the results of Gallardo *et al.* [75], where acoustic and optic modes have an asymmetric SW dispersion. Such nonreciprocity is induced by the dipolar interaction, namely, the dynamical dipolar energy is reduced only for one propagation direction, wherein the stray fields and dynamic magnetization form a flux-closure dynamical state seen in Ref. [75]. Then, the case for $N_L = 5$ depicted in Fig.5.2(b) is studied, showing a reciprocal band structure where the number of modes is equal to the N_L due to the degree of freedom available in the system. Such reciprocity behavior in the band structure is also seen in the other cases with an odd number of layers, as shown in Figs. 5.2(d) and 5.2(f), while in the cases with an even number of layers, the SW dispersion has an asymmetric behavior, as seen in Figs. 5.2(a), 5.2(c) and 5.2(e). The observed reciprocity in systems with even N_L and nonreciprocity in those with odd N_L can be intuitively understood through a simple picture involving a 180° rotation around the equilibrium magnetization axis. As illustrated in Fig. 5.2(a), when considering propagation towards the $-z$ direction (or $-k$ in SW dispersion), this is equivalent to rotating the system by 180° and then allowing wave propagation in the original positive wave vector direction. For systems with even N_L , this rotation results in a configuration where the propagation characteristics for $-k$ and $+k$ differ, indicating nonreciprocity. Conversely, as depicted in Fig. 5.2(b), for systems with odd N_L , the 180° rotated system appears identical to the original unrotated system. Therefore, the propagation for positive and negative wave vectors remains unchanged, leading to reciprocal behavior. This symmetry under rotation highlights the fundamental difference between systems with even and odd numbers of layers, explaining the observed propagation characteristics.

As the number of FM layers increases for even N_L , the degree of nonreciprocity is different in each branch. Therefore, only the two most non-reciprocal modes have been highlighted with blue and green colors for better visualization. Besides, as N_L increases, the low- and high-frequency modes (in the calculated frequency range) behave similarly for even and odd numbers of layers, evidencing a tendency toward a symmetrical SW dispersion. Also, such modes (with high and low frequencies) are concentrated into two densely populated branches, which are best appreciated for a large N_L [see Fig. 5.2(f) for instances]. These branches will be referred to as bulk modes (BMs) seen in [121,122], because such bands correspond to dynamic states that are localized in the whole nanostructure. Between these BMs, surface modes (SMs) are identified, which have a completely different nature depending on whether N_L is even or odd. If N_L is even, the surface modes have chiral properties, as shown in Fig. 5.2(e). In this case, the SMs behave as bulk modes for positive

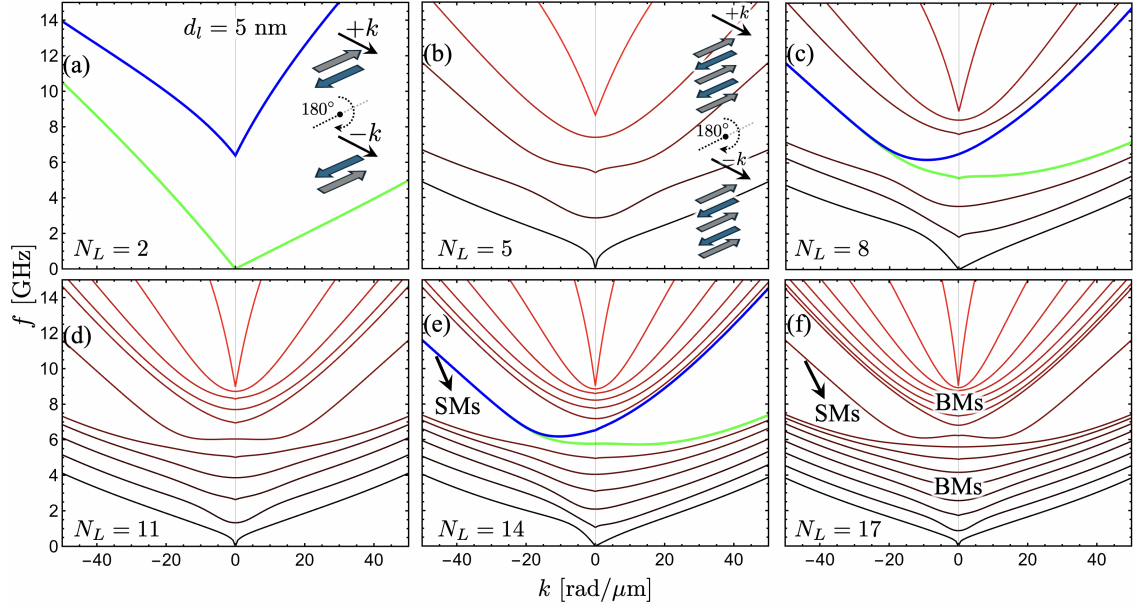


Figure 5.2: Spin wave dispersion as a function of N_L . The calculations are evaluated for $\delta_l = 5$ nm and a zero external field. Highlighted modes (blue and green curves) show the two most non-reciprocal modes. Bulk and surface modes are identified in SW bandstructure, which has chiral properties in the case of even N_L , while for odd N_L , the surface modes behave reciprocally under the inversion of the wave vector.

wave vectors, while for $k < 0$, these modes are degenerated in frequency for a large wave vector and are concentrated in the top or bottom part of the multilayer structure. If N_L is odd, as in the situation depicted in Fig. 5.2(f), the SM describes a symmetric dispersion, having the same dynamical energy for $\pm k$.

In response to the increase of interlayer exchange interaction constant in the dispersion relation, using Fig 5.2(e) as an example, the increment of J_{inter} provokes that the frequency of the modes is higher as expected. The frequency increment seen can be understood due to the antiparallel alignment between layers being stronger, to incite the excitation of the modes, more energy is necessary to make it possible. An important aspect of the previous results is the need for antiparallel alignment of the ferromagnetic layers, where the dipolar interaction creates chiral surface modes. Unlike natural antiferromagnets, the FM layers in synthetic antiferromagnets can be forced to align parallel by applying an in-plane external field. In this case, the chiral properties of the system are lost due to the lack of symmetry breaking along the thickness, making these nonreciprocal and

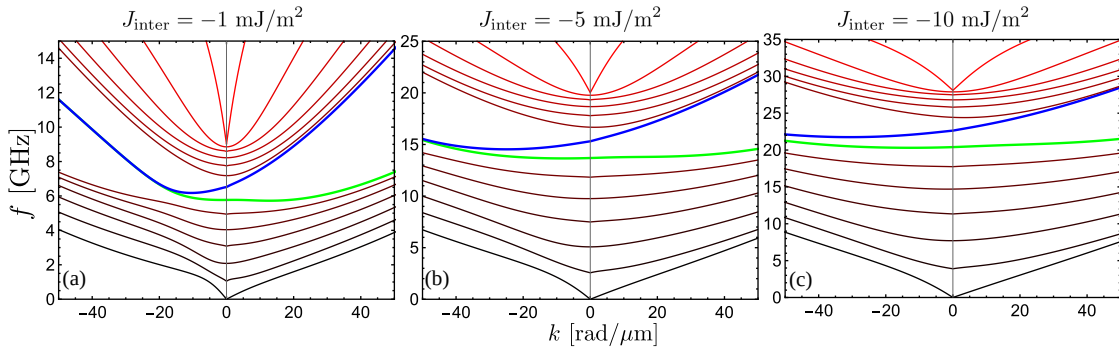


Figure 5.3: Spin wave dispersion as a function of interlayer exchange (a) $J_{\text{inter}} = -1$ mJ/m², (b) $J_{\text{inter}} = -5$ mJ/m² and (c) $J_{\text{inter}} = -10$ mJ/m². The calculations are evaluated for $d_l = 5$ nm, $N_L = 14$ nm, and zero external field.

localization properties reconfigurable.

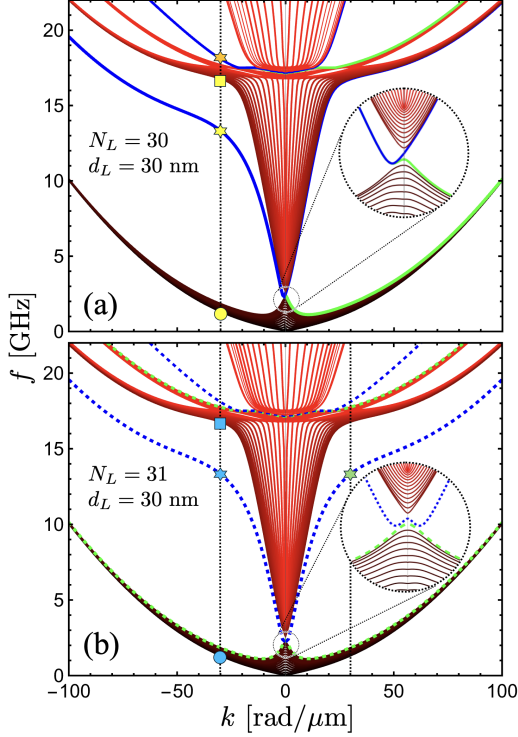


Figure 5.4: Spin-wave dispersion for $d_L = 30$ nm. In (a), the case with $N_L = 30$ is illustrated, while in (b), the number of layers is $N_L = 31$. Bulk modes are shown on a color scale from black to red, and the surface modes are highlighted in blue and green. The insets in (a) and (b) provide a magnified view of the modes within a small wave vector range, where the peculiar behavior of the surface modes can be appreciated. Circles, squares, and stars highlight the dynamic states evaluated at $|k| = 30 \text{ rad}/\mu\text{m}$, which are discussed in Fig. 5.6.

