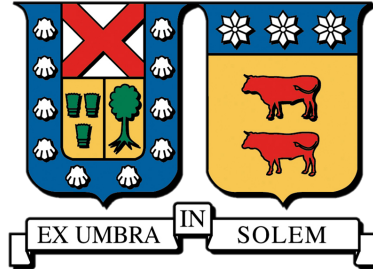


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NEUTRINO PORTAL FOR VECTOR DARK MATTER

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Abstract

In the Fermion Portal Vector Dark Matter (FPVDM) framework, we study how to connect the standard model with dark matter through neutrinos. In this framework the standard model is expanded with a $SU(2)_D \times U(1)_{Y_D}^g$ dark sector, where the $SU(2)_D$ gauge bosons are the dark matter candidates (specifically the $V^\pm$ components), this symmetry is broken by a dark scalar Φ_D , giving mass to the gauge bosons and leaving a remanent $\mathbb{Z}_2$ symmetry that ensures the stability of the candidate. The interaction with the standard model is done through VL-fermions that couple to neutrinos through a Yukawa-like term. We consider two models, with one and two families of VL-fermions, for which we analyze the phenomenology using restrictions, relic density, and direct and indirect detection of dark matter.

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Conventions

We are using the mostly minus Minkowky metric:

$$g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$$

So, for an on-shell particle we have $p^2 = m^2$.

Natural Units $\hbar = c = k_B = 1$ are used in this work, since that means that these fundamental constants can be transferred to units, then all is measure in powers of energy or GeV. A fairly complete list of conversions is provided by the appendix of [13].

Chapter 1

Introduction

The Standard Model of elementary particles (SM) is the most successful theory that describes the elementary block of the universe, quantum particles, and their interactions. However this Quantum Field Theory is incomplete, there are numerous aspects that we don't understand yet and the SM does not provide a satisfactory explanation, some of these questions are; stability of the vacuum, disparity in fermion masses, why three fermion generations?, why are more particles than antiparticles in the universe, can we unify the interactions as we did with the electroweak theory?, how can we include gravity as a renormalizable theory?, what are the masses of neutrino?, are neutrinos a Dirac or Majorana fermions?, what dark matter is made of?, and the list goes on. These questions motivate a whole area in fundamental research known as the physics beyond the standard model, and inspired most of the time by symmetries, as Weinberg [14] and Higgs [15],[16] did for building the SM and explain the weak bosons masses, respectively. Thus usually SM is expanded incorporating new symmetries and particles to answer these questions.

For answering the question of what dark matter is, there have been proposed many models and candidates. If we focus our attention on Wimp models we will notice that spin-1 candidates have been avoided in some compilations studies [17]. To bring back the stolen honor of vector candidates some models have been studied [18],[19],[20]. This time we focus our attention on a model in which gauge spin-1 particle is the dark matter candidate. We work within the FPVDM [21] framework in which the connection between the dark and standard sectors, is carried out through fermions, which separates processes such as the direct detection of the relic density, avoiding the tension caused by the classic Higgs-portal (Figure 1.1, a product of the experimental restrictions in which the coupling constant for DM must be large enough to achieve the relic density observed today.

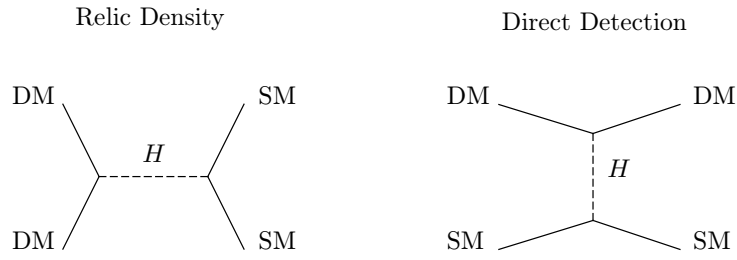


Figure 1.1: Standard Higgs-portal generic diagrams for relic density and Direct detection Processes.

We organized this Theses in the following way. Chapter 2 gives a very short review of the standard model of particle physics, which is the base for incorporate the dark matter as a fundamental particle, Chapter 3 covers the main aspects of a wimp dark matter that are used in this work, in Chapter 4 we develop the models and provides the phenomenology results, finally we summarize our the early conclusions and the future work in Chapter 5.

Chapter 2

Standard Model in a Nutshell

The most successful theory of physics, "The Standard Model of Particle Physics", is a quantum field theory, so the objects that appear in the equations are mathematical abstract objects that fill all the space which excitations are that we call particles¹. But these fields are Quantum fields, in the framework of second quantization, i.e., the states are in a Fock Space, such that they act on the particle states creating and annihilating particles and antiparticles. These quantum fields obey a commutation or anticommutation relation that tells us if they have a bosonic or fermionic statistic respectively, due to the spin-statistic theorem that relates to the spin of the particles. In particular for the standard model we have: spin-0, the Higgs boson; spin-1/2, all the fermions; and spin-1 all the gauge bosons which mediates the fundamental interactions.

Standard model is constructed as a gauge theory $SU(3)_C \times SU(2)_L \times U(1)_Y$. These are tensor products of group symmetries, they are important, because are the pillar on which field theories are built. They are symmetries in the sense that fields transform under them (when they do not transform we say that the field is a singlet of the symmetry group), for example, right-handed electrons transform as $e_R \rightarrow e^{-ig'Y\theta(x)}e_R$ under $U(1)_Y$, and when all the transformed fields are put back into the Lagrangian this remains invariant. Let's take a moment to talk a little about each symmetry; $SU(3)_C$ is the color group symmetry that acts only on quarks and is responsible for the strong interaction, the gluon G_μ^a is the mediator. $SU(2)_L$ is the weak, isospin or flavor symmetry that acts in all fermions but only on the left-handed part, therefore the standard model is a chiral theory, which distinguishes between the right and left part of fermion fields, this interaction is responsible for change the flavor of fermions allowing phenomena such as $\pm\beta$ decay, $W_\mu^\pm$ and Z_μ . Finally, $U(1)_Y$ is the hypercharge group symmetry and their mediator is B_μ . Unlike the other two symmetries, the last one is Abelian, so B_μ does not have a self-interaction, this gauge symmetry is

¹In the most rigorous and mathematical sense they are an irreducible, unitary representation of the Poincaré Group.

something like that "precursor" of electromagnetism $U(1)_{EM}$ that will appear when the electroweak $SU(2)_L \times U(1)$ symmetry is broken by the Higgs boson, in fact, photon A_μ and Z_μ boson are linear combinations of B_μ and W_μ^3 . Although the B_μ boson is not a physical field, hypercharge is necessary for writing the lagrangian with the gauge symmetry unbroken.

We can enounce now the standard model lagrangian, which will be the first ingredient of any Dark Matter particle model:

$$\begin{aligned}
\mathcal{L}_{SM} = & -\frac{1}{4}W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}G_{\mu\nu}^a G^{a\mu\nu} \\
& + \sum_{l=e,\nu,\tau} i(\bar{L}_l \not{D} L_l + \bar{l}_R \not{D} l_R) + \sum_{i=1}^3 \bar{Q}_i \not{D} Q_i + \sum_{q=\text{all quarks}} \bar{q}_R \not{D} q_R \\
& + |D_\mu H|^2 + \mu^2 H^\dagger H - \lambda(H^\dagger H)^2 \\
& - \sum_{l=e,\nu,\tau} y_l \bar{L}_l H l_R - \sum_{i=1}^3 \bar{Q}_i \left(y_{ii}^u i\sigma_2 H q_{uR}^i + \sum_{j=1,2,3} y_{ij}^d H q_{dR}^j \right) + h.c.
\end{aligned} \tag{2.1}$$

Where $W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g\epsilon^{abc}W_\mu^b W_\nu^c$ is the strength tensor of the isospin bosons $W^a = \{W^+, W^3, W^-\}$. Similarly $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ and $G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a + g_s f_{SU(3)}^{abc} G_\mu^b G_\nu^c$ are the strength tensor for hypercharge boson B_μ and gluons G_μ^a ($a = 1, 2, \dots, 8$ are color indices). $L_l = \begin{pmatrix} \nu_l \\ l \end{pmatrix}_L$ is the left lepton doublet, where $l = e, \mu, \tau$ are the electron, muon, and tau, respectively. The

last two charged leptons are "heavier electrons", only differ in mass, and are unstable. $Q_i = \begin{pmatrix} q_u^i \\ q_d^i \end{pmatrix}_L$ is the left quark doublet, where $i = 1, 2, 3$ is the generation, every generation or family is heavier than the previous one, explicitly $q_u^1 = u$, $q_u^2 = c$, $q_u^3 = t$, $q_d^1 = d$, $q_d^2 = s$ and $q_d^3 = b$ are up, charm, top, down, strange and bottom quarks, respectively. Due to weak interactions right and left-handed fermions are not equal they have different quantum numbers because right fields are singlets of $SU(2)_L$. Finally $H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ is the scalar Higgs field. The particles and their symmetries are summarized in table 2.1

The full covariant derivative is the following:

$$D_\mu = \partial_\mu - ig_s \frac{\lambda^a}{2} G_\mu^a - ig \left(\frac{T^\pm}{\sqrt{2}} W_\mu^\pm + T^3 W_\mu^3 \right) - ig' Y B_\mu \tag{2.2}$$

Where g_s , g and g' is the strong, $SU(2)_L$ and $U(1)_Y$ coupling constant respectively. λ^a are the

Gell-Mann matrices, generators of $SU(3)_C$; $T^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, $T^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ and $T^3 = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ are the $SU(2)_L$ generators. Finally Y is the hyper charge, the generator of $U(1)_Y$.

Field	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_{EM}$
$W_\mu = \begin{pmatrix} W_\mu^+ \\ W_\mu^3 \\ W_\mu^- \end{pmatrix}$	1	3	0	$\begin{matrix} 1 \\ 0 \\ -1 \end{matrix}$
B_μ	1	1	0	0
G_μ	8	1	0	0
$L_l = \begin{pmatrix} \nu_l \\ l \end{pmatrix}_L$, $l = e, \mu, \tau$	1	2	$-1/2$	$\begin{matrix} 0 \\ -1 \end{matrix}$
$l_R = e_R, \mu_R, \tau_R$	1	1	-1	-1
$Q_i = \begin{pmatrix} q_u^i \\ q_d^i \end{pmatrix}$, $i = 1, 2, 3$	3	2	$1/6$	$\begin{matrix} 2/3 \\ -1/3 \end{matrix}$
$q_R = \begin{matrix} u_R, c_R, t_R \\ d_R, s_R, b_R \end{matrix}$	3	1	$\begin{matrix} 2/3 \\ -1/3 \end{matrix}$	$\begin{matrix} 2/3 \\ -1/3 \end{matrix}$
$H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	1	2	$1/2$	$\begin{matrix} 1 \\ 0 \end{matrix}$

Table 2.1: Standard model fields and their symmetries, The bold numbers indicate if the field is an n-plet of the symmetry (**1** means that the field is a singlet and does not transform under such symmetry). For $U(1)_Y$ and $U(1)_{EM}$ we indicate the hypercharge and the electric charge, respectively.

As we mention before, the Higgs boson breaks the $SU(2)_L \times U(1)_Y$ symmetry acquiring a non-zero vacuum expectation value, $\langle H \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$, this allows fermions and gauge weak bosons to acquire mass, via Yukawa interactions fermion-Higgs-fermion and Higgs kinetic terms, respectively. Let's take a moment to develop this form of acquired mass via Symmetry breaking.

2.1 The Higgs Mechanisms

At some point in the, the $SU(2)_L$ symmetry was broken, since the expectation value of the Higgs field "moved" from the zero value, Figure 2.1. In the unitary gauge, we can write:

$$H = \begin{pmatrix} 0 \\ (v + h)/\sqrt{2} \end{pmatrix} \quad (2.3)$$

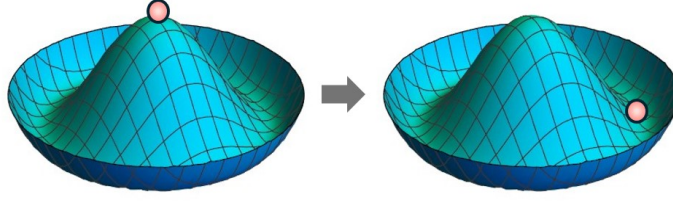


Figure 2.1: Phase transition of the Higgs potential.

in this way, massive terms for the gauge bosons in the Higgs kinetic term:

$$\begin{aligned}
 (D_\mu H)^\dagger D^\mu H &= (\partial_\mu H - igT^a W_\mu^a - i\frac{g'}{2}B_\mu)^\dagger (\partial^\mu H - igT^a W^{a\mu} - i\frac{g'}{2}B^\mu)H \\
 &\supset \frac{g^2 v^2}{4} W_\mu^+ W^{-\mu} + \frac{g^2 v^2}{8} (W^3)^2 + \frac{g'^2 v^2}{8} (B)^2 - \frac{gg'}{4} BW^3
 \end{aligned} \tag{2.4}$$

we see that all bosons have a massive term, but W^3 B have a mixed massive term because they are not physical fields, we come back to this fact in the next section.

Now fermions acquire mass from the Yukawa term, let's do an example with the electron:

$$y_e \bar{L}_e H e_R = \frac{y_e v}{\sqrt{2}} \bar{e}_L e_R + \frac{y_e}{\sqrt{2}} \bar{e}_L h e_R \tag{2.5}$$

We can identify the first term as a massive term. This is analogous for all SM fermions, with the exception of neutrinos, in the standard model they do not have mass, due to neutrino oscillations we know that they must have a tiny mass, however, we are not sure how is this massive term, this can be a Dirac term as all the standard model fermions, but the Yukawa coupling would be too small, $y_\nu \lesssim 10^{-11}$, in comparison with the others fermions. The neutrino is the only SM particle that can be a Majorana fermion, i.e., neutrino can be its own antiparticle under charge conjugation, this allows processes such as the Neutrinoless double beta decay, which has not been observed yet. Thus in principle neutrinos can have a Majorana mass term, for example $\frac{1}{2} M \bar{\nu}_L^c \nu_L$, where because the mass of Majorana is very large as a consequence the neutrinos will have a very small mass, like a kind of seesaw, and hence the name. In this work, we have a seesaw mechanism for neutrinos in the second model, but they still remain Dirac fermions (see section 4.3).

2.2 Non-Diagonal Fields

There are three interesting phenomena that are important to mention. First, there is an unbroken symmetry $U(1)_{EM}$ the electromagnetism and the conserved quantum number is the electric charge

$Q = T^3 + Y$. Second, when the symmetry breaks W_μ^3 and B_μ mix, so when we diagonalize we find the mass eigenstates, they are:

$$\begin{aligned} Z_\mu &= W_\mu^3 \cos \theta_W - B_\mu \sin \theta_W \\ A_\mu &= W_\mu^3 \sin \theta_W + B_\mu \cos \theta_W \end{aligned} \quad (2.6)$$

Where θ_W is the Weinberg angle. And third, quarks interact between families as we can observe from the $K^- \rightarrow \mu \bar{\nu}_\mu$ where the kaon is a bound state of $\bar{u}$ and s quarks, this interaction can be understood as a non-diagonal quark mass matrix. If we work in the base where u -like quarks are diagonal, then we must diagonalize the d -like quark mass matrix, the eigenstates are:

$$\begin{aligned} q_{dL}^i &= V_{ij}^\dagger q_{dL}^j \\ q_{dR}^i &= A_{ij}^\dagger q_{dR}^j \end{aligned} \quad (2.7)$$

Where V and A diagonalize the product $MM^\dagger$ and $M^\dagger M$ respectively. V is also known as the Cabibbo-Kobayashi-Maskawa matrix, since this matrix is unitary, this only changes the charged currents $\frac{g}{\sqrt{2}}(\bar{q}_{uL}^i W^+ V_{ij} q_{dL}^j + \bar{q}_{dL}^i V_{ij}^\dagger W^- q_{uL}^j)$, neutral currents remain unchanged, because $\bar{q}_{dL}^i q_{dL}^i = \bar{q}_{dL}^i q_{dL}^i$ (the same is true for right-handed quarks). This mathematical structure is similar in neutrino oscillation, although this is beyond SM physics, we will use this for the two generations NPVDM model. Neutrinos exhibit a similar behavior, interaction (or flavors) states are not mass eigenstates, therefore we use the Pontecorvo–Maki–Nakagawa–Sakata matrix to relate the states:

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix} \quad (2.8)$$

Where $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$ with θ_{ij} mixing angles between ν_i and ν_j states and δ_{CP} is the CP violation phase.

Chapter 3

Dark Matter

Dark Matter is the name of a gravitational phenomena that is observed in the universe. In 1930 Fritz Zwicky [22] was studying the coma cluster, he observed that the orbital speed of these galaxies moves faster than expected to keep the cluster gravitationally bound. To not contradict Newton's law of universal gravitation, Zwicky proposed that there should be some kind of substance or object that does not emit light to hold the cluster together gravitationally, he called it "Dunkle Materie" or Dark Matter in English. Unfortunately, no one took this idea seriously, but about 40 years later Vera Rubin and Kent Ford [23] observed the motion of stars in the Andromeda galaxy and found that the orbital speed remains almost constant when we move away from the galactic center, in contrast of what we expected from Newtonian gravity, where the speed of stars would decrease as we move away from the galaxy center. Without additional mass, these stars should have escaped the gravitational pull of the galaxy a long time ago. Again an additional invisible matter can explain the phenomena, Figure 3.1 shows the 21 Sc rotation curves measured by Rubin, Ford, and Thonnard [24].

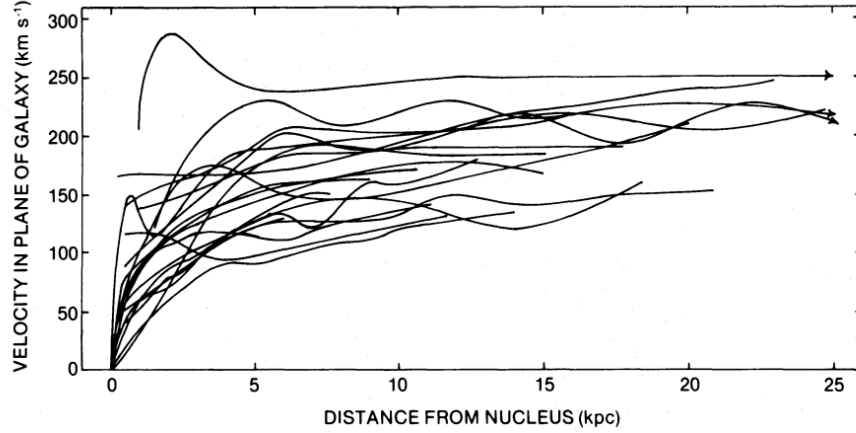


Figure 3.1: Rotation curves of spiral galaxies. Most galaxies show a flattening of the circular velocity at large radial distances.

This is only the beginning of history, today we know that we need this additional matter to explain many aspects of the universe, for making a short compilation, we can mention the Bullet cluster, two galaxy clusters that collided in the past, and the gas mix giving zones where the visible matter accumulates, but we can observe gravitational lenses in zones where we can see the visible matter, other evidence is the large-scale simulations of the universe where we need additional matter to explain why the universe is not so diluted and we can also observe evidence in the CMB from the baryon acoustic oscillations (BAO).

To explain what dark matter is, there are numerous kinds of candidates, and not all of them assume that dark matter is a fundamental particle, as the modified gravity, for name some of them: Fuzzy dark matter, Axions, Sterile Neutrino, Primordial Black Holes, Extra Dimensions, SIMPS, M.A.C.H.O.s, WIMPs, etc.

Today we know by simulations that DM (if is a particle) must be cold, i.e., a nonrelativistic particle, or at least warm, so what are we looking for is Cold Dark Matter (CDM), obviously Hot dark matter, as neutrinos, can contribute to the total amount of DM, but cold dark matter must be the dominant.

The rest of this Chapter is dedicated to some properties of WIMP (weakly interacting massive particles), the most popular DM candidate and declining in popularity in recent years. This kind of candidate assumes that DM is heavy, then nonrelativistic, and interacts very little with ordinary matter. WIMPs are of our interest because the models studied in this thesis are WIMPS models.

3.1 Relic Density

WIMPs candidates come with a proposal for the origin of dark matter, The following is the thermal assumption: In the early universe most of the particles were in thermal equilibrium. That means that creates and annihilates all kinds of particles and the temperature and energy were so high that there was no room for bound states such as atoms to exist. Then is reasonable to assume that WIMPs particles also were in equilibrium with this primordial soup, so unlike today reactions of creation and annihilation of DM were the daily bread at that time, but nothing is forever, the expansion of the universe separated the particles from the primordial soup, lowering the temperature and decreasing the energy available for interactions, therefore departures from the thermal equilibrium begin to occur. The first to decouple are heavy particles then increasingly lighter particles follow until neutrino decoupling, then Big Bang Nucleosynthesis takes place creating the first lighter atoms and finally, photons decouple being the last to decouple, those that form what we know today as the Cosmic Microwave Background (CMB)¹. Fig 3.2 shows a brief resume of the early history of our universe.

¹A further discussion of the early universe can be found in [1, 13]

Event	time t	redshift z	temperature T
Singularity	0	∞	∞
Quantum gravity	$\sim 10^{-43}$ s	–	$\sim 10^{18}$ GeV
Inflation	$\gtrsim 10^{-34}$ s	–	–
Baryogenesis	$\lesssim 20$ ps	$> 10^{15}$	> 100 GeV
EW phase transition	20 ps	10^{15}	100 GeV
QCD phase transition	20 μ s	10^{12}	150 MeV
Dark matter freeze-out	?	?	?
Neutrino decoupling	1 s	6×10^9	1 MeV
Electron-positron annihilation	6 s	2×10^9	500 keV
Big Bang nucleosynthesis	3 min	4×10^8	100 keV
Matter-radiation equality	60 kyr	3400	0.75 eV
Recombination	260–380 kyr	1100–1400	0.26–0.33 eV
Photon decoupling	380 kyr	1100	0.26 eV
Reionization	100–400 Myr	10–30	2.6–7.0 meV
Dark energy-matter equality	9 Gyr	0.4	0.33 meV
Present	13.8 Gyr	0	0.24 meV

Figure 3.2: Brief timetable of the thermal history of the universe, taken from Chapter 3 of [1]

WIMPs also have to decouple from the thermal bath, since they have a weak coupling (not necessary the weak coupling of $SU(2)_L$) as the universe cools there is a lower probability of interacting with dark matter, causing that the annihilations freeze out, and the number of DM in a commoving volume freezes in giving a constant "relic" density of dark matter that is observed today.

We can calculate when the decouple occurs and how much dark matter we going to observe today using the Boltzmann equation:

$$\hat{L}[f] = \hat{C}[f] \quad (3.1)$$

Where $\hat{L}$ is the Liouville operator, $\hat{C}$ is the collision operator, and f is the phase space density of a particle species. In covariant notation, this equation looks like:

$$\left(p^\mu \partial_\mu - \Gamma_{\nu\rho}^\mu p^\nu p^\rho \frac{\partial}{\partial p^\mu} \right) f = \hat{C}[f] \quad (3.2)$$

Where $\Gamma_{\nu\rho}^\mu$ is the affine connection that contains all the effects of gravity. For the FRW model, the universe is homogeneous and isotropic, so there is no preferred direction, which implies that $f = f(E, t)$, therefore:

$$\left(E\partial_t - \Gamma_{\nu\rho}^0 p^\nu p^\rho \frac{\partial}{\partial E} \right) f = \hat{C}[f] \quad (3.3)$$

Recall that non-zero terms of the affine connection are:

$$\begin{aligned} \Gamma_{jk}^i &= \frac{g^{ij}}{2} (\partial_k g_{ij} + \partial_j g_{ik} - \partial_l g_{jk}) \\ \Gamma_{ij}^0 &= -\frac{\dot{R}}{R} g_{ij} \\ \Gamma_{0j}^i &= \frac{\dot{R}}{R} \delta_j^i \end{aligned} \quad (3.4)$$

Using that $g_{ij} p^i p^j = -|\vec{p}|^2$ and $\dot{R}/R = H$, the Boltzmann equation is:

$$\left(\partial_t - H \frac{|\vec{p}|^2}{E} \frac{\partial}{\partial E} \right) f = \frac{\hat{C}[f]}{E} \quad (3.5)$$

Now we going to use that number density is:

$$n(t) = g \int \frac{d^3 p}{(2\pi)^3} f(E, t) \quad (3.6)$$

With g the internal (or spin) degrees of freedom. Adding the integral:

$$\dot{n} - Hg \int \frac{d\Omega dp}{(2\pi)^3} \frac{|\vec{p}|^4}{E} \frac{\partial}{\partial E} f = g \int \frac{d\Omega dp}{(2\pi)^3} \frac{\hat{C}[f]}{E} \quad (3.7)$$

Using $\frac{\partial f}{\partial E} = \frac{\partial f}{\partial |\vec{p}|} \frac{E}{|\vec{p}|}$ and integrating by parts we obtain for the second term in lhs:

$$-Hg \int \frac{d\Omega dp}{(2\pi)^3} \frac{|\vec{p}|^4}{E} \frac{\partial}{\partial E} f = 3Hg \int \frac{d\Omega dp}{(2\pi)^3} |\vec{p}|^2 f = 3Hn \quad (3.8)$$

Therefore we have:

$$\dot{n} + 3Hn = g \int \frac{d\Omega dp}{(2\pi)^3} \frac{\hat{C}[f]}{E} \quad (3.9)$$

Because we are interested in the relic of Dark Matter we want n_χ being χ a generic DM candidate. For collision operator we going to make some assumptions:

- Consider a 2 to 2 processes for DM annihilation $\psi\chi \rightarrow YX$, with XY generic non-DM particles and ψ can be DM (annihilation) or another particle (co-annihilation).

- Consider T or CP invariance, so $|\mathcal{M}_{i \rightarrow f}|^2 = |\mathcal{M}_{f \rightarrow i}|^2$
- Consider a Boltzmann statistics for all species in equilibrium $f_i(E_i) = e^{-(E_i - \mu_i)/T}$

Then our Boltzmann equation takes the following form:

$$\dot{n}_\chi + 3Hn_\chi = - \int d\Pi_X d\Pi_Y d\Pi_\psi d\Pi_\chi (2\pi)^4 \delta^4(p_X + p_Y - p_\chi - p_\psi) |\mathcal{M}|^2 (f_\chi f_\psi - f_X f_Y) \quad (3.10)$$

Where $d\Pi_i = \frac{g_i d^3 p_i}{(2\pi)^3 2E_i}$. Now we going to use that cross-section in the CM for the $ij \rightarrow kl$ process is:

$$\sigma_{ij} = \frac{1}{2E_i 2E_j v_{ij}} \int \frac{g_k d^3 p_k}{(2\pi)^3 2E_k} \frac{g_l d^3 p_l}{(2\pi)^3 2E_l} |\mathcal{M}|^2 (2\pi)^4 \delta^4(p_X + p_Y - p_\chi - p_\psi) \quad (3.11)$$

Therefore:

$$\dot{n}_\chi + 3Hn_\chi = - \int g_\chi \frac{d^3 p_\chi}{(2\pi)^3} g_\psi \frac{d^3 p_\psi}{(2\pi)^3} f_\chi f_\psi \sigma_{\chi\psi} v_{\chi\psi} + \int g_Y \frac{d^3 p_Y}{(2\pi)^3} g_X \frac{d^3 p_X}{(2\pi)^3} f_X f_Y \sigma_{XY} v_{XY} \quad (3.12)$$

These terms are very similar to an ensemble average, so we can write:

$$\dot{n}_\chi + 3Hn_\chi = -(\langle \sigma v \rangle_{\chi\psi} n_\chi n_\psi - \langle \sigma v \rangle_{XY} n_X n_Y) \quad (3.13)$$

This is a generic differential equation for the DM density in the universe. Let us perform a particular case for a better understanding. Suppose that the process is $\chi \bar{\chi} \rightarrow X \bar{X}$. Since the final particles are in equilibrium with the thermal bath they will have the same phase-space distribution function when DM was in equilibrium, so $f_X f_{\bar{X}} = f_\chi^{eq} f_{\bar{\chi}}^{eq}$. If there is no DM particle-antiparticle asymmetry then:

$$\langle \sigma v \rangle_{X\bar{X}} n_X n_{\bar{X}} = \langle \sigma v \rangle_{\chi\bar{\chi}} n_\chi^2 \quad (3.14)$$

So the Boltzmann equation takes the form

$$\dot{n}_\chi + 3Hn_\chi = - \langle \sigma v \rangle (n_\chi^2 - n_{eq}^2) \quad (3.15)$$

Now the DM number density decreases naturally with the universe expansion so to understand the effect of the decreasing due to the decoupling is useful to rescale the DM number density using the entropy per comoving volume s :

$$Y = \frac{n_\chi}{s} \quad (3.16)$$

²This becomes exact for a non-relativistic species in the limit $(m_i - \mu_i)T \gg 1$.

Using that the total entropy $S = R^3 s = \text{constant}$, we have

$$\begin{aligned}
\dot{Y} &= \frac{\dot{n}}{s} - \frac{n}{s^2} \dot{s} \\
&= \frac{\dot{n}}{s} - \frac{n}{s^2} \frac{d}{dt} \left(\frac{S}{R^3} \right) \\
&= \frac{\dot{n}}{s} + 3 \frac{n}{s^2} \frac{\dot{S}}{R^4} \\
&= \frac{\dot{n}}{s} + 3H \frac{n}{s}
\end{aligned} \tag{3.17}$$

So:

$$\dot{Y} = -s \langle \sigma v \rangle (Y^2 - Y_{eq}^2) \tag{3.18}$$

We usually rescale the time variable with $x = \frac{m_\chi}{T}$. During the radiation-dominated epoch, the relation between variables was:

$$t = 0.301 g_*^{-1/2} \frac{m_{pl}}{m_\chi^2} x^2 \tag{3.19}$$

Where $g_* = \sum_{i=bosons} g_i (T_i/T)^4 + 7/8 \sum_{i=fermions} g_i (T_i/T)^4$ is the total number of effectively massless degrees of freedom (spin degrees) for $m_i \ll T$ and $m_{pl} = 1.2211 \times 10^{19} [GeV]$ is the Planck mass, for further details see [13]. So with these considerations our equation finally becomes:

$$\begin{aligned}
\frac{dY}{dx} &= \dot{Y} \frac{dt}{dx} = -s 0.301 \frac{m_{pl}}{m_\chi} 2x \langle \sigma v \rangle (Y^2 - Y_{eq}^2) \\
\frac{dY}{dx} &= - \frac{s x}{H(m)} \langle \sigma v \rangle (Y^2 - Y_{eq}^2)
\end{aligned} \tag{3.20}$$

This equation describes the Freeze Out DM production Mechanisms, at some point x_f the annihilation freeze out and the dark matter density departure from the equilibrium leaving a constant relic density that is observed today by satellites as PLANCK [25]. Fig 3.3 shows the behavior of dark matter density.

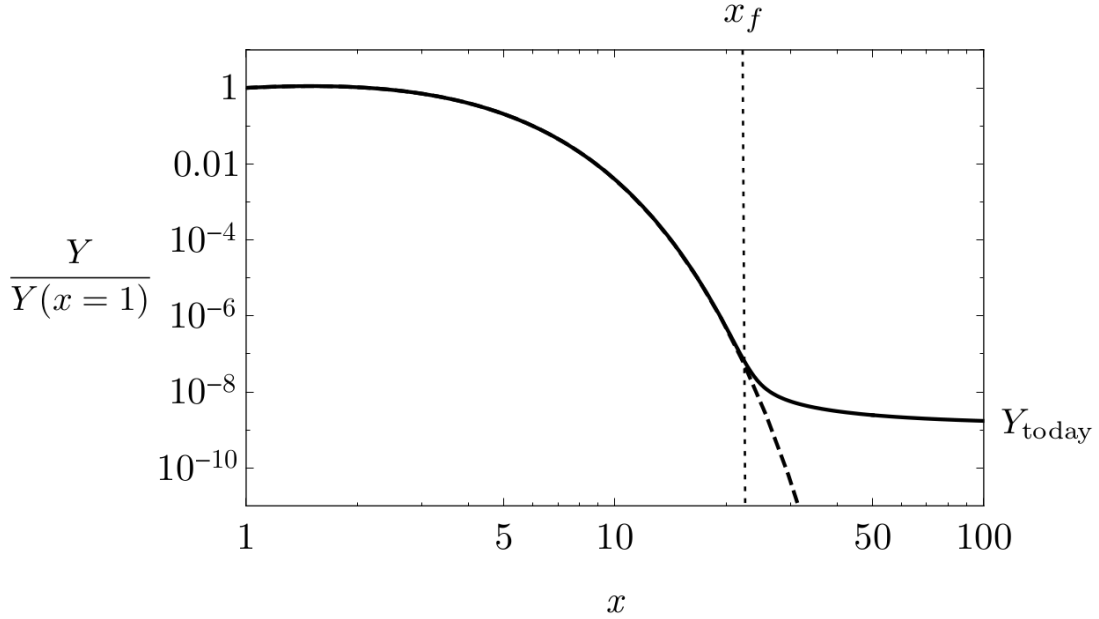


Figure 3.3: illustration of the DM number density Y as a function of x . x_f is the point where DM decouples. Image taken from [2].

Solving numerically the Boltzmann equation we can obtain $Y_{\text{today}} = Y_0$, with this quantity we can calculate the number density $n_{\chi 0} = s_0 Y_0$, the mass density $\rho_0 = m_\chi Y_0$, and the quantity of interest for this work, the dimensionless density $\Omega_\chi = \rho / \rho_c$, where $\rho_c = \frac{3H_0^2}{8\pi G}$ is the critical density of the universe with H_0 and G the Hubble constant and gravitational constant respectively. Due to the lack of precise knowledge of H_0 usually we hide our ignorance using $H_0 = 100h[kms^{-1}Mpc^{-1}]$ so the quantity that we really calculate is Ωh^2 , and the value measure by PLANCK is $\Omega h^2 = 0.12 \pm 0.001$, therefore our dark matter model should reproduce this number.

3.2 Direct Detection

If Dark Matter is a WIMP particle, means that interacts with the standard model with an interaction additional to the gravitational one. If this is the case, can we measure the scattering between dark matter and some standard-model particles?

The physics community has built numerous experiments to hunt down dark matter and measure the energy transmitted from dark matter to an atomic nucleus, some of them are XENON, LUX, Panda, PICO, etc. We can estimate the total DM number N that can we detect as the product of DM flux, the effective area of the target, and the exposure time of the experiment:

$$N = J t_{exp} A_{eff} \quad (3.21)$$

We can use that flux is $J = nv$, i.e., the dark matter number density n times the DM velocity v in the lab frame and the effective area $A_{eff} = N_T \sigma$ as the product of the number of target particles N_T in the experiment and the interaction cross section σ between DM and the target, this last one depend of the specific model. So

$$N = n_\chi t_{exp} N_T v \sigma \quad (3.22)$$

To determine the DM spectrum recoil we can simply take de derivative to N :

$$\frac{dN}{dE_R} = n_\chi t_{exp} N_T v \frac{d\sigma}{dE_R} \quad (3.23)$$

Obviously, not all DM particles have the same velocity, so we will have to take an average considering a speed distribution function $f(v)$:

$$\frac{dN}{dE_R} = n_\chi t_{exp} N_T \int_{v_{min}}^{v_{esc}} d^3v f(v) v \frac{d\sigma}{dE_R} \quad (3.24)$$

Is useful to rewrite some parameters, $n_\chi = \frac{\rho_0}{m_\chi}$ with ρ the local (in our region of the Milky Way) mass density of DM (this depend of the astrophysics distribution profile), $N_T = \frac{M_T}{m_N}$ where M_T is the total target mass of the experiment and m_N the nucleus mass, finally we note that N increase if we increase t_{exp} because there is more time for DM interact with a nucleus and something similar occur if we increase M_T , due to there are more targets to interact, we going to join this phenomenon in a single parameter called the exposure $\epsilon = t_{exp} M_T$, with these considerations we obtain:

$$\frac{dN}{dE_R} = \epsilon \frac{\rho_0}{m_N m_\chi} \int_{v_{min}}^{v_{esc}} d^3v f(v) v \frac{d\sigma}{dE_R} \quad (3.25)$$

Now the number of observable events per unit target mass per unit time or the event rate is defined as N/ϵ , so the differential event rate is:

$$\frac{dR}{dE_R} = \frac{\rho_0}{m_N m_\chi} \int_{v_{min}}^{v_{esc}} d^3v f(v) v \frac{d\sigma}{dE_R} \quad (3.26)$$

Considering the scattering with the nucleus is coherently, thus the nucleus moves as a whole, and the differential cross section can be written as:

$$\frac{d\sigma}{dE_R} = \frac{m_N}{2\mu^2 v^2} (\sigma_0^{SI} F_{SI}^2 + \sigma_0^{SD} F_{SD}^2) \quad (3.27)$$

Where m_N is the nucleus mass, $\mu = \frac{m_\chi m_N}{m_\chi + m_N}$ is the reduced mass between the nucleus and DM,

$\sigma_0^{SI,SD}$ are the spin-independent and spin-dependent cross sections at zero momentum transfer and $F_{SI,SD}$ are the respective form factors, which encode the dependence on the momentum transfer $q = \sqrt{2m_N E_R}$ and the nuclear structure. In the rest of this section we will care about $\sigma_0^{SI,SD}$, although factors like $f(v)$ or F are important for the calculation of the event rate, the part of the equation that is model dependent is σ_0 , here is the particle physics input. Now let's talk about some important things about the cross-section.