Once the influence of the quantity of FM layers coupled in the system and how the interlayer exchange interaction has a direct impact on the surface modes have been studied, the influence of dynamic dipolar interaction for multilayered SAFs of two different cases is analyzed. Namely, the $N_L = 30$ and 31 are studied, where the thickness of the FM layers are $d_l = 30$ nm. Fig. 5.4(a) shows the case $N_L = 30$, where the inset magnifies the behavior of the surface modes at small wave vectors. In this dispersion, the bulk modes are concentrated in a reduced frequency range as the thickness of the whole structure increases. For a large thickness of the system, the bulk modes tend to have the same frequency. This behavior can be anticipated since the mode quantization due to the confinement along the thickness becomes irrelevant for a large thickness of the multilayer structure. Indeed, in a standard physical picture, the perpendicular standing modes in a thick film exhibit different frequencies due to a term of the type $k_\perp = n\pi/d$ [123], being n integer number associated with the nodes of the modes. Thus, as N_L increases, such a term does not significantly influence the frequency so that all modes have similar dynamical energy of frequency. In contrast, the surface modes are still clearly identified in the band structure. For the case shown in Fig. 5.4(a), the chiral behavior is enhanced, and SMs are also present for a high-frequency range, evidencing a similar behavior reported in similar systems [?, 124, 125]. In the situation for $N_L = 31$ [Fig. 5.4(b)], the surface modes are also present, with their respective reciprocal features. Again, similarly to the cases exposed in Fig. 5.2, the range of negative wave vectors is similar for $N_L = 30$ and 31, while for $+k$, only bulk modes are excited for $N_L = 30$.

The proposed model also allows evaluating the depth profile of the dynamic response amplitude. The corresponding dynamic magnetization is shown in Fig. 5.5. In Fig. 5.5(a), the case of $N_L = 30$ is shown, as explained in Fig. 5.2, for an even number of layers coupled, the symmetry along the thickness is broken, producing a chiral behavior in the spin wave dispersion. In Fig. 5.5(b), the symmetry has been kept due to the number of layers coupled. For both cases, the green mode at $k = 30 \text{ rad } \mu\text{m}^{-1}$ shows the amplitude distribution is concentrated in the center of the magnetic structure, and this mode corresponds to an out-of-phase one. On the other side, when the green mode is evaluated at $k = -30 \text{ rad } \mu\text{m}^{-1}$, the $N_L = 31$ case is observed the same behavior for the amplitude distribution. It has the same frequency as for positive k , meanwhile, for $N_L = 30$, the surface mode is excited with higher spin wave amplitude in comparison with the previous cases, the oscillation distribution of the mode is highly localized on the surface of the multilayer structure due dipolar interaction.

The spin-wave profiles for a fixed wave vector are now calculated to demonstrate the surface

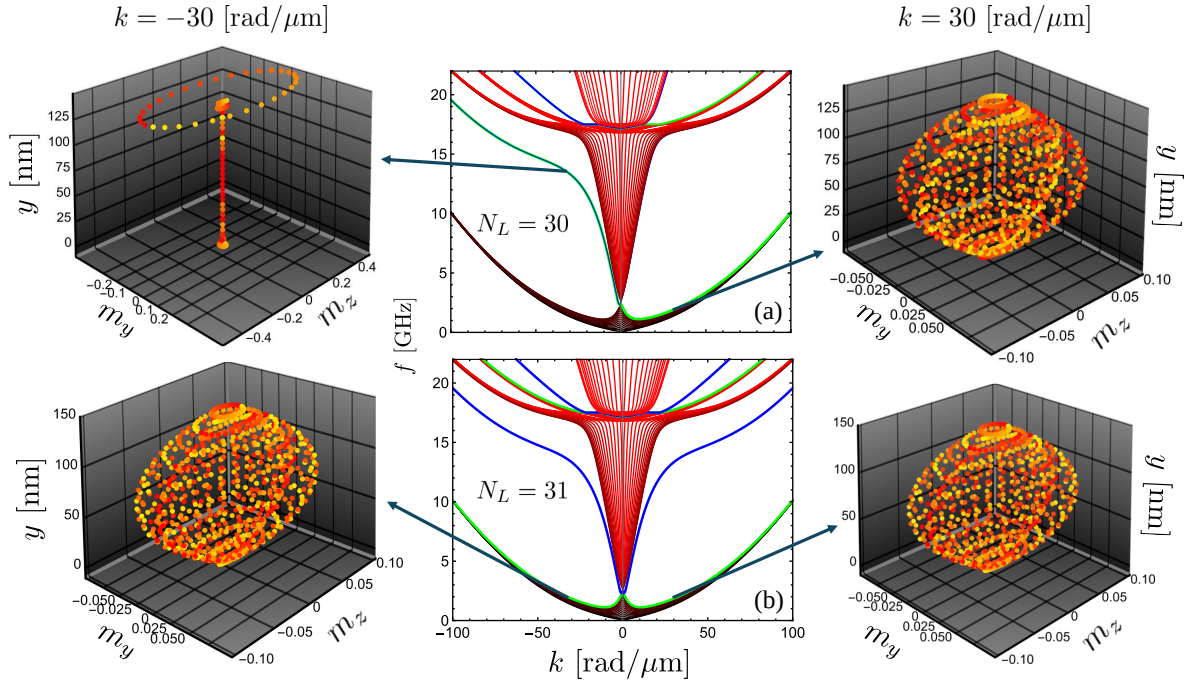


Figure 5.5: Figure (a) shows the spin wave dispersion of $N_L = 30$. In (b) depicts the case where $N_L = 31$, both cases have the same thickness. The right and left figures show the precessional amplitudes for the respective cases at $k = \pm 30$ rad μm^{-1} , where the time $t = 0$ has been highlighted with yellow dots to see the phase of the modes. From yellow to red color represents the direction of the precession of the magnetization.

and bulk nature of the modes. In Fig. 5.6(a), an illustration of two antiparallel magnetizations is shown, where the equilibrium magnetization component points along $\pm x$, and m_z (m_y) represents the in-plane (out-of-plane) dynamic magnetization component. Figs. 5.6(b)–(h) show some modes evaluated at $k = -30$ rad/ μm , $N_L = 30$, and $d_L = 30$ nm. The in-plane component m_z of the two lowest frequency modes is depicted in Figs. 5.6(b) and 5.6(c) (see circle in Fig. 5.4(a)). In these calculations, a small variation of the dynamic magnetization inside each ferromagnetic layer is observed, while the overall modulation of the magnetization along the entire structure follows the typical quantization structure. Specifically, the low-frequency mode tends to be uniform, while the second mode has a node at the center of the multilayer. Due to the significant thickness of the multilayer $d_T = N_L d_L + (N_L - 1)s = 911.6$ nm, the dynamical energy, or frequency, of these two bulk modes is quite similar, as shown in Fig. 5.4(a) (see yellow circle), where the first fourteen low-frequency modes have a similar frequency at $k = -30$ rad/ μm . In Fig. 5.6(d), the seventeenth mode (see yellow square marker in Fig. 5.4(a)) is plotted. This mode and the next thirteen modes correspond to those with one node inside each ferromagnetic layer. This is observed in Fig. 5.6(d), where within each FM film, the magnetization component m_z has negative and positive values. Because such modes are excited throughout the entire thickness of the system, they are also referred to as bulk modes.

Regarding the surface modes for $N_L = 30$, there are no such modes for positive wave vectors, as shown in Fig. 5.4(a). Therefore, the low- and high-frequency surface modes are calculated for the negative wave vector $k = -30$ rad/ μm . Figs. 5.6(e)–(f) show the low-frequency surface modes, which are degenerate in frequency, as shown in Fig. 5.4(a) (see yellow star). As observed in Figs. 5.6(e)–(f), these modes are strongly localized at the top or bottom of the nanosystem with practically the same dynamic magnetization profile. The insets in Figs. 5.6(e)–(f) show the three-dimensional magnetization oscillations of the outer layers, where the surface mode is localized. It is noted that the dynamic magnetization inside each FM layer changes along the normal direction but always with an in-phase oscillation. For the high-frequency surface mode (see orange star in Fig. 5.4(a)), similar properties of the surface modes are obtained, as illustrated in Figs. 5.6(g)–