Spin-Independent contribution arises from scalar $\bar{q}q$ and vector $\bar{q}\gamma^\mu q$ coupling in the effective lagrangian with DM and quarks, while Spin-Dependent arises from Axial-Vector coupling $\bar{q}\gamma^\mu q$. The effective lagrangian for vector DM can be written as³:

$$\mathcal{L}_{eff}^{\chi-q} = \alpha_q^S \chi^{*\mu} \chi_\mu \bar{q}q + \alpha_q^V \chi^{*\mu} \overleftrightarrow{\partial}_\nu \chi_\mu \bar{q}\gamma^\nu q + \alpha_q^A \epsilon^{\mu\nu\rho\sigma} \chi_\rho^* \overleftrightarrow{\partial}_\mu \chi_\nu \bar{q}\gamma_\sigma \gamma_5 q \quad (3.28)$$

For Spin-Independent the cross section takes the form:

$$\sigma_{0,\text{scalar}}^{SI} = \frac{4\mu^2}{\pi} (Zf^p + (A-Z)f^n)^2 \quad (3.29)$$

f^p and f^n are the effective coupling to dark matter to protons and neutrons (see [2]), since usually they are similar ($f^p \approx f^n$) it is said that $\sigma_0^{SI} \propto A^2$. For Vector contribution only valence quarks contribute [27]:

$$\sigma_{0,\text{Vector}}^{SI} = \frac{\mu^2}{64\pi} ((2\alpha_u^V + \alpha_n^V)Z + (2\alpha_d^V + \alpha_u^V)(A-Z))^2 \quad (3.30)$$

With $\alpha_{u,d}^V$ the coupling of DM- u, d in the effective lagrangian.

For Spin-Dependent contribution, the cross-section can be written as [28]

$$\sigma_0^{SD} = \frac{32\mu^2}{3\pi} J(J+1) \left| \sum_q \alpha_q^A \lambda_q^N \right|^2 \quad (3.31)$$

Where λ_q^N is the constant that relates spin matrix elements to the angular momentum of nucleus $\langle N | \bar{q}\gamma_\mu \gamma_5 q | N \rangle = 2\lambda_q^N \langle N | J_N | N \rangle$ and J is the total nuclear spin. Following [27] λ_q^N can be written as:

$$\lambda_q^N \simeq \frac{\Delta_q^p \langle S_p \rangle + \Delta_q^n \langle S_n \rangle}{J} \quad (3.32)$$

Where $\langle S_{p,n} \rangle$ are the expectation values of the proton and neutron spins within the nucleus, and $\langle i | \bar{q}\gamma_\mu \gamma_5 q | i \rangle = 2s_\mu \Delta_q^i$ [26], $i = p, n$, with s_μ the nucleon spin. Using $a_{p,n} = \sum_{q=u,d,s} \alpha_q^A \Delta_q^{p,n}$ we can

³An operator of the form $(\chi_\mu \chi_\nu^* - \chi_\mu^* \chi_\nu) \bar{q} \sigma^{\mu\nu} q$ is also considered in [26].

write:

$$\sigma_0^{SD} = \frac{32}{3\pi} \mu^2 \frac{(J+1)}{J} (a_p \langle S_p \rangle + a_n \langle S_n \rangle)^2 \quad (3.33)$$

Since the direct detection experiments use different targets, a more "universal" quantity is more useful to impose constraints on DM models. So we will use $\sigma_{p,n}$, the WIMP-nucleon cross-section. Following [29] and the notation of [30], they are defined as:

$$\sigma_{p,n} = \sigma \frac{\mu_{p,n}^2}{\mu^2} \frac{C_{p,n}}{C} \quad (3.34)$$

Here, $\mu_{p,n}$ are the reduced mass of DM-proton and DM-neutron, respectively. C is the enhancement factor. For spin-independent, this is:

$$C_{SI} = (f_p Z + f_n (A - Z))^2 + \frac{(2\alpha_u^V + \alpha_n^V)Z + ((2\alpha_d^V + \alpha_u^V)(A - Z))^2}{256} \quad (3.35)$$

And for spin dependent⁴:

$$C_{SD} = \frac{(J+1)}{J} (a_p \langle S_p \rangle + a_n \langle S_n \rangle)^2 \quad (3.36)$$

And $C_{p,n}$ are the enhancement factor for a single proton or neutron, e.g., for proton spin-dependent cross section this would be $C_p^{SD} = 3a_p^2/2$, where $\langle S_{n,p} \rangle = \frac{1}{2}$ and $J = 1/2$ for the proton.

As we mentioned before $\sigma_{p,n}$ are the important quantities for particle physics models, due to the restrictions imposed by the experiments in terms of these nucleon cross sections. Figure 3.4 shows some results of the direct detection experiments taken from [3, 4, 5]

⁴There is a well motivated alternative extracting method presented in [30].

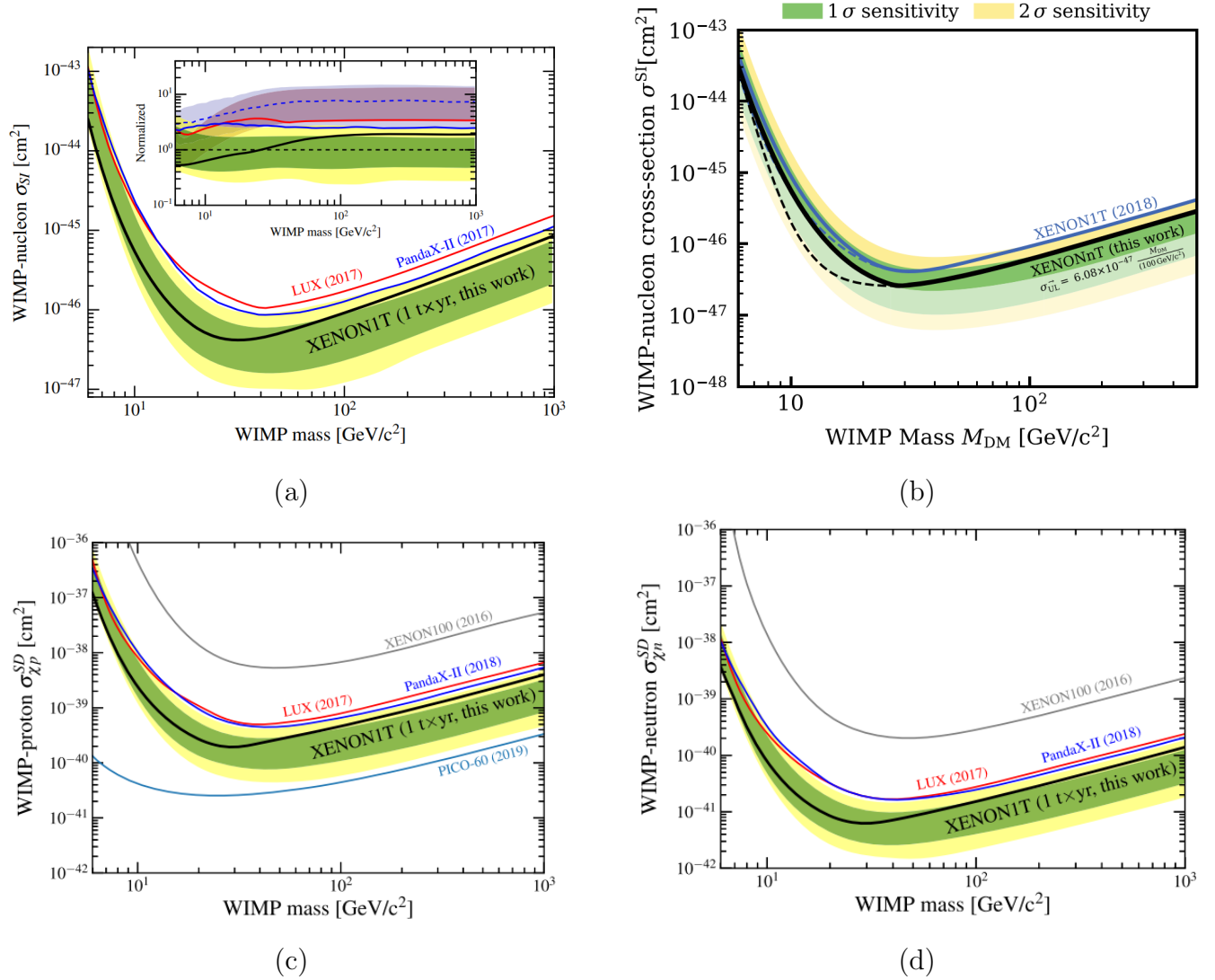


Figure 3.4: Direct Detection Limits for spin-dependent (c and d) and independent (a and b) WIMP-nucleon cross-section from XENON1T[3, 4], XENONnT[5], XENON100[6] LUX[7, 8], PandaX-II[9, 10] and PICO-60[11].

3.3 Indirect Detection

If dark matter is a WIMP particle implies that it can be annihilated, however, as we already mentioned in section 3.1 to keep constant the relic abundance dark matter can not be annihilated due to the freeze out. This is almost true for a WIMP candidate, but in sufficiently dense regions, dark matter has enough energy to annihilate into photons, regions like the galactic center. For this annihilation, a direct process DM to gamma rays is not necessary, because many particles could emit photons as sub-products. If we consider all these channels, then the annihilation rate per

particle is

$$\sum_i n_\chi \langle \sigma_i v \rangle \quad (3.37)$$

Now we can obtain the total rate in a volume $dV = l^2 dl d\omega$ by a multiplication of the number density in the volume:

$$\frac{1}{2} \sum_i n_\chi \langle \sigma_i v \rangle n_\chi l^2 dl d\Omega \quad (3.38)$$

A factor of two is put in the equation to avoid double counting of annihilation events. We can calculate the number of photons observed, per unit of time, per area, and energy that comes from the volume V , in the following way:

$$\frac{dN_{\gamma,obs}}{dE_\gamma dA dt} = \int_{\Delta\Omega} \int_{l.o.s} \frac{1}{4\pi l^2} \frac{1}{2} \sum_i \langle \sigma_i v \rangle \frac{dN_i}{dE_\gamma} n_\chi^2 l^2 dl d\Omega \quad (3.39)$$

Where dN_i/dE_γ is the spectrum of gamma rays produced per annihilation, the number of photons at a given energy E_γ produced in the i th annihilation channel, note that this is the photon flux observed, so if we assume that gamma rays produce in all directions, then the number of photons we observe is a fraction of the total diluted by the area of a sphere of radius l , hence the $4\pi l^2$ factor. We can reorganize this equation by rewriting the number density $n = \rho_\chi/m_\chi$:

$$\frac{d\Phi}{dE_\gamma} = \frac{1}{8\pi} \underbrace{\sum_i \frac{\langle \sigma_i v \rangle}{m_\chi^2} \frac{dN_i}{dE_\gamma}}_{\text{particle physics input}} \underbrace{\int_{\Delta\Omega} \int_{l.o.s} \rho_\chi^2 dl d\Omega}_{\text{astrophysics input}} \quad (3.40)$$

separating the part that depends on particle physics, the one of our interest, from astrophysics, also called the J -factor. Just to provide context the astrophysics part contains uncertainties due to we do not know for certain how the dark matter profile near the galactic center, Figure 3.5 shows some famous density profiles.

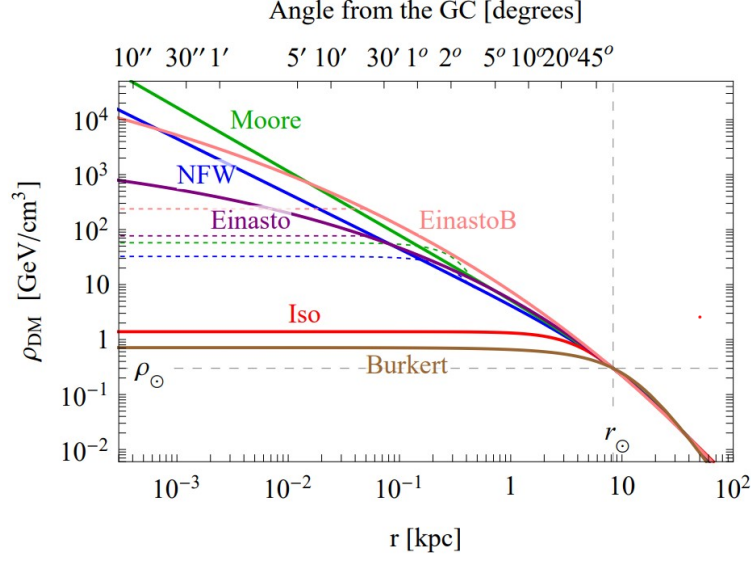


Figure 3.5: DM density profiles, image taken from [12], more details about the profiles also in this article.

As we mentioned before, we are interested in testing our specific model, and for that reason we are interested in the particle physics input, $\frac{\langle\sigma_{i v}\rangle}{m_{\chi}^2} \frac{dN_i}{dE_{\gamma}}$, but the photon spectrum depends on standard model particles which produce photons after DM annihilates to these particles. This part is well studied and we can find codes for the specific spectrum in [12] (and also J -factors), Figure 3.6 shows some spectrum as a function of the energy taken from the same study.

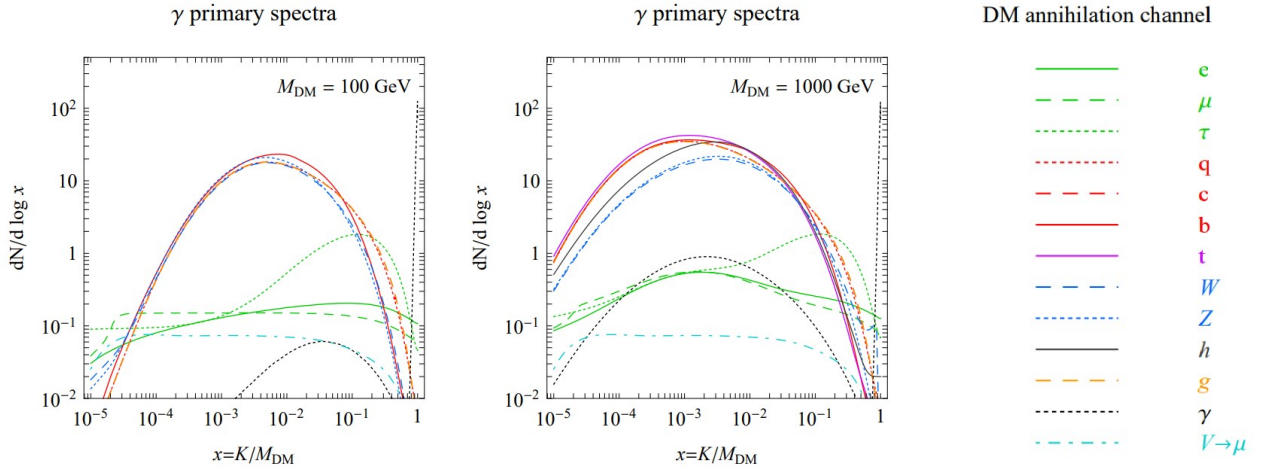


Figure 3.6: Spectra of photons for various DM annihilation channels.

Then the quantity of our interest, which depends exclusively on our model is $\langle\sigma v\rangle$, we already have upper limits to the sensibility of experiments, we will use the CTA projected limits [31] and

H.E.S.S. limits [32], depicted in Figure 3.7:

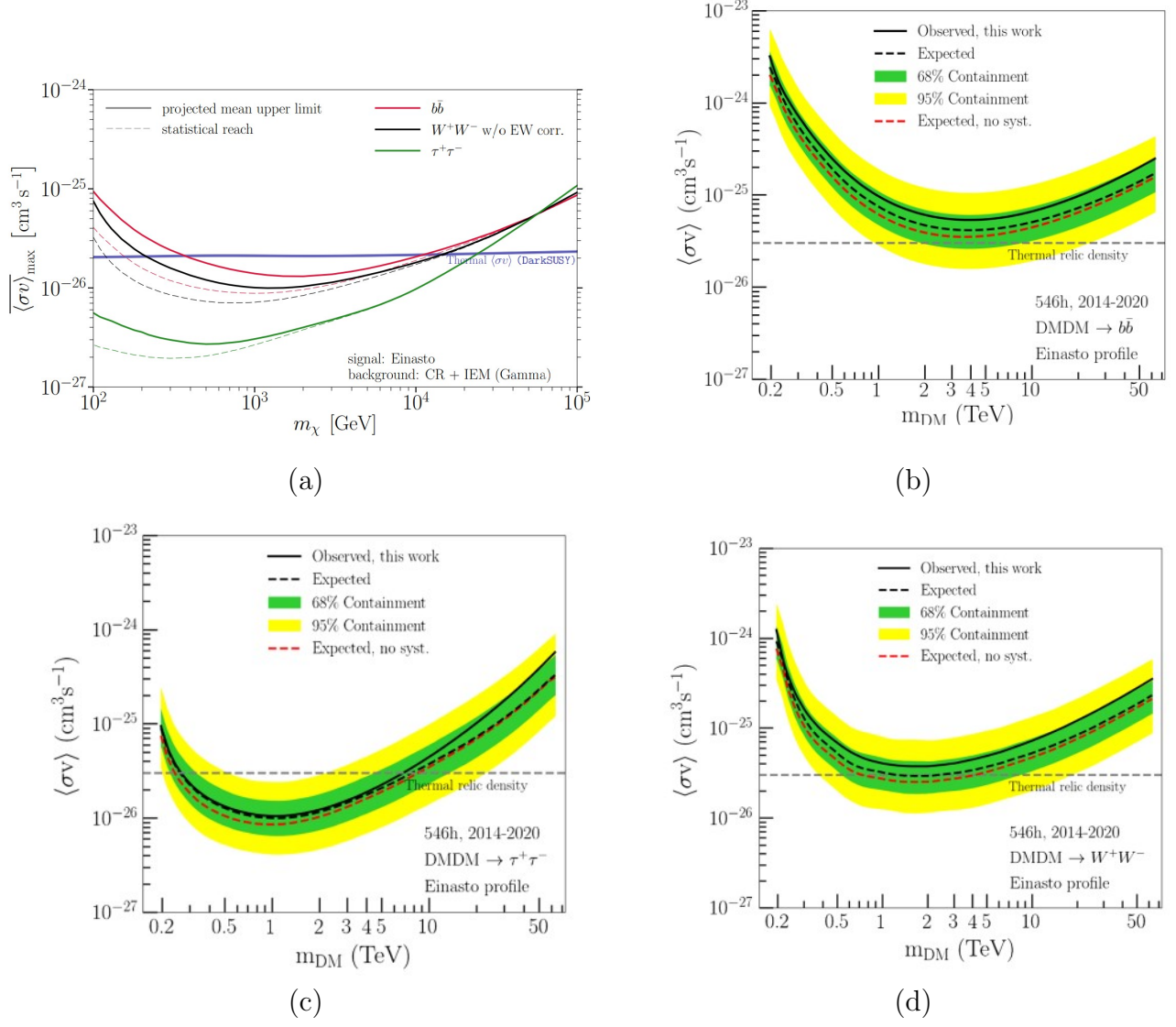


Figure 3.7: DM annihilation signal for channels W^+W^- , $\tau^+\tau^-$ and $b\bar{b}$. (a) Shows Expected sensitivity of CTA. (b), (c) and (d) show the upper Limits of HESS.