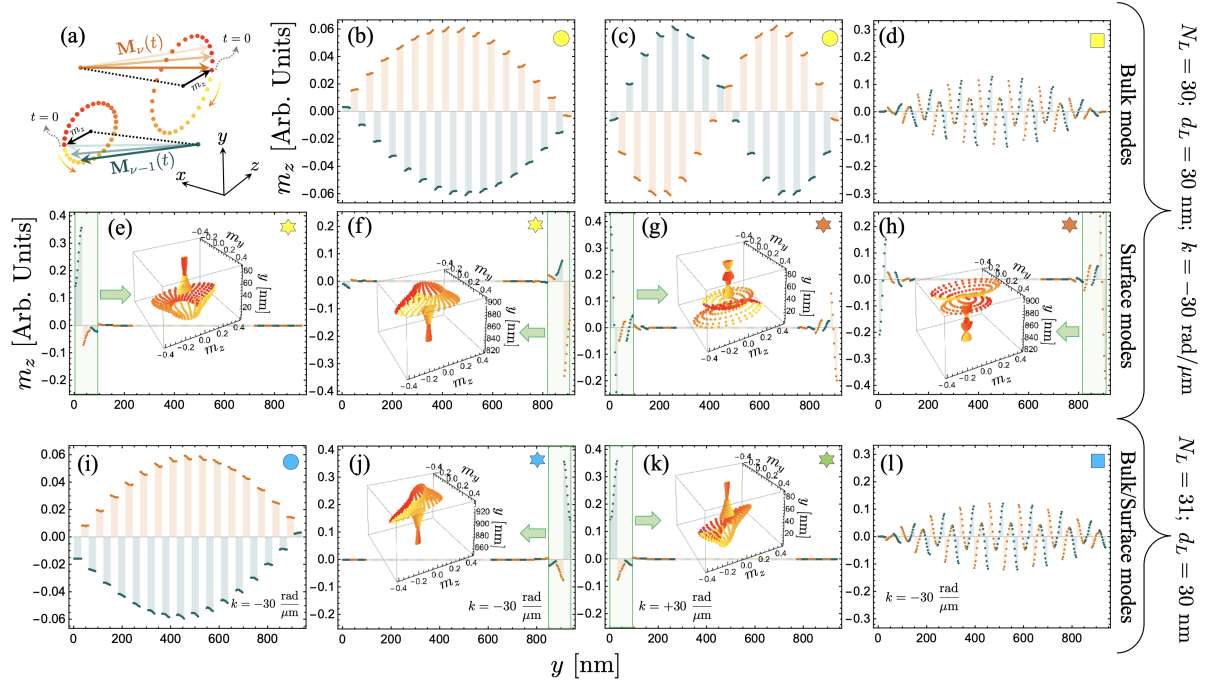


Figure 5.6: Spin-wave profiles for the bulk and surface modes highlighted by markers (circles, squares, and stars) in Fig. 5.4. In (a), a schematic representation of the magnetization oscillations is depicted. In (b)–(h), the case with $N_L = 30$, $d_L = 30$, and $k = -30$ rad/ μm is shown, where the bulk ((b)–(d)) and surface ((e)–(h)) modes are plotted as a function of the multilayer thickness. In (i)–(l), the system with $N_L = 31$ and $d_L = 30$ is considered. Panels (i) and (l) show the bulk modes, evaluated at $k = -30$ rad/ μm , against the thickness of the structure. In (j) and (k), the low-frequency surface waves are depicted for $k = -30$ rad/ μm and $k = +30$ rad/ μm , respectively.

(h). Nonetheless, the magnetic oscillations within each ferromagnetic layer are now out-of-phase, with a node close to the center of the FM film. Therefore, with increasing frequency, additional surface modes are expected to appear following the standard node distribution. For the positive wave vectors, as mentioned before, such surface modes do not exist. Namely, these modes have a unidirectional nature, which is relevant from both a fundamental and practical point of view.

The spin-wave profiles for the case $N_L = 31$ and $d_L = 30$ nm are shown in Figs. 5.6(i)–(l). Fig. 5.6(i) shows the low-frequency mode (see cyan circle in Fig. 5.4(b)), which has a similar magnetization distribution to the one shown in Fig. 5.6(b). For this reason, the low-frequency bulk modes are quite similar in the cases $N_L = 30$ and $N_L = 31$. This similarity is also observed for the high-frequency bulk modes, as shown in Figs. 5.6(d) and 5.6(l). Concerning the surface modes, it is noted that the surface mode evaluated at $k = -30$ rad/ μm (see cyan star in Fig. 5.4(b)) is completely similar to the one obtained in Fig. 5.6(f), while the surface mode calculated for a positive wave vector $k = +30$ rad/ μm (green star in Fig. 5.4(b)) is similar to the one calculated in Fig. 5.6(e). Namely, under an external excitation in the frequency range of these surface modes, the case $N_L = 31$ will exhibit two counterpropagating surface modes (with propagation along $\pm z$), with one localized on top and the other at the bottom of the system. Nevertheless, for the case $N_L = 30$, surface modes will propagate only along $-z$ due to the magnetochiral effects induced by the dipolar interaction.

An important aspect of the previous results is the need for antiparallel alignment of the ferromagnetic layers, where the dipolar interaction creates chiral surface modes. Unlike natural antiferromagnets, the FM layers in synthetic antiferromagnets can be forced to align parallel by applying an external field. In this case, the chiral properties of the system are lost due to the lack of symmetry breaking along the thickness, making these nonreciprocal properties reconfigurable. This is shown in Fig. 5.7, where the equilibrium magnetization angles of each FM layer are calculated as a function of the external field for $N_L = 8$ and $d_L = 5$ nm, with the external field applied along the z axis (see Fig. 5.1). As the external field increases, the magnetizations transition from

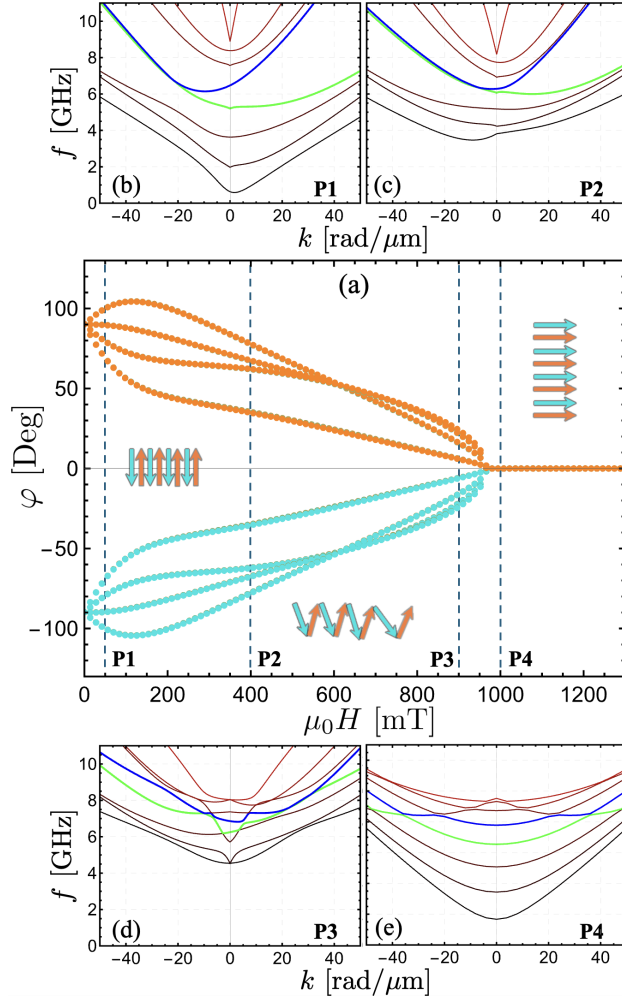


Figure 5.7: Equilibrium in-plane angle φ as a function of the external field, which is applied along the z axis. The angle φ is measured from the z axis, so that $\varphi = -90^\circ$ ($\varphi = 90^\circ$) means that the magnetization is oriented along the $-x$ (x) axis. Points (P.1), (P.2), (P.3), and (P.4) denote the equilibrium states used in the calculations of the dispersions illustrated in (b), (c), (d), and (e), respectively.

antiparallel to parallel, reaching such a parallel state at a critical field H_c . For $H < H_c$, spin flop states are observed, as illustrated in Fig. 5.7(a). The SW dispersion for equilibrium states P₁, P₂, P₃, and P₄ (Figs. 5.7(b)-(e)) shows that the chiral surface mode becomes reciprocal when the layers align parallel. The surface property also vanishes, and all modes are excited throughout the thickness of the structure when $\varphi_n = 0$ (not shown). In Fig. 5.7(d), most modes have a minimum frequency at zero wave vector, while a symmetric mode with a minimum at a nonzero wave vector behaves differently. This mode, known as the backward volume mode, is characterized by a range of wave vectors (for $k > 0$) where the group velocity is negative (small k range) and then positive (large k range). This mode presents in-phase precession along the thickness and, hence, is excited to high frequencies due to the negative interlayer exchange ($J_{\text{inter}} < 0$).

The results show the spin-wave dynamics in multilayered synthetic antiferromagnets, where the FM layers interact via interlayer dipolar and exchange interactions. Through the use of the dynamic matrix method, which allows the manipulation of the properties of FM layers along the thickness. Consequently, for thick ferromagnetic films, the intra- and inter-layer dipole-dipole interaction is intensified, influencing the band structure of spin waves. This study reveals that for an odd number of layers, the spin-wave band structure is symmetric under wave vector inversion, meanwhile, for an even number of layers, the spin-wave dispersion becomes asymmetric. With an increasing number of layers, the bands segregate into groups associated with bulk excitations. However, in the case of an even number of layers, a chiral surface mode emerges between the bulk modes, representing a mode excited at the top or bottom of the multilayered structure. The spin wave dispersions are further examined in the presence of an in-plane external field, which produces a transition from antiferromagnetic to ferromagnetic states of the equilibrium magnetization. Notably, it is demonstrated that the chiral surface modes are suppressed in the ferromagnetic equilibrium configuration.

Chapter 6

Conclusion and future view

The present work has devoted to the fundamental understanding of spin waves in ferromagnetic films. This thesis has been separated into two major parts: The analysis of ferromagnetic resonance for films with non-collinear magnetization and, the spin wave modes that are strongly localized on the surface of synthetic antiferromagnets. In this thesis has presented a theoretical aspect of spin waves dynamics in ferromagnetic films, including spin textures and symmetry breaking.

The proposal is based on the advantages of magnonics in technology applications and explores the dynamical matrix method, which uses the geometry of the sample to simplify the calculations. The method used consists of an eigenvalue problem to determine the system's frequency under the assumptions of the linear regime response and the small deviations of the magnetization.

As results of the study of the ferromagnetic resonance for the film with nonuniform anisotropy along its thickness and a synthetic antiferromagnet are the following:

1. The equilibrium magnetization develops a twisted state along the thickness under the application of an external field. This twisted configuration affects the resonance frequency of the system and the nucleation field.
2. In the case of the synthetic antiferromagnet composed of thick ferromagnetic films, a rotated equilibrium magnetization state is observed that mimics the role of the biquadratic interactions, suggesting that the biquadratic exchange interaction in thick coupled films can be easily overestimated if the magnetization texture formed along the thickness is not considered.

By the other hand, when spin waves are propagate in a synthetic antiferromagnet:

1. A notable interplay between the number of ferromagnetic layers and their thickness on the nonreciprocity of the spin-waves modes. The clear distinction in dispersion behavior between systems with even and odd N_L induces nonreciprocal asymmetric dispersion due to dynamic dipolar interactions and odd N_L results in a reciprocal behavior, highlights a crucial symmetry aspect of these structures.
2. The emergence and concentration of bulk modes, for a given value of the wave vector, within a narrow frequency range as N_L increases shows the influence of system thickness on such bulk modes. The observed chiral properties of surface modes in systems with even N_L add another ingredient of complexity and potential functionality to these systems.
3. Furthermore, the demonstrated tunability of these magnetic properties through external fields, enabling transitions from antiparallel to parallel alignment, showcases the reconfigurability and practical adaptability of these synthetic antiferromagnets, opening avenues for advanced applications in magnonics devices.

Finally, in the light of the results and the developed ideas, future research will consider:

1. Explore the curved systems, where many magnetochiral effects have been reported, where at least, in circular nanotubes, the physical origin is the dipolar interaction and the curvature induced geometrical charge [79, 126]. Then, trying to introduce any graduation in their properties like anisotropy or saturation magnetization could affect the bands in dispersion relation.

2. Taking advantage of the adaptability of dynamic matrix method, explore magnonic systems where multiple layers are coupled to observe any nonreciprocal behavior or flat bands in its band structure.

Appendix A

Dispersion relation

Start with the Maxwell equations in absence of currents in the magnetostatic limit [21]:

$$\nabla \times \mathbf{H} = 0; \quad \nabla \cdot \mathbf{B} = 0; \quad \nabla \times \mathbf{E} = -\dot{\mathbf{B}}, \quad (\text{A.1})$$

using the magnetic induction given by

$$\mathbf{B} = \mu_0(1 + \chi)\mathbf{H}. \quad (\text{A.2})$$

Here, it is introduced χ as the high-frequency susceptibility derived from the Landau-Lifshitz equation without loss. Now, using the magnetostatic potential ψ in $\mathbf{H} = -\nabla\psi$ allows to write down Walker's equation as [24]

$$(1 + \chi) \left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right] + \frac{\partial^2 \psi}{\partial z^2} = 0.$$

Assuming that the spatially modulated potential is proportional to a uniform plane wave, i.e $\psi \propto \exp(i\mathbf{k} \cdot \mathbf{r})$, is possible to find the dispersion relation

$$(1 + \chi)(k_x^2 + k_y^2) + k_z^2 = 0. \quad (\text{A.3})$$

From this, the different magnetostatic modes are obtained by choice of appropriate boundary conditions and relations for the different components of k [127]. In the following, is going to differentiate the tree different magnetostatic modes found in thin films, where is neglected the anisotropy and exchange interaction. The different modes are characterized by the angle between the magnetization and the in-plane wave vector $k_{\parallel}$

- **Magnetostatic surface wave(MSSW):** The name of this wave stems from the fact the amplitude decays exponentially from the surface. It is often called Damon-Eshbach (DE) mode as they were first described by Damon and Eshbach [128]. This wave is observed when $k_{\parallel}$ is perpendicular to the magnetization and $\mathbf{M}$ lies in the sample plane. The dispersion relation for this wave is given by [24]

$$\omega_{DE}^2 = \omega_H(\omega_H + \omega_{eff}) + \frac{\omega_{eff}^2}{4}(1 - e^{-2k_{\parallel}d}). \quad (\text{A.4})$$

- **Magnetostatic backward volume wave(MSBVW):** This modes is called a backward volume(BV) mode cause it exhibits a negative group velocity (see Fig. A.1). In contrast to DE, this mode's amplitude is distributed uniformly throughout the film's volume, which is why it is called a volume mode. The configuration of $k_{\parallel}$ and $\mathbf{M}$ is collinear to each other while being in the sample plane. The dispersion relation is given by [129]

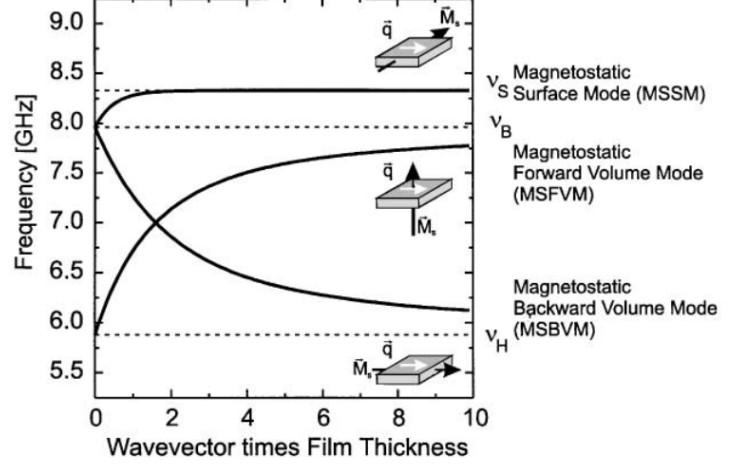
$$\omega_{BV}^2 = \omega_H \left[\omega_H + \omega_{eff} \left(\frac{1 - e^{-k_{\parallel}d}}{k_{\parallel}d} \right) \right]. \quad (\text{A.5})$$

- **Magnetostatic forward volume wave(MSFVW):** On the other side for the last case, here the forward volume mode has a positive group velocity and the amplitude is uniformly

distributed throughout the film's volume. Here, the magnetization points in the out-of-plane direction, i.e. perpendicular to the film plane. It is worth mentioning that this mode does not depend on the direction of the in-plane wave vector $k_{\parallel}$. The dispersion relation is isotropic and given by [24]

$$\omega_{FV}^2 = \omega_H \left[\omega_H + \omega_{eff} \left(1 - \frac{1 - e^{-k_{\parallel}d}}{k_{\parallel}d} \right) \right]. \quad (\text{A.6})$$

Figure A.1: Figure from [130]. Typology of spin wave modes in a magnetic film a function of the directions of the magnetization, $\mathbf{M}_s$, and the in-plane wavevector, $\mathbf{q}$. v_s : frequency of the surface spin wave on a semi-infinite half space, v_B , v_H : frequencies of the volume spin waves with the wavevector perpendicular and parallel to the external field, respectively.



Appendix B

Biquadratic exchange interaction

Biquadratic exchange field

The behaviour of this energy is similar to the bilinear interlayer interaction, it appears to the neighboring layers and its given by

$$\epsilon_{bq}^{(\eta\nu)} = -J_{bq} \left(\frac{\mathbf{M}_\eta \cdot \mathbf{M}_\nu}{M_{s_\eta} M_{s_\nu}} \right)^2 \quad (\text{B.1})$$

Similar to linear exchange case, is necessary to project the basis of layer ν into layer η . Thus, the energy density becomes

$$\begin{aligned} \epsilon_{bq}^{(\eta\nu)} = & -\frac{J_{bq}}{M_S^{(\eta)} M_S^{(\nu)}} (M_S^{(\eta)} M_S^{(\nu)} \cos(\varphi_\nu - \varphi_\eta) + M_S^{(\eta)} m_{X_\nu} \sin(\varphi_\nu - \varphi_\eta) \\ & - m_{X_\eta} M_S^{(\nu)} \sin(\varphi_\nu - \varphi_\eta) + m_{X_\eta} m_{X_\nu} \cos(\varphi_\nu - \varphi_\eta) + m_{Y_\eta} m_{Y_\nu})^2. \end{aligned}$$

Then, the effective field of biquadratic exchange components are given by

$$\begin{aligned} h_{Y_\eta}^{\text{bq}(\eta)}(\mathbf{r}, t) = & \sum_\nu (\delta_\eta^\nu \delta_{\eta+1}^\nu + \delta_{\eta+1}^\nu \delta_\eta^\nu) \frac{J_{bq}}{d\mu_0 M_S^{(\eta)} M_S^{(\nu)}} m_{Y_\nu}^{(\nu)}(\mathbf{r}, t) + \\ & \sum_\nu (\delta_\eta^\nu \delta_{\eta+1}^\nu + \delta_{\eta+1}^\nu \delta_\eta^\nu) \frac{2J_{bq} \cos(\varphi_\eta - \varphi_\nu)}{d\mu_0 M_S^{(\eta)} M_S^{(\nu)}} m_{Y_\nu}^{(\nu)}(\mathbf{r}, t) + \\ & \sum_\nu (\delta_\eta^\nu \delta_{\eta+1}^\nu + \delta_{\eta+1}^\nu \delta_\eta^\nu) \frac{2J_{bq}}{d\mu_0 M_S^{(\eta)}} \sin 2(\varphi_\eta - \varphi_\nu) m_{X_\eta}^{(\eta)}(\mathbf{r}, t) \\ h_{X_\eta}^{\text{bq}(\eta)}(\mathbf{r}, t) = & \sum_\nu (\delta_\eta^\nu \delta_{\eta+1}^\nu + \delta_{\eta+1}^\nu \delta_\eta^\nu) \frac{J_{bq} \cos(\varphi_\eta - \varphi_\nu)}{d\mu_0 M_S^{(\eta)} M_S^{(\nu)}} m_{X_\nu}^{(\nu)}(\mathbf{r}, t) + \\ & \sum_\nu (\delta_\eta^\nu \delta_{\eta+1}^\nu + \delta_{\eta+1}^\nu \delta_\eta^\nu) \frac{J_{bq} \cos[2(\varphi_\eta - \varphi_\nu)]}{d\mu_0 M_S^{(\eta)} M_S^{(\nu)}} m_{X_\nu}^{(\nu)}(\mathbf{r}, t), \end{aligned}$$

and

$$\begin{aligned} H_{Z_\eta}^{\text{bq}0(\eta)}(\mathbf{r}) = & \sum_\nu (\delta_\eta^\nu \delta_{\eta+1}^\nu + \delta_{\eta+1}^\nu \delta_\eta^\nu) \frac{J_{bq} \cos(\varphi_\eta - \varphi_\nu)}{d\mu_0 M_S^{(\eta)} M_S^{(\nu)}} + \\ & \sum_\nu (\delta_\eta^\nu \delta_{\eta+1}^\nu + \delta_{\eta+1}^\nu \delta_\eta^\nu) \frac{2J_{bq} \cos^2(\varphi_\eta - \varphi_\nu)}{d\mu_0 M_S^{(\eta)} M_S^{(\nu)}} \end{aligned} \quad (\text{B.2})$$

$$H_{X_\eta}^{\text{bq}0(\eta)}(\mathbf{r}) = \sum_\nu (\delta_\eta^\nu \delta_{\eta+1}^\nu + \delta_{\eta+1}^\nu \delta_\eta^\nu) \frac{J_{bq} \sin[2(\varphi_\eta - \varphi_\nu)]}{d\mu_0 M_S^{(\nu)}}. \quad (\text{B.3})$$