Chapter 4

Neutrino Portal for Vector Dark Matter Model

4.1 FPVDM Framework

The Fermion Portal Vector Dark Matter¹ (FPVDM) model is built in the following way. First, we consider that our dark matter particle has spin-1, which corresponds to a gauge field. So we going to add a new local gauge symmetry for which our field is the gauge boson, this means, there is a group of transformations under our field transform and the Lagrangian is invariant, thus we choose a group for these transformations, let's choose the minimal non-Abelian common group for that, $SU(2)_D$ group. We are adding a new Symmetry to the standard model in which all the Standard model fields are singlets under $SU(2)_D$ and this new field is a singlet of all the standard model symmetries. This vector field is now a triplet of the new group and can be represented as:

$$V_\mu = \begin{pmatrix} V_\mu^+ \\ V_\mu^0 \\ V_\mu^- \end{pmatrix} \quad (4.1)$$

This particle needs to be massive for being a dark matter candidate but is a gauge field and a massive term is not allowed because violates the gauge symmetries, so we need to break with the help of a new scalar field that is going to be a doublet of $SU(2)_D$ and will acquire a v.e.v:

$$\Phi_D = \begin{pmatrix} \varphi_{1/2} \\ \varphi_{-1/2} \end{pmatrix} \rightarrow \langle \Phi_D \rangle = \begin{pmatrix} 0 \\ v_D/\sqrt{2} \end{pmatrix} \quad (4.2)$$

¹In this section we construct only the general parts that we need of the model, other aspects like gauge $U(1)_D$ field, vector, and scalar mix can be found in detail in the original article [21].

Now if we couple through the covariant derivative $D_\mu = \partial_\mu - ig_D \left(\frac{1}{\sqrt{2}} V_\mu^\pm T_D^\pm + V_\mu^0 T_D^3 \right)$ where

$$T_D^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}; \quad T_D^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}; \quad T_D^3 = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

When the symmetry breaks, all fields in V_μ will have a mass of $M_V = \frac{v_D g_D}{2}$.

The particle needs to interact in some way with the standard model and in this kind of model the connection is made by an SM fermion which we call f^{SM} (can be any quark or lepton of SM), but for connecting the sectors we going to need the key ingredient, a new fermion:

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \quad (4.3)$$

This is a vector-like fermion, so the left and right-handed parts transform in the same way. These additional fields also form a doublet of $SU(2)_D$ and thus can interact with V_μ through covariant derivatives and to connect with the SM sector the doublet of VL-fermions couples to a SM field f^{SM} through a Yukawa term $-y' \bar{\Psi}_L \Phi_D f_R^{SM} + h.c.$

With all of these things in mind, we can construct the dark lagrangian density, which at the moment has the following form:

$$\mathcal{L}_D = -\frac{1}{4}(V_{\mu\nu}^a)^2 + \bar{\Psi}(i\not{D} - M_\Psi)\Psi + |D_\mu \Phi_D|^2 + \mu_D^2 \Phi_D^\dagger \Phi_D - \lambda_D (\Phi_D^\dagger \Phi_D)^2 - y' \bar{\Psi}_L \Phi_D f_R^{SM} + h.c. \quad (4.4)$$

Where $V_{\mu\nu}^a = \partial_\mu V_\nu^a - \partial_\nu V_\mu^a + g_D \epsilon^{abc} V_\mu^b V_\nu^c$.

4.1.1 $\mathbb{Z}_2$ Symmetry, Stability and Dark Charge

If we pay attention to the interaction between VL-fermions and bosons and then expand the term:

$$i\bar{\Psi}(-ig_D \gamma^\mu V_\mu)\Psi = \frac{g_D}{2}(\bar{\psi}_1 V^0 \psi_1 - \bar{\psi}_2 V^0 \psi_2 + \sqrt{2}\bar{\psi}_1 V^+ \psi_2 + \sqrt{2}\bar{\psi}_2 V^- \psi_1) \quad (4.5)$$

We'll notice that the vector is unstable, it can decay to ψ_1 and ψ_2 and this is not too good for a dark matter candidate. But we can make the fermion heavier and kinematically V_μ can't decay. Ok, this is not bad, because V_μ is a WIMP and must have a mass greater than $1GeV$ and if we put a VL-fermion mass of $M_\Psi > 5GeV$ the decay can't occur. However ψ_2 interact with f^{SM} , which can have a mass of a few GeV and is possible that $\psi_2 \rightarrow f^{SM}$. This makes V^0 irremediably unstable so this particle can not be dark matter, but we can place our hopes in $V^\pm$ because it

couples only to one ψ_2 and if we take $M_{\psi_1} > M_V$ there would be no problem. This is fine, but we need to be consistent in our Lagrangian, usually, the Lagrangian is constructed by proposing a symmetry and then adding all terms that are invariant under that symmetry and are renormalizable (if we don't want an effective theory) or just the terms that make the theory minimal, but in our case, there are no reason for not include another Yukawa term that takes the top particle of the VL-fermion doublet $y''\bar{\Psi}_L i\sigma_2 \Phi_D f_R^{SM}$, This term will cause that $\psi_1 \rightarrow f^{SM}$ making all the vector particles unstable. To fix that problem we can impose an additional symmetry, $\mathbb{Z}_2$, in which the fields transform in the following way:

$$\psi_1 \rightarrow -\psi_1; \quad \psi_2 \rightarrow \psi_2; \quad \varphi_{1/2} \rightarrow -\varphi_{1/2}; \quad \varphi_{-1/2} \rightarrow \varphi_{-1/2}; \quad V_\mu^\pm \rightarrow -V_\mu^\pm; \quad V_\mu^0 \rightarrow V_\mu^0. \quad (4.6)$$

In summary, the top components of the doublet acquire a minus sing, so we call them "odd" and the bottom component does not change, we call them "even", for the triplet only the middle component remains unchanged and the other two acquire a minus sing. The rest of the SM particles don't change under this transformation. In this way the $y''\bar{\Psi}_L i\sigma_2 \Phi_D f_R^{SM}$ is forbidden and $\mathbb{Z}_2$ give stability to the model.

The problem is that the multiples do not transform in the same way, i.e., there is another broken symmetry, which once broken causes this strange parity of these fields. So we going to expand our symmetry to $SU(2)_D \times U(1)_{Y_D}^g$, this is analogous to the electroweak model with the difference that the g in $U(1)_{Y_D}^g$ make explicit that the transformation is global preventing another massless gauge dark particle. So when Φ_D breaks $SU(2)_D \times U(1)_{Y_D}^g$ there will be a conserved symmetry $U(1)_D^g : e^{iQ_D\theta}$ where $Q_D = T_D^3 + Y_D$ is the dark charge, in analogy with electromagnetism this charge must be conserved in every process and this makes more explicit the notation of V_μ the super indices refer tho their dark charge. Now the $\mathbb{Z}_2 : (-1)^{Q_D}$ and is a subgroup of $U(1)_D^g$ where $\theta = \pi$, so all even fields have $Q_D = 0$ and odd fields $Q_D \neq 0$. Table 4.1 summarizes the fields and their symmetries.

4.1.2 Scalar Sector

In the scalar potential a scalar mixing renormalizable term is allowed by the symmetry $\lambda_{mix}(\Phi_D^\dagger \Phi_D)(H^\dagger H)$, but we want to study the phenomenology of the fermion portal, so we going to choose $\lambda_{mix} = 0$, this election will allow us to eliminate a free parameter in the space parameter of the model and the scalar will not mix at tree level² (A general scalar mixing between the Higgs field and the new scalar is developed in [21]).

²the fermions that couples to both scalars will make appear the scalar mixing at one-loop level but this interaction will be suppressed in comparison with a tree-level one.

Fields	$SU(2)_L$	$U(1)_Y$	$SU(2)_D$	$U(1)_{Y_D}^g$	$\mathbb{Z}_2$
$\Phi_D = \begin{pmatrix} \varphi_{1/2} \\ \varphi_{-1/2} \end{pmatrix}$	1	0	2	1/2	$\begin{smallmatrix} - \\ + \end{smallmatrix}$
$V_\mu = \begin{pmatrix} V_\mu^+ \\ V_\mu^0 \\ V_\mu^- \end{pmatrix}$	1	0	3	0	$\begin{smallmatrix} - \\ + \\ - \end{smallmatrix}$
$f_R^{SM} = \begin{pmatrix} q_{uR}, & \nu_{lR} \\ q_{dR}, & l_R \end{pmatrix}$	1	$\begin{smallmatrix} 2/3, & 0 \\ -1/3, & -1 \end{smallmatrix}$	1	0	$\begin{smallmatrix} + \\ - \end{smallmatrix}$
$\Psi_D = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$	1	$Y_{f_R^{SM}}$	2	1/2	$\begin{smallmatrix} - \\ + \end{smallmatrix}$

Table 4.1: Fields in the dark lagrangian and their symmetries.

Without scalar-mixing the dark scalar potential and the Higgs potential acquire an independent v.e.v. when the symmetry breaks. For the dark Scalar, this is:

$$V(\Phi_D) = -\mu_D^2 \Phi_D^\dagger \Phi_D + \lambda_D (\Phi_D^\dagger \Phi_D)^2 \rightarrow V(\Phi_D)_{min} = -\frac{\mu_D^2}{2} v_D^2 + \frac{\lambda_D}{4} v_D^4 \quad (4.7)$$

Minimizing the potential:

$$\frac{dV(\Phi_D)_{min}}{dv_D} = -\mu_D^2 v_D + \lambda_D v_D^3 = 0 \quad (4.8)$$

Since $v \neq 0$:

$$\mu_D^2 = \lambda_D v_D^2 = \frac{M_{H_D}^2}{2} \quad (4.9)$$

Using from the vector mass that $v_D = \frac{2M_V}{g_D}$ we can rewrite $\lambda_D = \frac{g_D^2 M_{H_D}^2}{8M_V^2}$.

4.2 Neutrino Portal Vector Dark Matter With One Ψ Generation

In the minimal scenario for the Neutrino Portal Vector Dark Matter model we connect the VL-fermions to only one generation and don't care about neutrino oscillations, however, we need to consider that neutrino has a tiny mass. The lagrangian is as follows

$$\begin{aligned} \mathcal{L} = \mathcal{L}_{SM} - \frac{1}{4}(V_{\mu\nu}^a)^2 + \bar{\Psi}(i\not{D} - M_\Psi)\Psi + |D_\mu \Phi_D|^2 + \mu_D^2 \Phi_D^\dagger \Phi_D - \lambda_D (\Phi_D^\dagger \Phi_D)^2 \\ + i\bar{\nu}_R \not{\partial} \nu_R - y_\nu \bar{L} i\sigma_2 H \nu_R - y' \bar{\Psi}_L \Phi_D \nu_R + h.c. \end{aligned} \quad (4.10)$$

We are forced to include the right neutrino to connect both sectors giving mass to a left neutrino³. We choose that is the electronic neutrino, which acquires said mas. Table 4.2 summarizes the fields for this extension model.

Fields	$SU(2)_L$	$U(1)_Y$	$SU(2)_D$	$U(1)_{Y_D}^g$	$\mathbb{Z}_2$
$\Phi_D = \begin{pmatrix} \varphi_{1/2} \\ \varphi_{-1/2} \end{pmatrix}$	1	0	2	1/2	$\begin{matrix} - \\ + \end{matrix}$
$V_\mu = \begin{pmatrix} V_\mu^+ \\ V_\mu^0 \\ V_\mu^- \end{pmatrix}$	1	0	3	0	$\begin{matrix} - \\ + \\ - \end{matrix}$
$L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$	2	-1/2	1	0	+
ν_R	1	0	1	0	+
$\Psi_D = \begin{pmatrix} n_D \\ N \end{pmatrix}$	1	0	2	1/2	$\begin{matrix} - \\ + \end{matrix}$

Table 4.2: Fields in the dark lagrangian and their symmetries.

4.2.1 Fermion Sector

In the fermion sector when the scalars acquire their v.e.v.s, massive terms will arise for fermions, but due to the new Yukawa interaction $\psi_2 = N$ will mix with ν , for that reason we need to diagonalize the mass matrix to find the mass eigenstates N' and ν' :

$$-\mathcal{L}_{m_f} = M_\Psi(\bar{n}_D n_D + \bar{N} N) + \frac{y_\nu v}{2} \bar{\nu}_L \nu_R + \frac{y' v_D}{2} \bar{N}_L \nu_R + h.c. \quad (4.11)$$

Notice that n_D do not mix, and we can rewrite the fields that mix in matrix form:

$$\begin{pmatrix} \bar{\nu}_L & \bar{N}_L \end{pmatrix} \begin{pmatrix} \frac{y_\nu v}{2} & 0 \\ \frac{y' v_D}{2} & M_\Psi \end{pmatrix} \begin{pmatrix} \nu_R \\ N_R \end{pmatrix} = \begin{pmatrix} \bar{\nu}_L & \bar{N}_L \end{pmatrix} \mathcal{M}_f \begin{pmatrix} \nu_R \\ N_R \end{pmatrix} \quad (4.12)$$

$$\begin{pmatrix} \bar{\nu}_L & \bar{N}_L \end{pmatrix} V_L V_L^\dagger \mathcal{M}_f V_R V_R^\dagger \begin{pmatrix} \nu_R \\ N_R \end{pmatrix} = \begin{pmatrix} \bar{\nu}'_L & \bar{N}'_L \end{pmatrix} \mathcal{M}_f^d \begin{pmatrix} \nu'_R \\ N'_R \end{pmatrix} \quad (4.13)$$

Where $V_i = \begin{pmatrix} \cos \theta_i & \sin \theta_i \\ -\sin \theta_i & \cos \theta_i \end{pmatrix}$, with $i = R, L$ are rotation matrices that diagonalize the matrices

³Remember that in the standard model neutrinos are Weyl fermions, they do not have mass, but we know from neutrino oscillations that they must be massive, however, we have neglected the oscillations to study dark matter phenomenology.

$\mathcal{M}_f \mathcal{M}_f^\dagger$ and $\mathcal{M}_f^\dagger \mathcal{M}_f$, V_R and V_L respectively. Then

$$V_L^\dagger \mathcal{M}_f V_R = \mathcal{M}_f^d = \begin{pmatrix} m_\nu & 0 \\ 0 & m_N \end{pmatrix} \quad (4.14)$$

Solving the eigensystem we find the mass eigenvalues:

$$m_{\nu,N}^2 = \frac{1}{4} \left[y_\nu^2 v^2 + y' v_D^2 + 2M_\Psi \mp \sqrt{(y_\nu v^2 + y' v_D^2 + 2M_\Psi^2)^2 - 8y^2 v^2 M_\Psi^2} \right] \quad (4.15)$$

And the fields in terms of mass eigenstates (ν'_i and N'_i) and vice-versa are:

$$\begin{pmatrix} \nu'_i \\ N'_i \end{pmatrix} = V_i^\dagger \begin{pmatrix} \nu_i \\ N_i \end{pmatrix} = \begin{pmatrix} \cos \theta_i \nu_i - \sin \theta_i N_i \\ \sin \theta_i \nu_i + \cos \theta_i N_i \end{pmatrix} \quad (4.16)$$

$$\begin{pmatrix} \nu_i \\ N_i \end{pmatrix} = V_i \begin{pmatrix} \nu'_i \\ N'_i \end{pmatrix} = \begin{pmatrix} \cos \theta_i \nu'_i + \sin \theta_i N'_i \\ -\sin \theta_i \nu'_i + \cos \theta_i N'_i \end{pmatrix} \quad (4.17)$$

4.2.2 Parameter Space

If we use the masses as free parameters we can rewrite the other coupling constants as:

$$y_\nu = \sqrt{2} \frac{m_\nu m_N}{M_\Psi v}, \quad y' = g_D \frac{\sqrt{(m_N^2 - M_\Psi^2)(M_\Psi^2 - m_\nu^2)}}{\sqrt{2} M_\Psi M_V}, \quad \sin^2 \theta_L = \frac{m_\nu^2}{M_\Psi^2} \frac{m_N^2 - M_\Psi^2}{m_N^2 - m_\nu^2}, \quad \sin^2 \theta_R = \frac{m_N^2 - M_\Psi^2}{m_N^2 - m_\nu^2}. \quad (4.18)$$

Due to the mass hierarchy $M_V < M_\Psi < m_N$ and the fact that the neutrino mass is very small $m_\nu \sim 10^{-10} \text{ GeV}$, $\sin^2 \theta_i$ less than one. We also note if $M_\Psi = m_N$, we are in the completely decoupled case where there is no connection between the standard model and the dark sector.