Matrix elements

The components of $\tilde{\mathbf{A}}$ are determined by the energetic interactions present in the magnetic nanostructure. The energetic terms include Zeeman, demagnetizing, in-plane uniaxial anisotropy, and interlayer exchange terms. In this work, the interlayer terms arise only in the case of a synthetic

antiferromagnet, where the bilinear and biquadratic exchange interactions are accounted. The energy density associated with the bilinear and biquadratic terms is expressed as:

$$\epsilon_{ij} = -J_{\text{bl}} \frac{\mathbf{M}_i \cdot \mathbf{M}_j}{M_{s_i} M_{s_j}} - J_{\text{bq}} \left(\frac{\mathbf{M}_i \cdot \mathbf{M}_j}{M_{s_i} M_{s_j}} \right)^2,$$

Here, i and j represent the sublayers at the interface of the two FM films (denoted as α and $\alpha+1$). When J_{bl} is the dominant term, and its value is negative, the magnetization of the films will be antiparallel at zero external field. Conversely, if J_{bq} is the dominant term and also negative, the magnetization of the layers will tend to be perpendicular to each other. The effective interlayer exchange fields acting on the i -th layer can be obtained from $\mathbf{H}_i^{\text{ex}} = -(\mu_0 d_i)^{-1} \partial_{\mathbf{M}_i} \epsilon_{ij}$. In the same way, the effective fields associated with the Zeeman, demagnetizing, and uniaxial anisotropy can be obtained (see details in Ref. [75])

By considering the dynamic effective field components as

$$h_{\xi_n}^e(\mathbf{r}) = \sum_{\xi'} \sum_{\eta} T_{\xi_n}^{\xi'} m_{\xi'},$$

with $\xi_n = X_n, Y_n$, the matrix elements of $\tilde{\mathbf{A}}$ can be derived as follows

$$A_{Y_n Y_\eta} = \begin{cases} M_{s_n} T_{X_n}^{Y_\eta} & n \neq \eta \\ M_{s_n} T_{X_n}^{Y_n} & n = \eta, \end{cases}$$

$$A_{X_n X_\eta} = \begin{cases} -M_{s_n} T_{Y_n}^{X_\eta} & n \neq \eta \\ -M_{s_n} T_{Y_n}^{X_n} & n = \eta, \end{cases}$$

$$A_{Y_n X_\eta} = \begin{cases} M_{s_n} T_{X_n}^{Y_\eta} & n \neq \eta \\ M_{s_n} T_{X_n}^{X_n} - H_{Z_n}^{\text{e0}} & n = \eta, \end{cases}$$

and

$$A_{X_n Y_\eta} = \begin{cases} -M_{s_n} T_{Y_n}^{Y_\eta} & n \neq \eta \\ -M_{s_n} T_{Y_n}^{Y_n} + H_{Z_n}^{\text{e0}} & n = \eta. \end{cases}$$

The $T_{\xi_n}^{\xi'}$ terms are $T_{X_n}^{Y_n} = T_{Y_n}^{X_n} = T_{X_n}^{Y_\eta} = T_{Y_n}^{X_\eta} = 0$, $T_{Y_n}^{Y_n} = -1$; and

$$T_{X_n}^{X_n} = \sum_{\eta} \frac{2J_{\text{bq}}}{d_n \mu_0 M_{s_n}} \sin^2(\varphi_n - \varphi_\eta) (\delta_n^\alpha \delta_\eta^{\alpha+1} + \delta_n^{\alpha+1} \delta_\eta^\alpha),$$

$$T_{X_n}^{X_\eta} = \frac{J_{\text{bq}} \cos[2(\varphi_n - \varphi_\eta)]}{d_n \mu_0 M_{s_n} M_{s_\eta}} (\delta_n^\alpha \delta_\eta^{\alpha+1} + \delta_n^{\alpha+1} \delta_\eta^\alpha),$$

$$T_{Y_n}^{Y_\eta} = \frac{2J_{\text{bq}} \cos(\varphi_n - \varphi_\eta)}{d_n \mu_0 M_{s_n} M_{s_\eta}} (\delta_n^\alpha \delta_\eta^{\alpha+1} + \delta_n^{\alpha+1} \delta_\eta^\alpha).$$

In the previous equations, H_{un} is the uniaxial anisotropy field, where the easy axis corresponds to x . Besides, it should be noted that J_{I} is null at the layers α and $\alpha+1$, where the interlayer terms (bilinear and biquadratic) are present. Note that the expression $A_{X_n Y_n} = M_{s_n} + H_{Z_n}^{\text{e0}}$, where M_{s_n} accounts for the contribution of the demagnetizing field resulting from surface charges at the top and bottom of the sublayers, while the term $H_{Z_n}^{\text{e0}}$ corresponds to the Z_n -component of the effective static field, and it is defined as follows:

$$\begin{aligned} H_{Z_n}^{\text{bq0}} &= H_{Z_n}^{\text{e0}} + \sum_{\eta} \frac{J_{\text{bq}} \cos(\varphi_n - \varphi_\eta)}{d_n \mu_0 M_{s_n}} (\delta_n^\alpha \delta_\eta^{\alpha+1} + \delta_n^{\alpha+1} \delta_\eta^\alpha) \\ &+ \sum_{\eta} \frac{2J_{\text{bq}} \cos^2(\varphi_n - \varphi_\eta)}{d_n \mu_0 M_{s_n}} (\delta_n^\alpha \delta_\eta^{\alpha+1} + \delta_n^{\alpha+1} \delta_\eta^\alpha). \end{aligned}$$

Bibliography

- [1] E. Du Trémolet de Lacheisserie, D. Gignoux, and M. Schlenker. *Magnetism*. Number v. 1 in Collection Grenoble Sciences. Springer, 2005.
- [2] Alexander Mikerov. From history of electrical engineering: Electricity and magnetism in old, middle ages and renaissance. *Proceedings of the Russia Northwest Section, SPbGETU, Spb.*, 3, 01 2012.
- [3] R. Goyal. *Nanomaterials and Nanocomposites: Synthesis, Properties, Characterization Techniques, and Applications*. CRC Press, 10 2017.
- [4] C A F Vaz, J A C Bland, and G Lauhoff. Magnetism in ultrathin film structures. *Reports on Progress in Physics*, 71(5):056501, apr 2008.
- [5] Ching Tsang, R.E. Fontana, Tsann Lin, D.E. Heim, V.S. Speriosu, B.A. Gurney, and M.L. Williams. Design, fabrication and testing of spin-valve read heads for high density recording. *IEEE Transactions on Magnetism*, 30(6):3801–3806, 1994.
- [6] G. Binasch, P. Grünberg, F. Saurenbach, and W. Zinn. Enhanced magnetoresistance in layered magnetic structures with antiferromagnetic interlayer exchange. *Phys. Rev. B*, 39:4828–4830, Mar 1989.
- [7] M. N. Baibich, J. M. Broto, A. Fert, F. Nguyen Van Dau, F. Petroff, P. Etienne, G. Creuzet, A. Friederich, and J. Chazelas. Giant magnetoresistance of (001)fe/(001)cr magnetic superlattices. *Phys. Rev. Lett.*, 61:2472–2475, Nov 1988.
- [8] Shinji Yuasa, Taro Nagahama, Akio Fukushima, Yoshishige Suzuki, and Koji Ando. Giant room-temperature magnetoresistance in single-crystal fe/mgo/fe magnetic tunnel junctions. *Nature materials*, 3:868–71, 01 2005.
- [9] Stuart Parkin, Christian Kaiser, Alex Panchula, Philip Rice, Brian Hughes, Mahesh Samant, and See-Hun Yang. Giant tunneling magnetoresistance at room temperature with mgo(100) tunnel barriers. *Nature materials*, 3:862–7, 01 2005.
- [10] Gary A. Prinz. Magnetoelectronics. *Science*, 282(5394):1660–1663, 1998.
- [11] S. A. Wolf, D. D. Awschalom, R. A. Buhrman, J. M. Daughton, S. von Molnár, M. L. Roukes, A. Y. Chtchelkanova, and D. M. Treger. Spintronics: A spin-based electronics vision for the future. *Science*, 294(5546):1488–1495, 2001.
- [12] Igor Žutić, Jaroslav Fabian, and S. Das Sarma. Spintronics: Fundamentals and applications. *Rev. Mod. Phys.*, 76:323–410, Apr 2004.
- [13] Amalio Fernández-Pacheco, Robert Streubel, Olivier Fruchart, Riccardo Hertel, Peter Fischer, and Russell Cowburn. Three-dimensional nanomagnetism. *Nature Communications*, 06 2017.
- [14] Alberto P. Guimarães. *Principles of Nanomagnetism*. NanoScience and Technology. Springer International Publishing, 2017.
- [15] A. N. Bogdanov and U. K. Rößler. Chiral symmetry breaking in magnetic thin films and multilayers. *Phys. Rev. Lett.*, 87:037203, Jun 2001.

- [16] A. Bogdanov and A. Hubert. Thermodynamically stable magnetic vortex states in magnetic crystals. *Journal of Magnetism and Magnetic Materials*, 138(3):255–269, 1994.
- [17] Markus Garst, Johannes Waizner, and Dirk Grundler. Collective spin excitations of helices and magnetic skyrmions: review and perspectives of magnonics in non-centrosymmetric magnets. *Journal of Physics D: Applied Physics*, 50(29):293002, jun 2017.
- [18] M. Labrune and L. Belliard. Stripe domains in multilayers: Micromagnetic simulations. *physica status solidi (a)*, 174(2):483–497, 1999.
- [19] Carlos Javier Garcia Cervera. *Magnetic domains and magnetic domain walls*. New York University, 1999.
- [20] W.F. Brown. *Micromagnetics*. Interscience tracts on physics and astronomy. Interscience Publishers, 1963.
- [21] David J. Griffiths. *Introduction to Electrodynamics*. Cambridge University Press, 5 edition, 2023.
- [22] A. Aharoni. *Introduction to the Theory of Ferromagnetism*. International Series of Monographs on Physics. Clarendon Press, 2000.
- [23] A.G. Gurevich and G.A. Melkov. *Magnetization Oscillations and Waves*. Taylor & Francis, 1996.
- [24] D.D. Stancil and A. Prabhakar. *Spin Waves: Theory and Applications*. Springer US, 2009.
- [25] C. Ríos-Venegas, F. Brevis, R. A. Gallardo, and P. Landeros. Dynamic origin of conical helix magnetization textures stabilized by dzyaloshinskii-moriya interaction. *Phys. Rev. B*, 105:224403, Jun 2022.
- [26] Thomas Schwarze. *Spin Waves in 2D and 3D Magnonic Crystals: From Nanostructured Ferromagnetic Materials to Chiral Helimagnets*. 2013.
- [27] J. M. D. Coey. *Magnetism and magnetic materials / J.M.D. Coey*. Cambridge University Press, Cambridge, 2010.
- [28] Steven H. Simon. *The Oxford Solid State Basics / Steven H. Simon*. Oxford University Press, Oxford, first edition. edition, 2013.
- [29] F. Bloch. Zur Theorie des Ferromagnetismus. *Z. Phys.*, 61:206, 1930.
- [30] T. Holstein and H. Primakoff. Field dependence of the intrinsic domain magnetization of a ferromagnet. *Phys. Rev.*, 58:1098–1113, Dec 1940.
- [31] Freeman J. Dyson. General theory of spin-wave interactions. *Phys. Rev.*, 102:1217–1230, Jun 1956.
- [32] B. N. Brockhouse. Scattering of neutrons by spin waves in magnetite. *Phys. Rev.*, 106:859–864, Jun 1957.
- [33] P. Grünberg. Magnetostatic spin-wave modes of a heterogeneous ferromagnetic double layer. *Journal of Applied Physics*, 52(11):6824–6829, 11 1981.
- [34] Anjan Barman, Gianluca Gubbiotti, S Ladak, A O Adeyeye, M Krawczyk, J Gräfe, C Adelman, S Cotofana, A Naeemi, V I Vasyuchka, B Hillebrands, S A Nikitov, H Yu, D Grundler, A V Sadovnikov, A A Grachev, S E Sheshukova, J-Y Duquesne, M Marangolo, G Csaba, W Porod, V E Demidov, S Urazhdin, S O Demokritov, E Albisetti, D Petti, R Bertacco, H Schultheiss, V V Kruglyak, V D Poimanov, S Sahoo, J Sinha, H Yang, M Müntenzenberg, T Moriyama, S Mizukami, P Landeros, R A Gallardo, G Carlotti, J-V Kim, R L Stamps, R E Camley, B Rana, Y Otani, W Yu, T Yu, G E W Bauer, C Back, G S Uhrig, O V Dobrovolskiy, B Budinska, H Qin, S van Dijken, A V Chumak, A Khitun, D E Nikonov, I A Young, B W Zingsem, and M Winklhofer. The 2021 magnonics roadmap. *Journal of Physics: Condensed Matter*, 33(41):413001, aug 2021.

- [35] Benedetta Flebus, Dirk Grundler, Bivas Rana, YoshiChika Otani, Igor Barsukov, Anjan Barman, Gianluca Gubbiotti, Pedro Landeros, Johan Akerman, Ursula Ebels, Philipp Pirro, Vladislav E Demidov, Katrin Schultheiss, Gyorgy Csaba, Qi Wang, Florin Ciubotaru, Dmitri E Nikonov, Ping Che, Riccardo Hertel, Teruo Ono, Dmytro Afanasiev, Johan Mentink, Theo Rasing, Burkard Hillebrands, Silvia Viola Kusminskiy, Wei Zhang, Chunhui Rita Du, Aurore Finco, Toeno van der Sar, Yunqiu Kelly Luo, Yoichi Shiotani, Joseph Sklenar, Tao Yu, and Jinwei Rao. The 2024 magnonics roadmap. *Journal of Physics: Condensed Matter*, 36(36):363501, jun 2024.
- [36] S.M. Rezende. *Fundamentals of Magnonics*. Lecture Notes in Physics. Springer International Publishing, 2020.
- [37] David J. Griffiths and Darrell F. Schroeter. *Introduction to Quantum Mechanics*. Cambridge University Press, 3 edition, 2018.
- [38] Ronald S Indeck, Marcel W Muller, and Laleh Avazpour. *Magnetic Recording Measurements*. Elsevier, 2018.
- [39] Charles Kittel. On the theory of ferromagnetic resonance absorption. *Phys. Rev.*, 73:155–161, Jan 1948.
- [40] Yves Henry, Olga Gladii, and Matthieu Bailleul. Propagating spin-wave normal modes: A dynamic matrix approach using plane-wave demagnetizing tensors, 2016.
- [41] Massimiliano d’Aquino. Computation of magnetization normal oscillation modes in complex micromagnetic systems. *IFAC Proceedings Volumes*, 45(2):504–509, 2012. 7th Vienna International Conference on Mathematical Modelling.
- [42] Florian Bruckner, Massimiliano d’Aquino, Claudio Serpico, Claas Abert, Christoph Vogler, and Dieter Suess. Large scale finite-element simulation of micromagnetic thermal noise. *Journal of Magnetism and Magnetic Materials*, 475:408–414, 2019.
- [43] M. Grimsditch, L. Giovannini, F. Montoncello, F. Nizzoli, Gary K. Leaf, and Hans G. Kaper. Magnetic normal modes in ferromagnetic nanoparticles: A dynamical matrix approach. *Phys. Rev. B*, 70:054409, Aug 2004.
- [44] M. N. Baibich, J. M. Broto, A. Fert, F. Nguyen Van Dau, F. Petroff, P. Etienne, G. Creuzet, A. Friederich, and J. Chazelas. Giant magnetoresistance of (001)fe/(001)cr magnetic superlattices. *Phys. Rev. Lett.*, 61:2472–2475, Nov 1988.
- [45] M T Johnson, P J H Bloemen, F J A den Broeder, and J J de Vries. Magnetic anisotropy in metallic multilayers. *Reports on Progress in Physics*, 59(11):1409, nov 1996.
- [46] Volker Sluka, Tobias Schneider, Rodolfo Gallardo, Attila Kákay, Markus Weigand, Tobias Warnatz, Roland Mattheis, Alejandro Roldán-Molina, Pedro Landeros, Vasil Tiberkevich, Andrei Slavin, Artur Erbe, Alina Deac, Jürgen Lindner, Jörg Raabe, Jürgen Fassbender, and Sebastian Wintz. Emission and propagation of 1d and 2d spin waves with nanoscale wavelengths in anisotropic spin textures. *Nature Nanotechnology*, 14, 04 2019.
- [47] R.H Kodama. Magnetic nanoparticles. *Journal of Magnetism and Magnetic Materials*, 200(1):359–372, 1999.
- [48] Sergei P Gubin, Yurii A Koksharov, G B Khomutov, and Gleb Yu Yurkov. Magnetic nanoparticles: preparation, structure and properties. *Russian Chemical Reviews*, 74(6):489, jun 2005.
- [49] An-Hui Lu, E.L. Salabas, and Ferdi Schüth. Magnetic nanoparticles: Synthesis, protection, functionalization, and application. *Angewandte Chemie International Edition*, 46(8):1222–1244, 2007.
- [50] L. Sun, Y. Hao, C.-L. Chien, and P. C. Searson. Tuning the properties of magnetic nanowires. *IBM Journal of Research and Development*, 49(1):79–102, 2005.
- [51] P. Landeros, S. Allende, J. Escrig, E. Salcedo, D. Altbir, and E. E. Vogel. Reversal modes in magnetic nanotubes. *Applied Physics Letters*, 90(10):102501, 03 2007.