Using the optical theorem, coupling constants can not be much larger than 4π , such we have a restriction for $r = \frac{m_N}{M_\Psi}$:

$$y_\nu = \sqrt{2} \frac{m_\nu}{v} r \leq 4\pi \quad \Rightarrow \quad 1 < r \leq \frac{4\pi v}{\sqrt{2} m_\nu} \quad (4.19)$$

A similar applies to y' , neglecting the mass of the neutrino:

$$y' \approx \frac{g_D m_N}{\sqrt{2} M_V} \sqrt{r^2 - 1} \leq 4\pi \quad (4.20)$$

This leads to the in-equation

$$m_N^4 - m_N^2 M_\Psi^2 - M_\Psi^2 \left(\frac{4\pi\sqrt{2}M_V}{g_D} \right)^2 \leq 0 \quad (4.21)$$

And give us a more restrictive constrain⁴ for the maximum value of the m_N , as we can see from figure 4.1. The restriction for m_N is as follows:

$$m_N \leq M_\Psi \sqrt{\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{1}{M_\Psi^2} \left(4\pi\sqrt{2}\frac{M_V}{g_D} \right)^2}} \quad (4.22)$$

We can also invert this relationship for a minimum value of M_Ψ , calling $C = 4\pi\sqrt{2}\frac{M_V}{g_D}$ we can write:

$$M_\Psi \geq \sqrt{\frac{-4(C^2 + m_N^2) + \sqrt{(4(C^2 + m_N^2))^2 + 16m_N^4}}{2}} \quad (4.23)$$

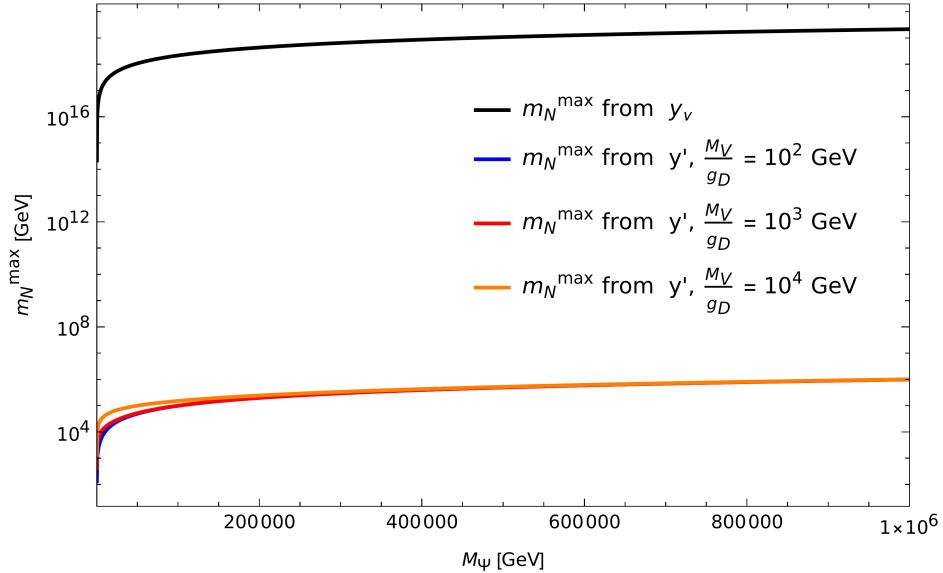


Figure 4.1: Maximum values for m_N from unitarity constrains of Yukawa couplings.

Finally, the unitarity of massive gauge bosons imposes a restriction on the mass of the dark scalar, M_{H_D} . In analogy with the standard model, where the Higgs boson propagator contributes

⁴Due to the dependence of the ratio $\frac{M_V}{g_D}$ in the 4.22 equation, it would be possible to make it more restrictive than 4.20, but we would require a ratio $\frac{M_V}{g_D} \sim 10^{30} \text{ GeV}$, this would be outside the mass range of the wimps in the case where M_V is very large, and if g_D is too small the relic density would be higher than observed.

to the process $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$ fixing the unitarity problem due to the longitudinal polarization of W boson, in our case dark Higgs have the same mathematical structure interaction Lagrangian $\mathcal{L}_{\Phi_D V}$ than standard model has in $\mathcal{L}_{HW}$, then we have this restriction for the mass of the scalar from partial wave decomposition:

$$M_{H_D}^2 \leq \frac{32\pi M_V^2}{\sqrt{2}g_D^2}. \quad (4.24)$$

Then, to have a scalar that can reach 1000 GeV (considering $g_D \sim 0.5^5$) we are going to set $M_V > 100 \text{ GeV}$ in the phenomenology simulations.

In summary, we have a 5-dimensional parameter-space $\{g_D, M_V, M_\Psi, m_N, M_{H_D}\}$ with the following constraints:

$$\begin{aligned} -4\pi &\leq g_D \leq 4\pi \\ 100 \text{ GeV} &\leq M_V \leq M_\Psi \\ M_V &\wedge \sqrt{\frac{-4(C^2 + m_N^2) + \sqrt{(4(C^2 + m_N^2))^2 + 16m_N^4}}{2}} \leq M_\Psi < m_N \\ M_\Psi < m_N &\leq M_\Psi \sqrt{\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{1}{M_\Psi^2} \left(4\pi\sqrt{2}\frac{M_V}{g_D}\right)^2}} \\ M_{H_D} &\leq \sqrt{\frac{32\pi M_V^2}{\sqrt{2}g_D^2}} \end{aligned} \quad (4.25)$$

4.2.3 Loop Induced interactions

In this model, Dark Matter does not couple either quarks or photons in the lagrangian, so we can not calculate direct and indirect cross sections at tree level. These interactions appear due to quantum corrections, we are considering the first corrections in perturbation theory, the one-loop corrections. Fortunately, the corrections are already calculated in [21] for the quark top FPVDM model, then we use those results and carefully modify them for this model. The next sub-sub-sections explain the loop implementation.

⁵This value is motivated by relic density, see Figure 4.4

VVZ Induced Vertex

We implemented a effective interaction between $V^\pm$ and Z bosons into the **LanHEP** [33] code through the following effective lagrangian:

$$\begin{aligned}
\mathcal{L}_{eff} = & F_{Z1} i (\partial_\mu V_\nu^+ V^{-\nu} Z^\mu - \partial_\mu V_\nu^- V^{+\nu} Z^\mu) \\
& + \frac{F_{Z2}}{M_V^2} i (\partial_\mu V_\alpha^+ V_\beta^- - \partial_\mu V_\beta^- V_\alpha^+) (i \partial^\alpha i \partial^\beta Z^\mu) \\
& + F_{Z3} i (\partial_\alpha Z_\beta - \partial_\beta Z_\alpha) V^{+\alpha} V^{-\beta} \\
& - F_{Z5} i \epsilon^{\alpha\beta\mu\rho} (i \partial_\rho V_\alpha^+ V_\beta^- - i \partial_\rho V_\beta^- V_\alpha^+) Z_\mu \\
& - \frac{F_{Z8}}{M_V^2} i (\epsilon^{\mu\alpha\rho\sigma} (i \partial_\rho V_\alpha^+ V_\beta^- - i \partial_\rho V_\beta^- V_\alpha^+) i \partial^\beta i \partial_\sigma Z_\mu - \epsilon^{\mu\beta\rho\sigma} (i \partial_\rho V_\alpha^+ V_\beta^- - i \partial_\rho V_\beta^- V_\alpha^+) i \partial^\alpha i \partial_\sigma Z_\mu)
\end{aligned} \tag{4.26}$$

Where F_{Zi} are form factors (further details in Appendix A), which are calculated from loop diagrams in Figure 4.2:

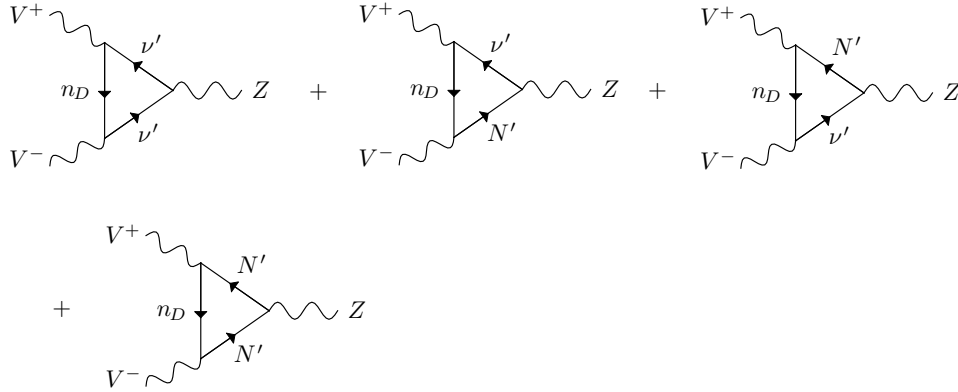


Figure 4.2: Triangle diagrams contribution to VVZ vertex.

Once implemented the effective lagrangian we obtain the effective vertex $V^{\alpha\beta\delta}(p_1, p_2, q)$, with p_1, p_2 and q the 4-momentum of V_α^+, V_β^- and Z_δ respectively:

$$\begin{aligned}
V^{\alpha\beta\delta} = & F_{1Z} (p_1 - p_2)^\delta g^{\alpha\beta} + \frac{F_{2Z}}{M_V^2} (p_1 - p_2)^\delta q^\alpha q^\beta + F_{3Z} (q^\alpha g^{\delta\beta} - q^\beta g^{\delta\alpha}) \\
& - i F_{5Z} \epsilon^{\alpha\beta\mu\rho} (p_1 - p_2)_\rho - i \frac{F_{8Z}}{M_V^2} (\epsilon^{\delta\alpha\rho\sigma} q^\beta - \epsilon^{\delta\beta\rho\sigma} q^\alpha) q_\sigma (p_1 - p_2)_\rho
\end{aligned} \tag{4.27}$$

Kinetic Mixing

At one loop level, Z boson and V^0 can mix their kinetic terms, thinking of the correction as an effective theory we can write:

$$\mathcal{L}_{KM} = -\frac{1}{4}Z_{\mu\nu}^2 + \frac{1}{2}m_Z^2 Z^2 - \frac{1}{4}(V_{\mu\nu}^0)^2 + \frac{1}{2}m_{V^0}^2 (V^0)^2 - \frac{\epsilon_{ZV}}{2}V_{\mu\nu}^0 Z^{\mu\nu} + \frac{1}{2}\Delta m_{ZV}^2 Z_\mu V^\mu + j_Z^\mu Z_\mu + j_{V^0}^\mu V_\mu^0 \quad (4.28)$$

Here $A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ for $A = Z, V^0$, ϵ_{ZV} and Δm_{ZV}^2 are the mixing parameter calculated from the loops (the values are provided in the Appendix B) in Figure 4.3, such that, it would be small, and finally j_A^μ are the fermionic currents connected to the bosons.

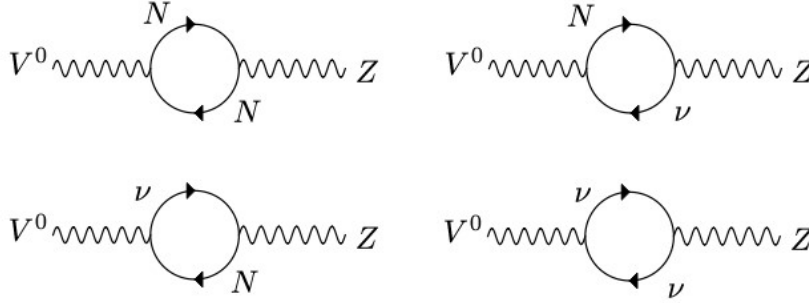


Figure 4.3: Feynman diagrams that contribute to the kinetic Mixing between V^0 and Z boson.

Then we can get rid of the mixing term weighted by ϵ_{ZV} rotating the states by the following matrix:

$$V^{KM} = \begin{pmatrix} 1 & -\frac{\epsilon_{ZV}}{\sqrt{1-\epsilon_{ZV}^2}} \\ 0 & \frac{1}{\sqrt{1-\epsilon_{ZV}^2}} \end{pmatrix} \quad (4.29)$$

therefore:

$$\begin{pmatrix} Z \\ V^0 \end{pmatrix} = V^{KM} \begin{pmatrix} Z' \\ V'^0 \end{pmatrix} \quad (4.30)$$

Now is easy to check that this election, which is not unique, eliminates the mixing term between fields Z and V^0 , However, the currents are affected for this rotation, such that:

$$j_Z^\mu Z'_\mu + \left(\frac{j_{V^0}^\mu}{\sqrt{1-\epsilon_{ZV}^2}} + \frac{j_Z \epsilon_{ZV}}{\sqrt{1-\epsilon_{ZV}^2}} \right) V'^0_\mu \quad (4.31)$$

Besides that there is no kinetic mix between fields, they aren't eigenstates since they are still coupled by a non-diagonal mass term, therefore, we must diagonalize the mass matrix:

$$M_{ZV}^2 = \begin{pmatrix} m_Z^2 & \frac{1}{2}\Delta m_{ZV}^2 \\ \frac{1}{2}\Delta m_{ZV}^2 & m_V^2 \end{pmatrix} \quad (4.32)$$

When we rotate the states, this matrix is transformed too:

$$M_{ZV}^{\prime 2} = (V^{KM})^T M_{ZV}^2 V^{KM} = \begin{pmatrix} m_Z^2 & \frac{\Delta m_{ZV}^2 - 2m_Z^2 \epsilon_{ZV}}{2\sqrt{1-\epsilon_{ZV}^2}} \\ \frac{\Delta m_{ZV}^2 - 2m_Z^2 \epsilon_{ZV}}{2\sqrt{1-\epsilon_{ZV}^2}} & \frac{m_V^2 - \Delta m_{ZV}^2 \epsilon_{ZV} + m_Z^2 \epsilon_{ZV}^2}{1-\epsilon_{ZV}^2} \end{pmatrix} \quad (4.33)$$

Now we can diagonalize through a rotation with angle θ_{ZV}

$$R(\theta_{ZV}) M_{ZV} R(\theta)^\dagger \quad (4.34)$$

Since the matrix is diagonal, the non-diagonal elements must be zero, then we obtain the value:

$$\tan 2\theta_{ZV} = 2 \frac{(\epsilon_{ZV} m_Z^2 - \frac{1}{2}\Delta m_{ZV}^2) \sqrt{1-\epsilon_{ZV}^2}}{m_Z^2(1-2\epsilon^2) - m_V^2 + \epsilon_{ZV} \Delta m_{ZV}^2} \quad (4.35)$$

Taking $\epsilon^2 \simeq 0$ and the term $\Delta m_{ZV} \epsilon_{ZV}$ is a second order term, so we can neglect it too, then considering that the angle is small:

$$\theta_{ZV} \simeq \frac{\epsilon_{ZV} m_Z^2 - \frac{1}{2}\Delta m_{ZV}^2}{m_Z^2 - m_V^2} \quad (4.36)$$

Using this angle we can rewrite Δm_{ZV}^2 as

$$\Delta m_{ZV}^2 = 2m_Z^2 \epsilon_{ZV} - 2\theta_{ZV} (m_Z^2 - m_V^2) \quad (4.37)$$

And expanding in series of ϵ_{ZV} θ_{ZV} at second order the masses are corrected by:

$$\begin{aligned} \hat{m}_Z^2 &= m_Z^2 \left(1 + \theta_{ZV}^2 \left(1 - \frac{m_V^2}{m_Z^2} \right) \right) \\ \hat{m}_V^2 &= m_V^2 \left(1 + (\theta_{ZV} - \epsilon_{ZV})^2 \left(1 - \frac{m_Z^2}{m_V^2} \right) \right) \end{aligned} \quad (4.38)$$

Therefore, the "prime" states in terms of the mass eigenstates are:

$$\begin{pmatrix} Z' \\ V'^0 \end{pmatrix} = R^\dagger(\theta) \begin{pmatrix} \hat{Z} \\ \hat{V}^0 \end{pmatrix} = \begin{pmatrix} \cos \theta_{ZV} \hat{Z} + \sin \theta_{ZV} \hat{V}^0 \\ -\sin \theta_{ZV} \hat{Z} + \cos \theta_{ZV} \hat{V}^0 \end{pmatrix} \quad (4.39)$$

This, again, modifies the interaction terms with bosons:

$$\begin{aligned}
& \left[\left(\cos \theta_{ZV} + \frac{\epsilon_{ZV}}{\sqrt{1-\epsilon^2}} \sin \theta_{ZV} \right) j_Z^\mu - \frac{\sin \theta_{ZV}}{\sqrt{1-\epsilon_{ZV}^2}} j_{V^0}^\mu \right] \hat{Z}_\mu \\
& + \left[\frac{\cos \theta_{ZV}}{\sqrt{1-\epsilon_{ZV}^2}} j_{V^0}^\mu + \left(\sin \theta_{ZV} - \frac{\cos \theta_{ZV} \epsilon_{ZV}}{\sqrt{1-\epsilon_{ZV}^2}} \right) j_Z^\mu \right] \hat{V}_\mu^0
\end{aligned} \tag{4.40}$$

writing the currents as

$$\begin{aligned}
j_Z^\mu &= -i \frac{g}{c_W} \bar{\psi}_i \gamma^\mu (T_3^{ij} - s_W^2 Q^{ij}) \psi_j \\
j_{V^0}^\mu &= -i g_D \bar{\psi}_i \gamma^\mu T_{D3}^{ij} \psi_j
\end{aligned} \tag{4.41}$$

With ψ the fermions that couples to the respective current. Now we can extract these changes to modify the covariant derivative. We only consider leading order terms, since ϵ_{ZV} is small $\sqrt{1-\epsilon_{ZV}^2} \simeq 1$ and $\epsilon^2 \simeq 0$, also the θ_{ZV} is small, so $\sin \theta_{ZV} \simeq \theta_{ZV}$, $\cos \theta \simeq 1$ and finally $\epsilon_{ZV} \cdot \sin \theta_{ZV}$ is also order two, so we can also neglect this term. Now the full covariant derivative implemented in our model read as (from now on Z and V^0 will refer to $\hat{Z}$ and $\hat{V}^0$, respectively and the same applies for the masses):

$$\begin{aligned}
D_\mu \simeq & \partial_\mu - i g_s \frac{\lambda^a}{2} G_\mu^a - i g \frac{T^\pm}{\sqrt{2}} W_\mu^\pm - i \left[\frac{g}{c_W} (T^3 - s_W^2 Q) - g_D T_D^3 \theta_{ZV} \right] Z_\mu - i e Q A_\mu \\
& - i g_D \frac{T_D^\pm}{\sqrt{2}} V_\mu^\pm - i \left[g_D T_D^3 + (\theta_{ZV} - \epsilon_{ZV}) \frac{g}{c_W} (T^3 - s_W^2 Q) \right] V_\mu^0
\end{aligned} \tag{4.42}$$

4.2.4 Relic Density

We implemented this model in **LanHEP** [33] and used **MicrOMEGAs v5.3.41** [34] to calculate the phenomenology, in this case, the Dark-Matter relic density, i.e., the amount of dark matter density present in the universe after the decoupling of DM, for this work this will be the density of $V^\pm$. First, in order to understand the behavior of the model, we calculate the relic density, by varying the mass of the WIMP for some values of g_D , Figure 4.4, finding that if we want the model to saturate the relic density, we require $0.5 < g_D < 1$ for masses about 1 TeV in according to some kind of weak interaction, then if dark matter is more massive, g_D must be bigger. The principal channels (bigger than 1% involved in the Dark matter annihilation that contributes to the decoupling of the thermal bath in the early universe are shown in Figure 4.5. As we can see in Figure 4.6 channel (a) is the most important contribution around 90 – 70% depending on the vector mass, then as the mass of the WIMP increases (b) and (c) becomes more important and the (d) channel appears only when the mass of the vector M_V is very close of M_Ψ .

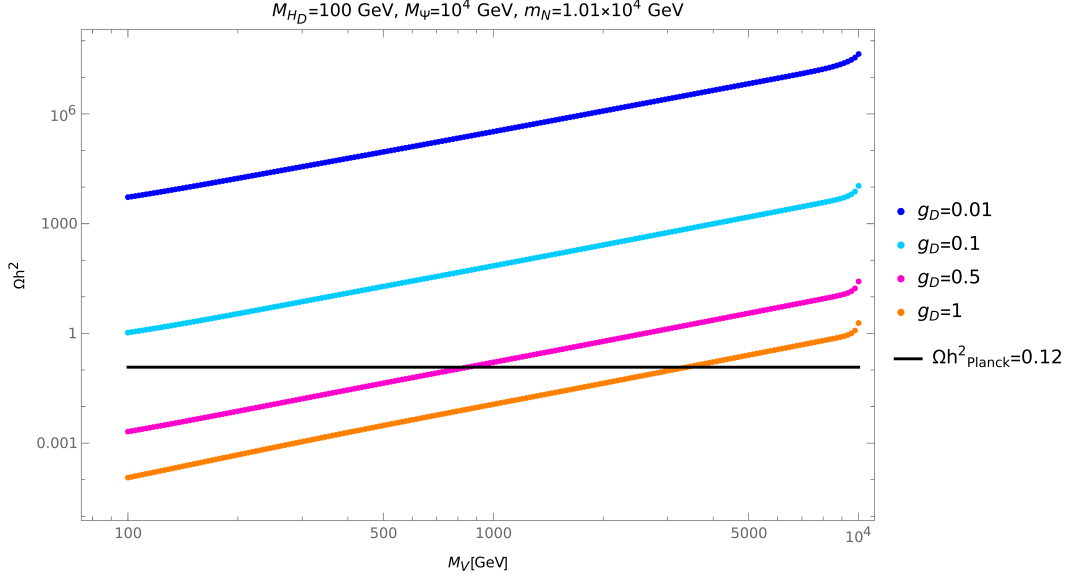


Figure 4.4: Relic density vs WIMP mass for different values of g_D . The black line shows the experimental data taken by the Planck satellite, i.e., the total amount of DM in the universe.

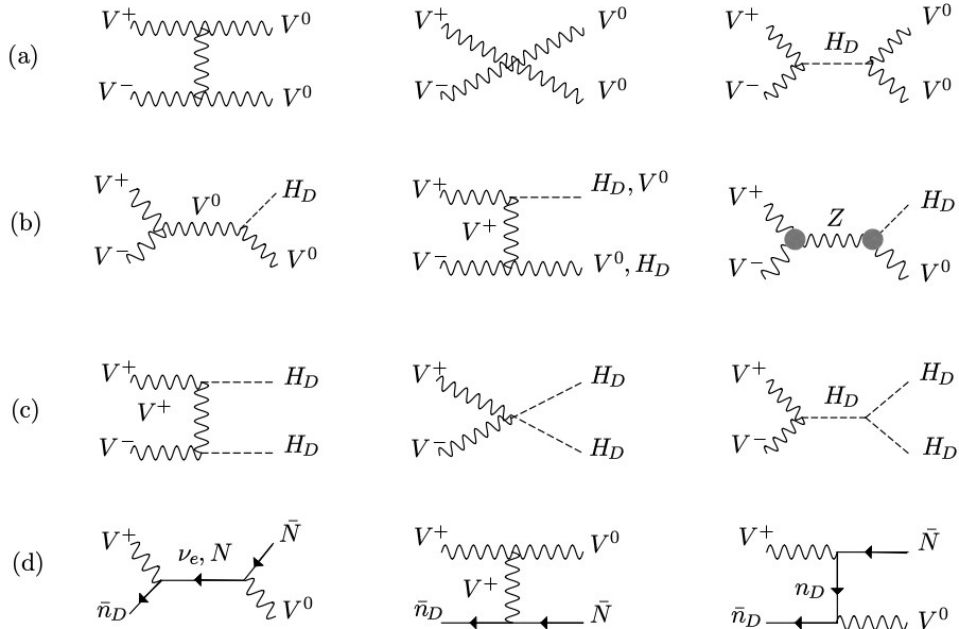


Figure 4.5: Main channels involved in the DM decouple, bigger than 1% of contribution to Ωh^2 . Gray dots make reference to the effective interactions discussed in the section above.

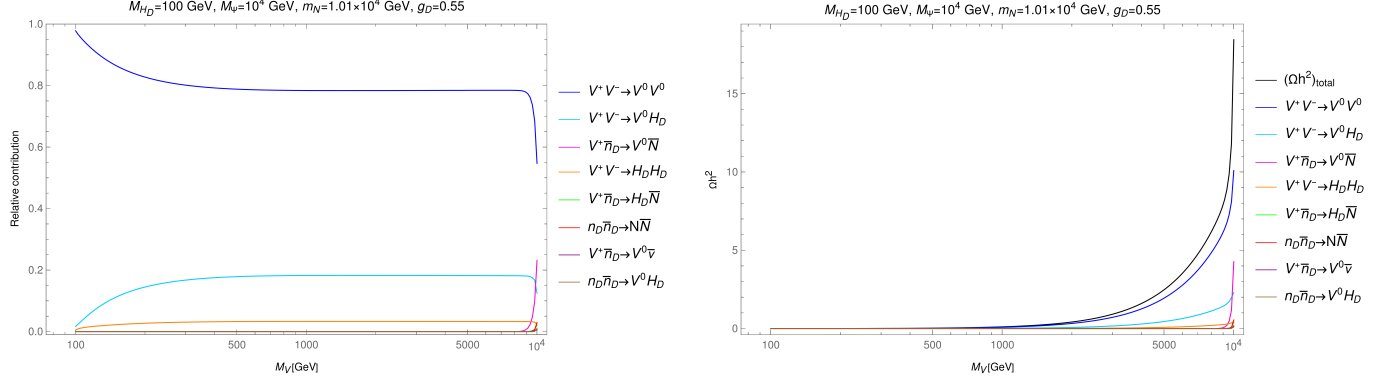


Figure 4.6: Left: Relative contribution for channels involved in relic the relic density calculation (bigger than 0.1%). Right: Total and partial Ωh^2 for channels with a contribution bigger than 0.1% to the total Ωh^2 .

We then calculate the relic density by scanning in different planes of the parameter space considering the unitarity constraints and around 40,000 points per graph. In each graph, we have plotted some contour lines in which the value of Ωh^2 remains constant and the dashed contour line is the value measured by Planck, $\Omega h^2 = 0.12$. This line separates the part of the model that is excluded because it is overabundant, with the sector that is closest to red being the excluded one. Some sectors of the plots are very close to blue, these blue sectors are sectors where some channels become resonant increasing the cross-section, and causing dark matter to annihilate even further, resulting in a reduction of the relic remaining after the dark matter decouples.

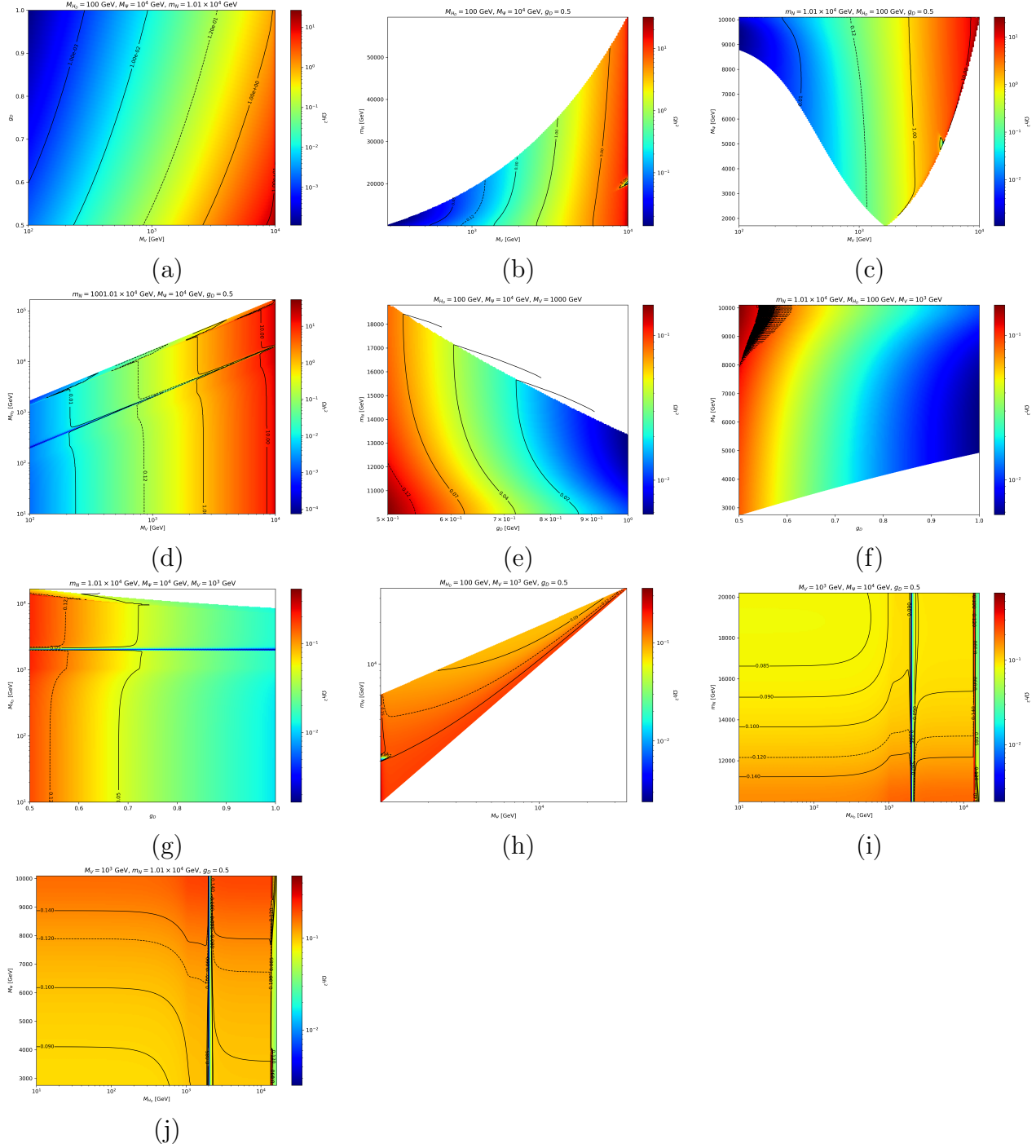


Figure 4.7: Relic density scan for projections on planes of the 5-dimensional parameter space for the standard values $M_V = 10^3$ GeV, $g = 5$, $m_N = 1.01 \times 10^4$ GeV, $M_\Psi = 10^4$ GeV and $M_{H_D} = 100$ GeV. In each graph, two of these parameters are varied leaving the rest fixed. (a), (b), (c), (d), (e), (f), (g), (h), (i) and (j) shows (M_V, g_D) , (M_V, m_N) , (M_V, M_Ψ) , (M_V, M_{H_D}) , (g_D, m_N) , (g_D, M_Ψ) , (g_D, M_{H_D}) , (M_Ψ, m_N) , (M_{H_D}, m_N) and (M_{H_D}, M_Ψ) planes, respectively. Black curves represent lines of constant value of relic density and the dashed one is the mean value measured by the Planck satellite $\Omega h^2 = 0.12$, points with values bigger than this are excluded for being overabundant. White zones are forbidden by unitary constrain with the exception of (c) where the region $M_V > M_\Psi$ is excluded because the DM candidate is n_D instead of vector $V^\pm$, and (h) where mass hierarchy impose $M_\Psi < m_N$.

4.2.5 Direct Detection

In this model, Dark Matter does not couple either quarks or photons in the lagrangian, so we can not calculate direct and indirect cross sections at tree level. As we mentioned before, these interactions arise from quantum corrections. We calculate the signal for spin-dependent and independent DM-nucleon cross section because does not depend on the nuclei targets of the direct detection experiments. These signals have two contributions, one from the kinetic mixing and the other from the triangle-loop. In both cases Z boson is who made the connection with quarks at the particle level, the Feynman diagrams are depicted in Figure 4.8.

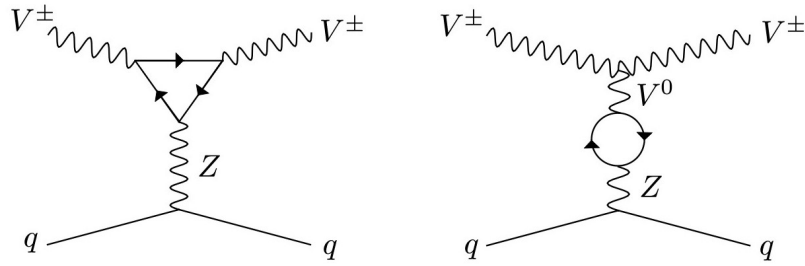


Figure 4.8: Feynman diagrams that contribute to the direct detection signature.

We used **micrOMEGAs** to calculate the cross sections, this software uses a complicated algorithm explained in detail in [26], which considers the quark content of the nucleons. The results for both contributions are shown in figure 4.9 we note that the Kinetic mixing is the dominant process for spin-independent cross-section, while in spin-dependent they are comparable, they are in the same order of magnitude.

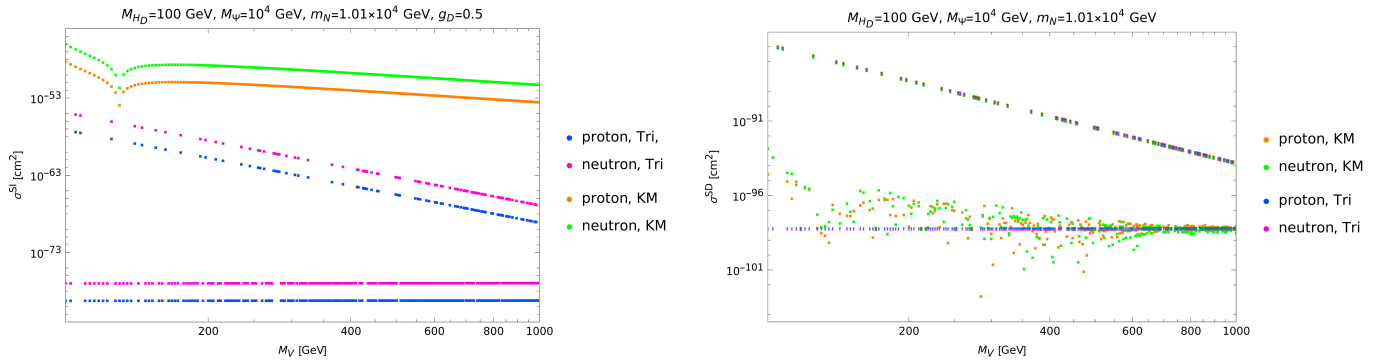


Figure 4.9: Contributions from Triangle Loop (Tri) and Kinetic Mixing (KM) to the DM-Nucleon cross-section. The left side shows the contributions for the spin-independent cross section and the Right side the spin-dependent one

Due to the fact that our dark matter candidate is, in general, under-abundant⁶ we follow the method used in [35] defining a scale factor:

$$\mathcal{F} = \begin{cases} 0 & \text{if } \Omega h^2 > 0.12 \\ \frac{\Omega h^2}{0.12} & \text{other case} \end{cases} \quad (4.43)$$

Since an under-abundant candidate means the model does not reproduce all the observed DM in the universe, this factor represents the probability that the dark matter particle be our vector candidate, such that

$$\sigma_{\text{DM-Nucleon}}^{\text{eff}} = \mathcal{F} \cdot \sigma_{\text{DM-Nucleon}} \quad (4.44)$$

Then, the measured quantity would be the effective cross-section $\sigma_{\text{DM-Nucleon}}^{\text{eff}}$, which is plotted in Figures 4.10 to 4.12. We observe in Figure 4.10.(a) and (b) that the variation of g_D is suppressed by the $\mathcal{F}$ since the relic depends strongly on the value of g_D if this coupling is bigger then DM annihilates more and then becomes under-abundant, lowering the scattering probability. From Figure 4.10.(c) we note that the points that saturate the relic are those that are closer to the M_Ψ value and a more promising cross-section is achieved when the masses of the fermions are closer to the value of the wimp mass, Figure.4.10.(e) and (f) tells us that for a bigger cross-section the masses of M_Ψ and m_N must be they should be as far away from each other as possible, this can be explained by the mixing angles of the fermions, which depends on the masses m_N and M_Ψ , and when these are equal, $\sin \theta_i$ ($I = R, L$) return a zero value decoupling the dark matter from standard model.

⁶We see in the previous section that can be over-abundant, but this point of the parameter space is excluded.