- [52] Daeyeon Lee, Robert E. Cohen, and Michael F. Rubner. Heterostructured magnetic nanotubes. *Langmuir*, 23(1):123–129, 2007. PMID: 17190494.
- [53] J. Alam, C. Bran, H. Chiriac, N. Lupu, T.A. Óvári, L.V. Panina, V. Rodionova, R. Varga, M. Vazquez, and A. Zhukov. Cylindrical micro and nanowires: Fabrication, properties and applications. *Journal of Magnetism and Magnetic Materials*, 513:167074, 2020.
- [54] Luc Piraux. Magnetic nanowires. *Applied Sciences*, 10(5), 2020.
- [55] B. Heinrich and J.F. Cochran. Ultrathin metallic magnetic films: magnetic anisotropies and exchange interactions. *Advances in Physics*, 42(5):523–639, 1993.
- [56] D.L. Mills, Sergio Rezende, Burkard Hillebrands, and Kamel Ounadjela. Spin dynamics in ultrathin magnetic films, spin dynamics in confined magnetic structures ii. *Topics Applied Phys.*, 87, 01 2003.
- [57] S.J Lee, S Souma, G Ihm, and K.J Chang. Magnetic quantum dots and magnetic edge states. *Physics Reports*, 394(1):1–40, 2004.
- [58] C. A. Downing and M. E. Portnoi. Magnetic quantum dots and rings in two dimensions. *Phys. Rev. B*, 94:045430, Jul 2016.
- [59] M. Krawczyk and H. Puzskarski. Plane-wave theory of three-dimensional magnonic crystals. *Phys. Rev. B*, 77:054437, Feb 2008.
- [60] G Gubbiotti, S Tacchi, M Madami, G Carlotti, A O Adeyeye, and M Kostylev. Brillouin light scattering studies of planar metallic magnonic crystals. *Journal of Physics D: Applied Physics*, 43(26):264003, jun 2010.
- [61] S. Tacchi, G. Duerr, J. W. Klos, M. Madami, S. Neusser, G. Gubbiotti, G. Carlotti, M. Krawczyk, and D. Grundler. Forbidden band gaps in the spin-wave spectrum of a two-dimensional bicomponent magnonic crystal. *Phys. Rev. Lett.*, 109:137202, Sep 2012.
- [62] A. V. Chumak, V. I. Vasyuchka, A. A. Serga, M. P. Kostylev, V. S. Tiberkevich, and B. Hillebrands. Storage-recovery phenomenon in magnonic crystal. *Phys. Rev. Lett.*, 108:257207, Jun 2012.
- [63] M. Langer, F. Röder, R. A. Gallardo, T. Schneider, S. Stienen, C. Gatel, R. Hübner, L. Bischoff, K. Lenz, J. Lindner, P. Landeros, and J. Fassbender. Role of internal demagnetizing field for the dynamics of a surface-modulated magnonic crystal. *Phys. Rev. B*, 95:184405, May 2017.
- [64] P. Alvarado-Seguel and R. A. Gallardo. Band structure of a one-dimensional bilayer magnonic crystal. *Phys. Rev. B*, 100:144415, Oct 2019.
- [65] Ivan K Schuller, S Kim, and C Leighton. Magnetic superlattices and multilayers. *Journal of Magnetism and Magnetic Materials*, 200(1):571–582, 1999.
- [66] V. V. Kruglyak, M. L. Sokolovskii, V. S. Tkachenko, and A. N. Kuchko. Spin-wave spectrum of a magnonic crystal with an isolated defect. *Journal of Applied Physics*, 99(8):08C906, 04 2006.
- [67] R. A. Gallardo, T. Schneider, A. Roldán-Molina, M. Langer, A. S. Núñez, K. Lenz, J. Lindner, and P. Landeros. Symmetry and localization properties of defect modes in magnonic superlattices. *Phys. Rev. B*, 97:174404, May 2018.
- [68] M. Dvornik, Y. Au, and Volodymyr Kruglyak. *Micromagnetic Simulations in Magnonics*, volume 125, pages 101–115. Springer, 01 2013.
- [69] Arne Vansteenkiste, Jonathan Leliaert, Mykola Dvornik, Mathias Helsen, Felipe Garcia-Sanchez, and Bartel Van Waeyenberge. The design and verification of MuMax3. *AIP Advances*, 4(10):107133, 10 2014.
- [70] Arne Vansteenkiste, Jonathan Leliaert, Mykola Dvornik, Mathias Helsen, Felipe Garcia-Sanchez, and Bartel Van Waeyenberge. The design and verification of MuMax3. *AIP Advances*, 4(10):107133, 10 2014.

- [71] T. Valet and A. Fert. Theory of the perpendicular magnetoresistance in magnetic multilayers. *Phys. Rev. B*, 48:7099–7113, Sep 1993.
- [72] K. Yu. Guslienko, S. O. Demokritov, B. Hillebrands, and A. N. Slavin. Effective dipolar boundary conditions for dynamic magnetization in thin magnetic stripes. *Phys. Rev. B*, 66:132402, Oct 2002.
- [73] K. Y. Guslienko, R. W. Chantrell, and A. N. Slavin. Dipolar localization of quantized spin-wave modes in thin rectangular magnetic elements. *Phys. Rev. B*, 68:024422, Jul 2003.
- [74] A.L. González, P. Landeros, and Álvaro S. Núñez. Spin wave spectrum of magnetic nanotubes. *Journal of Magnetism and Magnetic Materials*, 322(5):530–535, 2010.
- [75] R.A. Gallardo, T. Schneider, A.K. Chaurasiya, A. Oelschlägel, S.S.P.K. Arekapudi, A. Roldán-Molina, R. Hübner, K. Lenz, A. Barman, J. Fassbender, J. Lindner, O. Hellwig, and P. Landeros. Reconfigurable spin-wave nonreciprocity induced by dipolar interaction in a coupled ferromagnetic bilayer. *Phys. Rev. Appl.*, 12:034012, Sep 2019.
- [76] A F Franco. Intensity enhancement of ferromagnetic resonance modes in exchange coupled magnetic multilayers. *New Journal of Physics*, 22(1):013017, jan 2020.
- [77] R. A. Gallardo, P. Alvarado-Seguel, and P. Landeros. High spin-wave asymmetry and emergence of radial standing modes in thick ferromagnetic nanotubes. *Phys. Rev. B*, 105:104435, Mar 2022.
- [78] F H De Leeuw, R Van Den Doel, and U Enz. Dynamic properties of magnetic domain walls and magnetic bubbles. *Reports on Progress in Physics*, 43(6):689, jun 1980.
- [79] P. Landeros and Álvaro S. Núñez. Domain wall motion on magnetic nanotubes. *Journal of Applied Physics*, 108(3):033917, 08 2010.
- [80] Giovanni Finocchio, Felix Büttner, Riccardo Tomasello, Mario Carpentieri, and Mathias Kläui. Magnetic skyrmions: from fundamental to applications. *Journal of Physics D: Applied Physics*, 49(42):423001, sep 2016.
- [81] Gen Yin, Yufan Li, Lingyao Kong, Roger K. Lake, C. L. Chien, and Jiadong Zang. Topological charge analysis of ultrafast single skyrmion creation. *Phys. Rev. B*, 93:174403, May 2016.
- [82] Ran Cheng, Maxwell Li, Arjun Sapkota, Anish Rai, Ashok Pokhrel, Tim Mewes, Claudia Mewes, Di Xiao, Marc De Graef, and Vincent Sokalski. Magnetic domain wall skyrmions. *Phys. Rev. B*, 99:184412, May 2019.
- [83] William Fuller Brown. Thermal fluctuations of a single-domain particle. *Phys. Rev.*, 130:1677–1686, Jun 1963.
- [84] D. Fiorani. *Surface Effects in Magnetic Nanoparticles*. Nanostructure Science and Technology. Springer, 2005.
- [85] J. Mazo-Zuluaga, J. Restrepo, and J. Mejía-López. Effect of surface anisotropy on the magnetic properties of magnetite nanoparticles: A Heisenberg–Monte Carlo study. *Journal of Applied Physics*, 103(11):113906, 06 2008.
- [86] J. Jiménez-Bustamante, A. Lindner, H. N. Koyun, R. Salikhov, K. Lenz, J. Lindner, and R. A. Gallardo. Static and dynamic properties of noncollinear magnetized ferromagnetic films. *Phys. Rev. B*, 109:094403, Mar 2024.
- [87] R A Gallardo, P Alvarado-Seguel, T Schneider, C Gonzalez-Fuentes, A Roldán-Molina, K Lenz, J Lindner, and P Landeros. Spin-wave non-reciprocity in magnetization-graded ferromagnetic films. *New J. Phys.*, 21(3):033026, mar 2019.
- [88] Michael Farle. Ferromagnetic resonance of ultrathin metallic layers. *Reports on Progress in Physics*, 61(7):755, jul 1998.
- [89] H. Dötsch, Peter Röschmann, and W. Schilz. Ferrimagnetic resonance spectra of magnetic bubble films at low microwave frequencies. *Applied Physics*, 15:167–173, 02 1978.