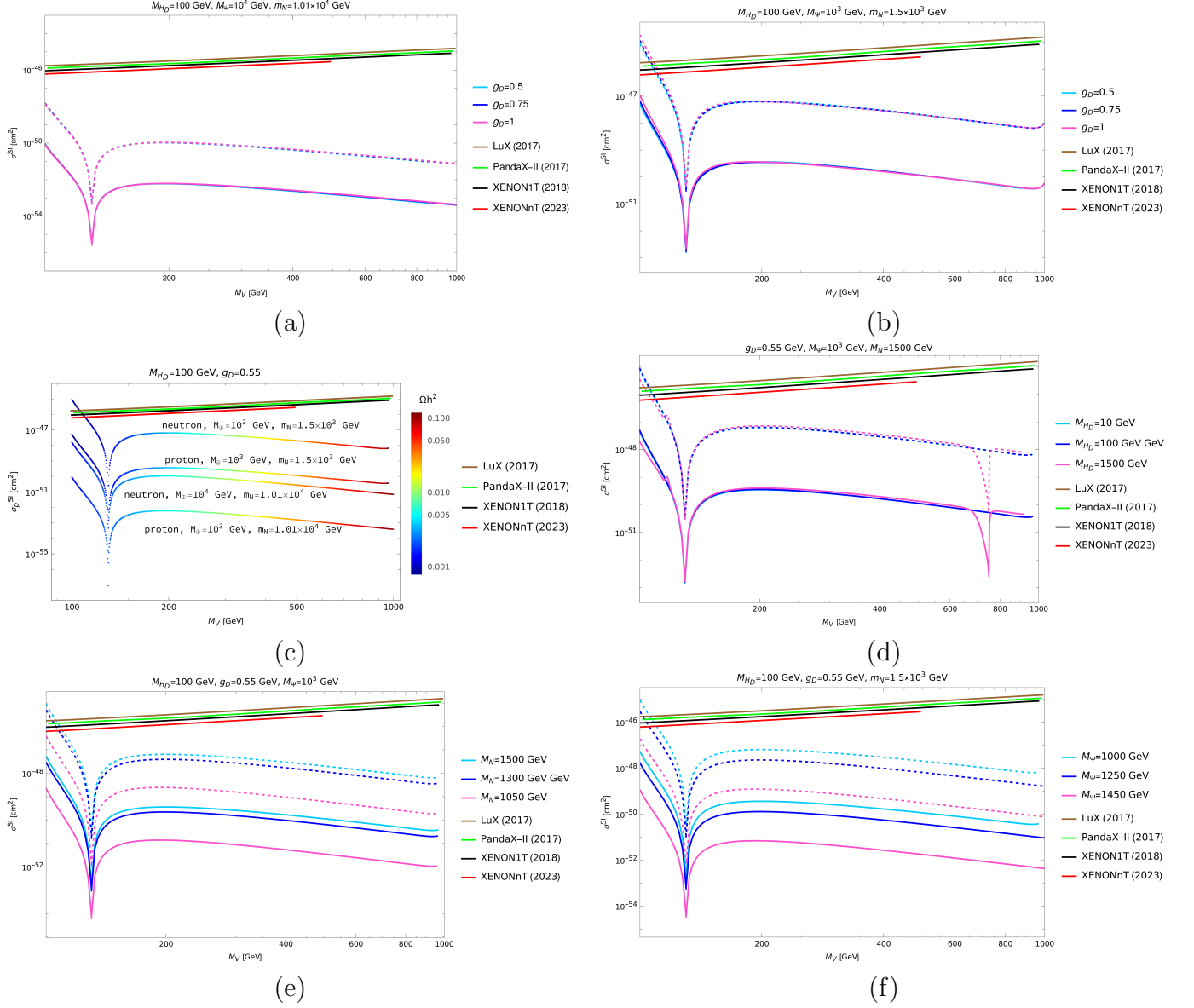


Figure 4.10: Graphs for spin-independent DM-Nucleon cross-section for different variation of parameters. Experimental upper limits, taken from [3, 5] are shown for Lux, PandaX-II, XENON1T and XENONnT. Solid lines represent proton cross-section, while dashed ones are for neutrons.

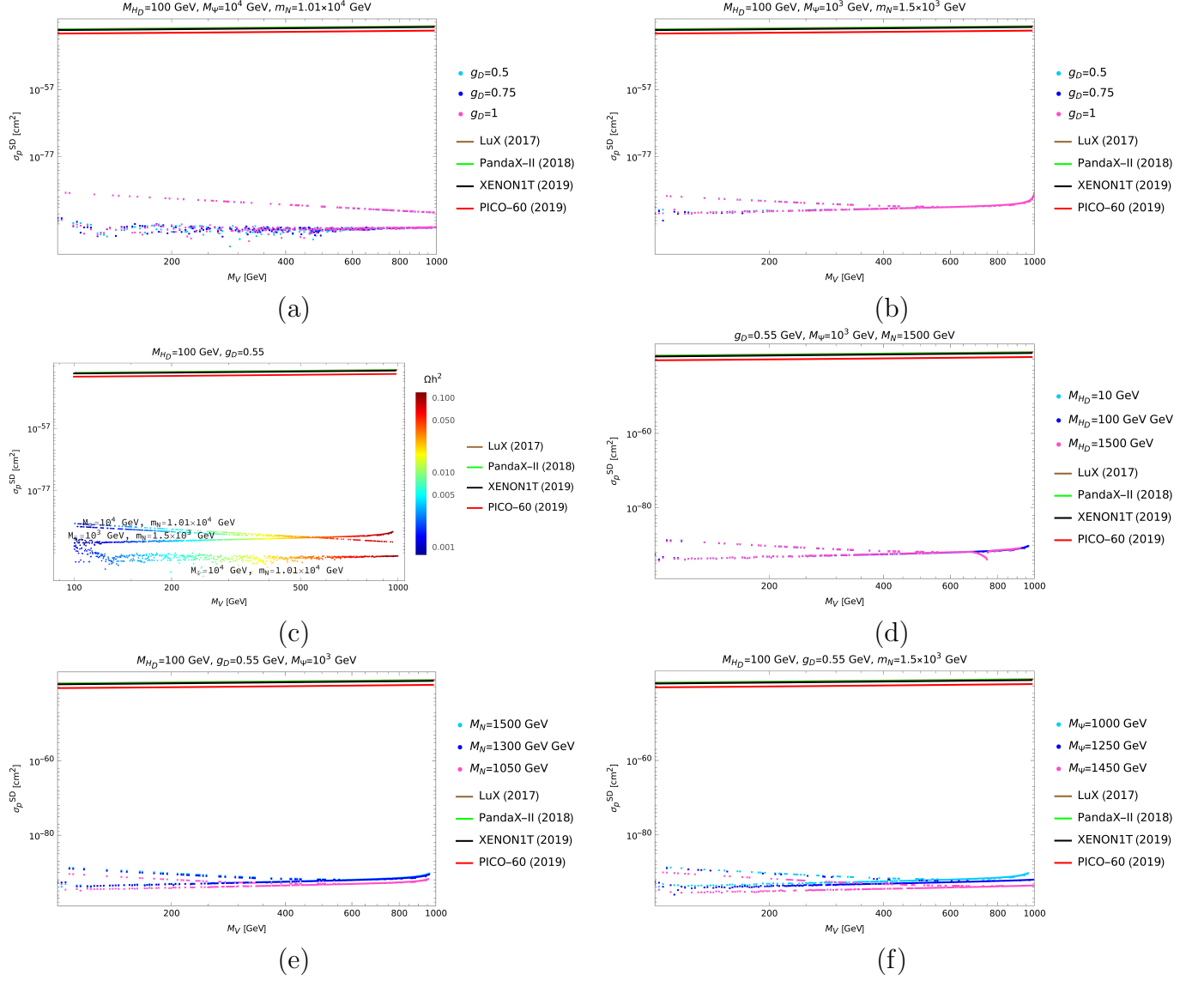


Figure 4.11: Graphs for spin-dependent DM-proton cross-section for different variation of parameters. Experimental upper limits, taken from [3], are shown for Lux, PandaX-II, XENON1T and PICO-60.

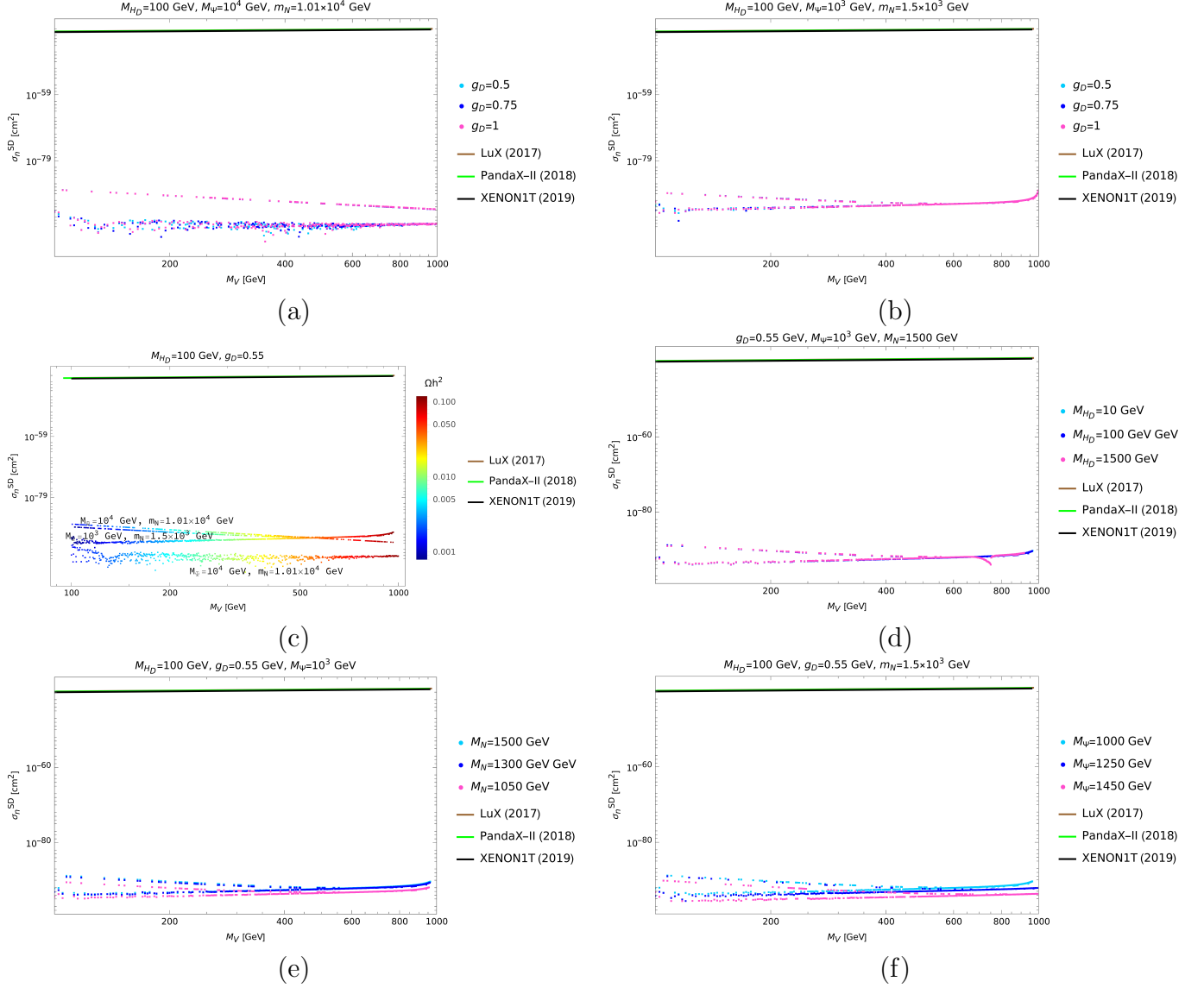


Figure 4.12: Graphs for spin-dependent DM-neutron cross-section for different variation of parameters. Experimental upper limits, taken from [3], are shown for Lux, PandaX-II, and XENON1T.

4.2.6 Indirect Detection

In this section, we present our results for Indirect detection signals. Considering that in regions dense enough in the universe, DM can annihilate to standard model particles, which then will produce photons that we would detect. Since particle physics input⁷ to the photon flux, correspond to the average cross section times the velocity $\langle\sigma v\rangle$. Some experiments like H.E.S.S [32] had

⁷Technically the photon spectrum of the i th-channel, $\frac{dN_i}{dE_\gamma}$, also correspond to the particle physics input, but as it depends on the decays, hadronization and other process, which are non-model dependent we take only take in consideration the model dependent part $expval\sigma v$.

measured upper limits for the DM annihilation cross section $\langle\sigma v\rangle$ (DM DM $\rightarrow$ SM SM), with SM SM = $\{b\bar{b}\}, \{\tau^+\tau^-\}, \{W^+W^-\}$, therefore we had calculated these quantities using the combined power of **MicrOMEGAs** and **CalcHEP** [36].

We used **MicrOMEGAs** to calculate $\langle\sigma v\rangle$, unfortunately, the main contributions, bigger than 1% are the process of the kind of dark matter annihilation to non-standard model particles. The main channels are the same that contribute to the relic density ((a), (b) and (c) from Figure 4.5) and channels that go direct to the standard model as $V^+V^- \rightarrow ZZ$, which is the bigger one, after neutrinos that do not contribute to the photon flux, its contribution is around $\sim 10^{-30}$ times the total cross-section. Therefore we have to think of another way to produce SM particles, but the particles of the main channels mentioned before are unstable, they can decay to SM or non-SM particles, for that reason we calculated the branching ratios of V^0 and H_D using **calcchep**. The calculated quantity is:

$$\langle\sigma v\rangle (V^+V^- \rightarrow \text{SM SM}) = \mathcal{F}^2 \sum_i \langle\sigma v\rangle_{\text{non-SM}}^i P_{\text{non-SM} \rightarrow \text{SM}}^i \quad (4.45)$$

Here, we again use the $\mathcal{F}$ factor, defined in the equation 4.43, this time we have two DM particles in the initial state, hence the square. $\langle\sigma v\rangle_{\text{non-SM}}$ are the values calculated by **MicrOMEGAs**. Then, the non-SM final states can decay, in the case of V^0 this particle can decay in many channels, so we consider the branching ratio of interest calculated with **Calcchep**, $Br(b\bar{b})$, $Br(\tau^+\tau^-)$ and $Br(W^+W^-)$, which depend on M_V . For the final state V^0V^0 we need to consider that at least one of these decays to the interest particles, for example, $b\bar{b}$, then in this case:

$$P_{\text{non-SM} \rightarrow b\bar{b}}^{V^0V^0} = 2Br(b\bar{b}) - Br(b\bar{b})^2. \quad (4.46)$$

"Dark Higgs" decay to $\nu\bar{\nu}$ 100% of the time, other Branching ratios are negligible, for this reason:

$$P_{\text{non-SM} \rightarrow b\bar{b}}^{V^0H_D} = Br(b\bar{b}) \quad (4.47)$$

$$P_{\text{non-SM} \rightarrow b\bar{b}}^{H_DH_D} = 0 \quad (4.48)$$

for the other particles, this is the same, but with the respective branching ratios. The results of indirect detection for DM annihilating to $b\bar{b}$, $\tau^-\tau^+$ and W^+W^- with the respective limits taken from [31, 32] are shown in Figures 4.13, 4.14 and 4.15, respectively. Where we can observe that $\langle\sigma v\rangle$ is small when g_D grows and is smaller when the values of M_Ψ and m_N are close.

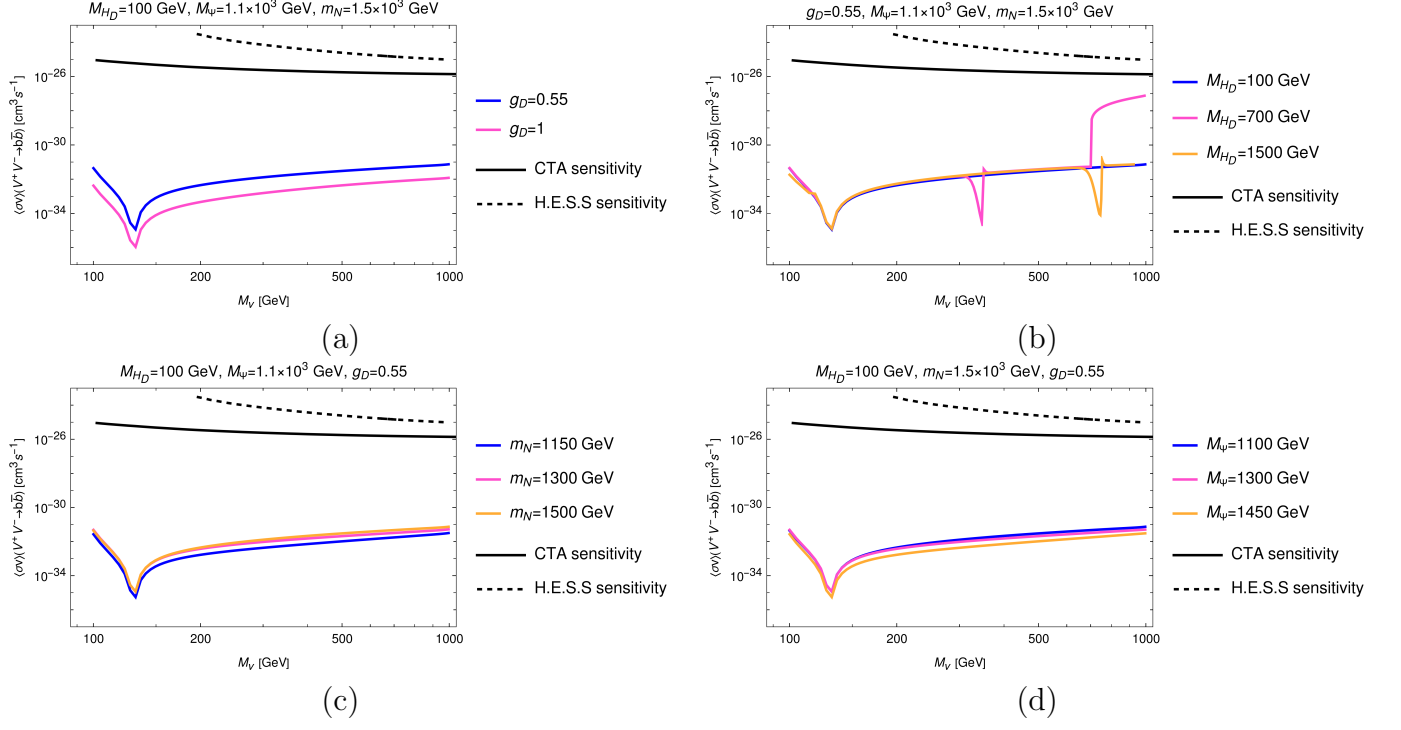


Figure 4.13: Average cross section times velocity for the annihilation process $V^+V^- \rightarrow b\bar{b}$ for different values of the parameter space. The solid black line shows the CTA expected sensitivity and the dashed black line the H.E.S.S. sensitivity..