- [90] Jing-guo Hu, Guo-jun Jin, and Yu-qiang Ma. Ferromagnetic resonance and exchange anisotropy in ferromagnetic/antiferromagnetic bilayers. *Journal of Applied Physics*, 91(4):2180–2185, 02 2002.
- [91] Xing Chen, Cuixiu Zheng, Sai Zhou, Yaowen Liu, and Zongzhi Zhang. Ferromagnetic resonance modes of a synthetic antiferromagnet at low magnetic fields. *Journal of Physics: Condensed Matter*, 34(1):015802, oct 2021.
- [92] S. Sorokin, R. A. Gallardo, C. Fowley, K. Lenz, A. Titova, G. Y. P. Atcheson, G. Denehy, K. Rode, J. Fassbender, J. Lindner, and A. M. Deac. Magnetization dynamics in synthetic antiferromagnets: Role of dynamical energy and mutual spin pumping. *Phys. Rev. B*, 101:144410, Apr 2020.
- [93] O. Gladii, R. Salikhov, O. Hellwig, H. Schultheiss, J. Lindner, and R. A. Gallardo. Spin-wave nonreciprocity at the spin-flop transition region in synthetic antiferromagnets. *Phys. Rev. B*, 107:104419, Mar 2023.
- [94] M. Rühlig, R. Schäfer, A. Hubert, R. Mosler, J. A. Wolf, S. Demokritov, and P. Grünberg. Domain observations on fcc Fe layered structures. evidence for a biquadratic coupling effect. *physica status solidi (a)*, 125(2):635–656, 1991.
- [95] S O Demokritov. Biquadratic interlayer coupling in layered magnetic systems. *Journal of Physics D: Applied Physics*, 31(8):925, apr 1998.
- [96] S. M. Rezende, C. Chesman, M. A. Lucena, M. C. de Moura, A. Azevedo, F. M. de Aguiar, and S. S. P. Parkin. Biquadratic coupling in sputtered Fe/Cr/Fe still in need of a new mechanism. *Journal of Applied Physics*, 85(8):5892–5894, 04 1999.
- [97] P. Vavassori, M. Grimsditch, and Eric E. Fullerton. Biquadratic exchange coupling in an unequal Fe/Cr/Fe(100) trilayer. *Journal of Magnetism and Magnetic Materials*, 223(3):284–292, 2001.
- [98] D. Mampallil, D. Aernout, and J. Meersschaut. Field-cooling effect in biquadratically interlayer-coupled fcc Fe trilayers. *Phys. Rev. B*, 75:060402, Feb 2007.
- [99] Mei Li, Jie Lu, and Wei He. Symmetry breaking induced magnon-magnon coupling in synthetic antiferromagnets. *Phys. Rev. B*, 103:064429, Feb 2021.
- [100] A. Sud, K. Yamamoto, K. Z. Suzuki, S. Mizukami, and H. Kurebayashi. Magnon-magnon coupling in synthetic ferrimagnets. *Phys. Rev. B*, 108:104407, Sep 2023.
- [101] Kay Yakushiji, Atsushi Sugihara, Akio Fukushima, Hitoshi Kubota, and Shinji Yuasa. Very strong antiferromagnetic interlayer exchange coupling with iridium spacer layer for perpendicular magnetic tunnel junctions. *Applied Physics Letters*, 110(9):092406, 02 2017.
- [102] Changting Dai and Fusheng Ma. Strong magnon–magnon coupling in synthetic antiferromagnets. *Applied Physics Letters*, 118(11):112405, 03 2021.
- [103] M. Belmeguenai, T. Martin, G. Woltersdorf, M. Maier, and G. Bayreuther. Frequency- and time-domain investigation of the dynamic properties of interlayer-exchange-coupled Ni₈₁Fe₁₉/Ni₈₁Fe₁₉ thin films. *Phys. Rev. B*, 76:104414, Sep 2007.
- [104] T. Martin, M. Belmeguenai, M. Maier, K. Perzlmaier, and G. Bayreuther. Pulsed inductive measurement of ultrafast magnetization dynamics in interlayer exchange coupled NiFeRuNiFe films. *Journal of Applied Physics*, 101(9):09C101, 03 2007.
- [105] M Belmeguenai, T Martin, G Woltersdorf, G Bayreuther, V Baltz, A K Suszka, and B J Hickey. Microwave spectroscopy with vector network analyzer for interlayer exchange-coupled symmetrical and asymmetrical NiFe/Ru/NiFe. *Journal of Physics: Condensed Matter*, 20(34):345206, aug 2008.
- [106] Subrojati Bosu, Yuya Sakuraba, Kesami Saito, Hai Wang, Seiji Mitani, and Koki Takanashi. Biquadratic exchange coupling in epitaxial Co₂MnSi/Cr/Fe trilayers. *IEEE Transactions on Magnetics*, 44(11):2620–2623, 2008.

- [107] Y. F. Chiang, Jared J. I. Wong, X. Tan, Yan Li, K. Pi, W. H. Wang, H. W. K. Tom, and R. K. Kawakami. Oxidation-induced biquadratic coupling in co/fe/mgo/fe(001). *Phys. Rev. B*, 79:184410, May 2009.
- [108] W X Xia, K Inoue, S Saito, and M Takahashi. Effect of rh spacer on synthetic-antiferromagnetic coupling in fecob/rh/fecob films. *Journal of Physics: Conference Series*, 266(1):012064, jan 2011.
- [109] Rembert Duine, Kyung-Jin Lee, Stuart Parkin, and Mark Stiles. Synthetic antiferromagnetic spintronics. *Nature Physics*, 14, 03 2018.
- [110] Kang Wang, Vineetha Bheemarasetty, and Gang Xiao. Spin textures in synthetic antiferromagnets: Challenges, opportunities, and future directions. *APL Materials*, 11(7):070902, 07 2023.
- [111] P. Grünberg, R. Schreiber, Y. Pang, U. Walz, M. B. Brodsky, and H. Sowers. Layered magnetic structures: Evidence for antiferromagnetic coupling of Fe layers across Cr interlayers. *Journal of Applied Physics*, 61(8):3750–3752, 04 1987.
- [112] P. Bruno and C. Chappert. Ruderman-kittel theory of oscillatory interlayer exchange coupling. *Phys. Rev. B*, 46:261–270, Jul 1992.
- [113] A. Fert, P. Grünberg, A. Barthélémy, F. Petroff, and W. Zinn. Layered magnetic structures: interlayer exchange coupling and giant magnetoresistance. *Journal of Magnetism and Magnetic Materials*, 140-144:1–8, 1995. International Conference on Magnetism.
- [114] M.D. Stiles. Interlayer exchange coupling. *Journal of Magnetism and Magnetic Materials*, 200(1):322–337, 1999.
- [115] S. S. P. Parkin. Systematic variation of the strength and oscillation period of indirect magnetic exchange coupling through the 3d, 4d, and 5d transition metals. *Phys. Rev. Lett.*, 67:3598–3601, Dec 1991.
- [116] Qu Yang, Lei Wang, Ziyao Zhou, Liqian Wang, Yijun Zhang, Shishun Zhao, Guohua Dong, Yuxin Cheng, Tai Min, Zhongqiang Hu, Wei Chen, Ke Xia, and Ming Liu. Ionic liquid gating control of rkkj interaction in fecob/ru/fecob and (pt/co)₂/ru/(co/pt)₂ multilayers. *Nature Communications*, 9, 03 2018.
- [117] Phunvira Chongthanaphisut, Seul-Ki Bac, Seonghoon Choi, Kyung Jae Lee, Jihoon Chang, Suho Choi, Sanghoon Lee, Moses Nnaji, Xiaoqing Liu, M. Dobrowolska, and J. Furdyna. Interlayer exchange coupling in ferromagnetic semiconductor trilayers with out-of-plane magnetic anisotropy. *Scientific Reports*, 9, 03 2019.
- [118] Binbin Chen, Haoran Xu, Chao Ma, Stefan Mattauch, Da Lan, Feng Jin, Zhuang Guo, Siyuan Wan, Pingfan Chen, Guanyin Gao, Feng Chen, Yixi Su, and Wenbin Wu. All-oxide-based synthetic antiferromagnets exhibiting layer-resolved magnetization reversal. *Science*, 357(6347):191–194, 2017.
- [119] Martin Lonsky and Axel Hoffmann. Dynamic fingerprints of synthetic antiferromagnet nanostructures with interfacial Dzyaloshinskii–Moriya interaction. *Journal of Applied Physics*, 132(4):043903, 07 2022.
- [120] Y. Henry, O. Gladii, and M. Bailleul. Propagating spin-wave normal modes: A dynamic matrix approach using plane-wave demagnetizing tensors. *arXiv:1611.06153*, 2016.
- [121] S A Owerre. Strain-induced topological magnon phase transitions: applications to kagome-lattice ferromagnets. *Journal of Physics: Condensed Matter*, 30(24):245803, may 2018.
- [122] Fu Yichen and Hong Qin. Topological phases and bulk-edge correspondence of magnetized cold plasmas. *Nature Communications*, 12:3924, 06 2021.
- [123] Sourav Sahoo, Sucheta Mondal, Gwilym Williams, Andrew May, Sam Ladak, and Anjan Barman. Ultrafast magnetization dynamics in a nanoscale three-dimensional cobalt tetrapod structure. *Nanoscale*, 10:9981–9986, 2018.

- [124] Ryohei Wakatsuki, Motohiko Ezawa, and Naoto Nagaosa. Domain wall of a ferromagnet on a three-dimensional topological insulator. *Scientific Reports*, 5, 12 2014.
- [125] Daniel Malz, Johannes Knolle, and Andreas Nunnenkamp. Topological magnon amplification. *Nature Communications*, 10, 09 2019.
- [126] Denis Sheka, Oleksandr Pylypovskyi, Pedro Landeros, Yuri Gaididei, Attila Kákay, and Denys Makarov. Nonlocal chiral symmetry breaking in curvilinear magnetic shells. *Communications Physics*, 3, 07 2020.
- [127] L. R. Walker. Magnetostatic modes in ferromagnetic resonance. *Phys. Rev.*, 105:390–399, Jan 1957.
- [128] R. W. Damon and J. R. Eshbach. Magnetostatic Modes of a Ferromagnetic Slab. *Journal of Applied Physics*, 31(5):S104–S105, 05 1960.
- [129] B A Kalinikos and A N Slavin. Theory of dipole-exchange spin wave spectrum for ferromagnetic films with mixed exchange boundary conditions. *Journal of Physics C: Solid State Physics*, 19(35):7013, dec 1986.
- [130] S.O. Demokritov, B. Hillebrands, and A.N. Slavin. Brillouin light scattering studies of confined spin waves: linear and nonlinear confinement. *Physics Reports*, 348(6):441–489, 2001.