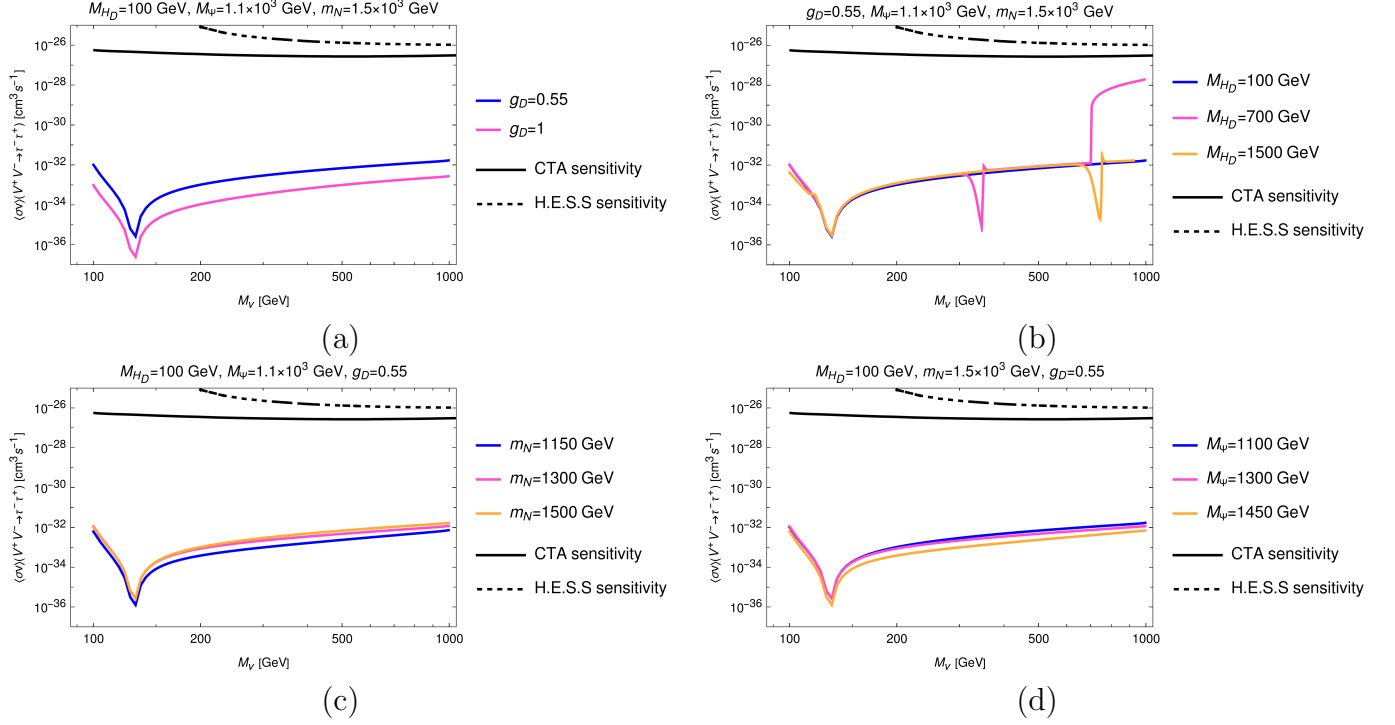


Figure 4.14: Average cross section times velocity for the annihilation process $V^+V^- \rightarrow \tau^+\tau^-$ for different values of the parameter space. The solid black line shows the CTA expected sensitivity and the dashed black line the H.E.S.S. sensitivity.

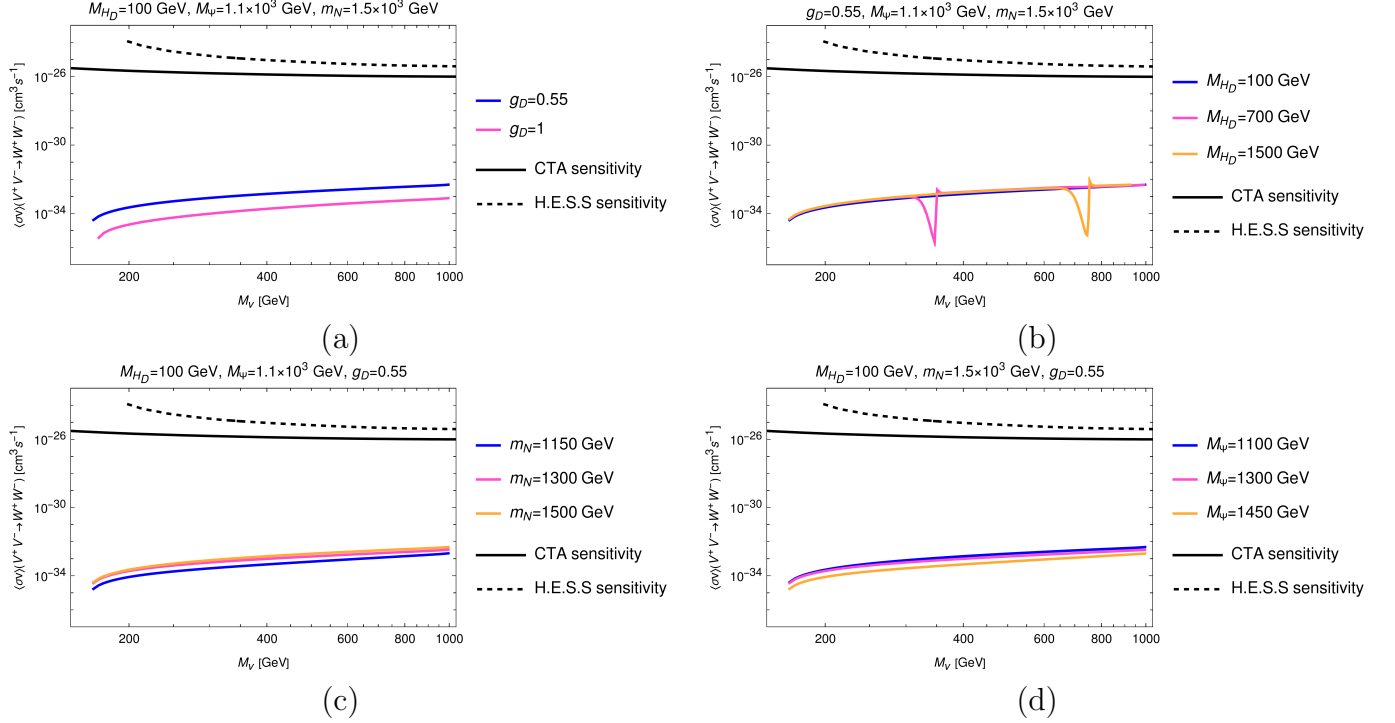


Figure 4.15: Average cross section times velocity for the annihilation process $V^+V^- \rightarrow W^+W^-$ for different values of the parameter space. The solid black line shows the CTA expected sensitivity and the dashed black line the H.E.S.S. sensitivity.

Figures 4.13.b and 4.14.b because there is an exception for $M_{H_D} = 700$ GeV, when H_D becomes massive enough to decay to heavier particles, which is the case, so the channels for DM annihilating to $V^0 H_D$ and $H_D H_D$ open later when DM has enough mass to produce an on-shell H_D boson and then this boson decays mainly in ZZ around 100% of the time, and Z presents larger branching ratios for the particles of interest. Since we need at least one of these particles to decay to the SM ones the equations 4.47 and 4.48 are modified to:

$$P_{\text{non-SM} \rightarrow b\bar{b}}^{V^0 V^0} = Br_V(b\bar{b}) + 2Br_Z(b\bar{b}) - 2Br_V(b\bar{b})^2 Br_Z(b\bar{b}) + Br_Z^2(b\bar{b}) Br_V(b\bar{b}). \quad (4.49)$$

and

$$P_{\text{non-SM} \rightarrow b\bar{b}}^{V^0 H_D} = 4Br_Z(b\bar{b}) - 6Br_Z^2(b\bar{b}) + 4Br_Z^3(b\bar{b}) - Br_Z^4(b\bar{b}) \quad (4.50)$$

Analogous for $\tau^+\tau^-$ channel (W^+W^- channel remains the same as before). We have used $Br_Z(b\bar{b}) = 15.1\%$ and $Br_Z(\tau^+\tau^-) = 3.4\%$ calculated with **CalcHEP** and coincides with the values presented by the PDG [37]. The pick where the value cross-section downs are due to the relic density becomes underabundant due to the resonance of a channel mediated by H_D , weighing on the $\mathcal{F}$ factor.

4.3 NPVDM with Two Ψ Generations

We now consider a more complete model that takes into account neutrino oscillations, but only with two families. Although we are not going to perform neutrino oscillation calculations, we will have to deal with the consequences of the oscillations when we diagonalize the mass matrix, because when we consider massive neutrinos the neutrino mass eigenstates mix their masses in the interaction states present in the Lagrangian.

$$\begin{aligned} \mathcal{L} = \mathcal{L}_{SM} - \frac{1}{4}(V_{\mu\nu}^a)^2 + \sum_k \bar{\Psi}_k (i\not{D} - M_\Psi) \Psi_k + |D_\mu \Phi_D|^2 + \mu_D^2 \Phi_D^\dagger \Phi_D - \lambda_D (\Phi_D^\dagger \Phi_D)^2 \\ + \sum_k i \bar{\nu}_{kR} \not{D} \nu_{kR} - \sum_{l,k} y_{lk}^\nu \bar{L}_l i \sigma_2 H \nu_{kR} - \sum_k y'_{kk} \bar{\Psi}_{kL} \Phi_D \nu_{kR} + h.c. \end{aligned} \quad (4.51)$$

Where $k = 1, 2$ and $l = e, \mu, \tau$.

4.3.1 Fermion Sector

Similarly to the minimal model case, the masses of VL-fermions mix with the neutrinos masses:

$$-\mathcal{L}_{m_f} = M_\Psi (\bar{n}_{Dk} n_{Dk} + \bar{N}_k N_k) + (m_D)_{lk} \bar{\nu}_{lL} \nu_{kL} + (M_D)_{kk} \bar{N}_{kL} \nu_{kR} + h.c. \quad (4.52)$$

Where the mass matrices are defined as:

$$\frac{y_\nu}{\sqrt{2}} = m_D = \begin{pmatrix} m_{e1} & m_{e2} \\ m_{\mu 1} & m_{\mu 2} \\ m_{\tau 1} & m_{\tau 2} \end{pmatrix} \quad \frac{y' v_D}{\sqrt{2}} = M_D = \begin{pmatrix} M_{D1} & 0 \\ 0 & M_{D2} \end{pmatrix}. \quad (4.53)$$

Using the following vector notation:

$$\nu_L = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_L \quad N_L = \begin{pmatrix} N_1 \\ N_2 \end{pmatrix}_L \quad \nu_R = \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}_R \quad N_R = \begin{pmatrix} N_1 \\ N_2 \end{pmatrix}_R \quad (4.54)$$

we can rewrite in matrix form:

$$\begin{pmatrix} \bar{\nu}_L & \bar{N}_L \end{pmatrix} \mathcal{M} \begin{pmatrix} N_R \\ \nu_R \end{pmatrix} = \begin{pmatrix} \bar{\nu}_L & \bar{N}_L \end{pmatrix} \begin{pmatrix} 0 & m_D \\ M_\Psi & M_D \end{pmatrix} \begin{pmatrix} N_R \\ \nu_R \end{pmatrix} \quad (4.55)$$

The mass matrix can be changed into a block diagonal form by unitary transformations of the left- and right-handed spinors $\mathbb{M}$

$$\begin{pmatrix} \mathbb{1} & -\frac{m_D}{M_D} \\ \frac{m_D^T}{M_D} & \mathbb{1} \end{pmatrix} \begin{pmatrix} 0 & m_D \\ M_\Psi & M_D \end{pmatrix} \begin{pmatrix} \mathbb{1} & \frac{M_\Psi}{M_D} \\ -\frac{M_\Psi}{M_D} & \mathbb{1} \end{pmatrix} = \begin{pmatrix} -\frac{M_\Psi}{M_D} m_D & -\left(\frac{M_\Psi}{M_D}\right)^2 m_D \\ -\frac{m_D^T M_\Psi}{M_D^2} m_D & \left(\frac{m_D^T m_D}{M_D^2} + \frac{M_\Psi^2}{M_D^2} + 1\right) M_D \end{pmatrix} \quad (4.56)$$

$$\simeq \begin{pmatrix} -m_\nu & 0 \\ 0 & M_D \end{pmatrix}$$

We have assumed that M_D is bigger than m_D and M_Ψ , then $\frac{M_\Psi}{M_D}$ ⁸ is small, but $\left(\frac{M_\Psi}{M_D}\right)^2 \rightarrow 0$ is negligible, therefore $m_\nu = \frac{M_\Psi}{M_D} m_D$ is the neutrino mass matrix and the elements are small because M_D is bigger, this is a seesaw mechanism.

Then in terms of mass eigenstates:

$$\begin{pmatrix} \bar{\nu}_L & \bar{N}_L \end{pmatrix} \mathcal{M} \begin{pmatrix} N_R \\ \nu_R \end{pmatrix} = \begin{pmatrix} \bar{\nu}'_L & \bar{N}'_L \end{pmatrix} \begin{pmatrix} U_{\text{PMNS}} & 0 \\ 0 & \mathbb{1} \end{pmatrix} \begin{pmatrix} -m_\nu & 0 \\ 0 & M_D \end{pmatrix} \begin{pmatrix} U_R^\dagger & 0 \\ 0 & \mathbb{1} \end{pmatrix} \begin{pmatrix} N'_R \\ \nu'_R \end{pmatrix} \quad (4.57)$$

$$= -\bar{\nu}'_L U_{\text{PMNS}} m_\nu U_R^\dagger N'_R + \bar{N}'_L M_D \nu'_R$$

Where U_R is the matrix that diagonalizes the product $m_\nu^\dagger m_\nu$ and U_{PMNS} is the Pontecorvo–Maki–Nakagawa–Sakata matrix that diagonalizes $m_\nu m_\nu^\dagger$

$$U_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix} \quad (4.58)$$

following the procedure used in [38] we can write m_ν as:

$$m_\nu = \begin{pmatrix} \frac{M_{\Psi 1}}{M_{D1}} & \frac{M_{\Psi 2}}{M_{D2}} \end{pmatrix} \begin{pmatrix} (m_D)_{11} \\ (m_D)_{21} \\ (m_D)_{31} \end{pmatrix} \begin{pmatrix} (m_D)_{12} \\ (m_D)_{22} \\ (m_D)_{32} \end{pmatrix} \quad (4.59)$$

$$= \begin{pmatrix} m_a \vec{u} & m_b \vec{v} \end{pmatrix}$$

Where $\vec{u}$ and $\vec{v}$ are 3-complex vectors with unit norm. Therefore the square product is:

$$m_\nu m_\nu^\dagger = m_a^2 \vec{u} \vec{u}^\dagger + m_b^2 \vec{v} \vec{v}^\dagger \quad (4.60)$$

⁸Understand $1/A$ as the inverse of the matrix A , A^{-1} .

Now the eigenvalues, i.e., the neutrino masses are:

$$\mu_0 = m_{\nu 1}^2 = 0, \quad \mu_{\pm} = m_{\nu 2,3}^2 = \frac{1}{2} \left(m_a^2 + m_b^2 \pm \sqrt{(m_a^2 - m_b^2)^2 + 4m_a^2 m_b^2 \rho^2} \right) \quad (4.61)$$

We can use $\rho = u^\dagger \cdot v \in [0, 1]$ as a free parameter. Then, the normalized eigenvectors are:

$$\begin{aligned} \hat{e}_0 &= \frac{\vec{u}^* \times \vec{v}^*}{\sqrt{1 - |\vec{u} \cdot \vec{v}|}} \\ \hat{e}_{\pm} &= \frac{\vec{e}_{\pm}}{|\vec{e}_{\pm}|} = \frac{A\vec{u} + B^{\pm}v}{\sqrt{A^2 + 2AB^{\pm}\rho + (B^{\pm})^2}} \end{aligned} \quad (4.62)$$

Where $A = 2m_a^2\rho$ and $B^{\pm} = m_b^2 - m_a^2 \pm \sqrt{(m_a^2 - m_b^2)^2 + 4m_a^2 m_b^2 \rho^2}$, as $\hat{e}_i$ are eigenvectors, then we can write $U_{\text{PMNS}}^\dagger = (\hat{e}_0 \quad \hat{e}_+ \quad \hat{e}_-)$, therefore the 3-vectors can be rewritten as:

$$\begin{aligned} v_j &= \frac{(U_{\text{PMNS}}^*)_{3j}|\vec{e}_+| - (U_{\text{PMNS}}^*)_{2j}|\vec{e}_-|}{2\sqrt{(m_a^2 - m_b^2)^2 + 4m_a^2 m_b^2 \rho^2}} \\ u_j &= \frac{(U_{\text{PMNS}}^*)_{3j}|\vec{e}_+| + (U_{\text{PMNS}}^*)_{2j}|\vec{e}_-|}{4m_a^2\rho} - \frac{m_b^2 - m_a^2}{2m_a^2\rho} v_j \end{aligned} \quad (4.63)$$

Diagonalizing $m_\nu^\dagger m_\nu$ we can also obtain the values of U_R :

$$U_R = \begin{pmatrix} -\frac{\sqrt{2}m_a m_b \rho (m_a^2 - m_b^2 + \sqrt{Q_{U_R}})^{-1/2}}{Q_{U_R}^{1/4}} & \frac{\sqrt{2}m_a m_b \rho (m_b^2 - m_a^2 + \sqrt{Q_{U_R}})^{-1/2}}{Q_{U_R}^{1/4}} \\ \frac{m_a^2 - m_b^2 + \sqrt{Q_{U_R}}}{\sqrt{2}Q_{U_R}^{1/4}} & \frac{m_b^2 - m_a^2 + \sqrt{Q_{U_R}}}{\sqrt{2}Q_{U_R}^{1/4}} \end{pmatrix} \quad (4.64)$$

With $Q_{U_R} = m_a^4 + m_b^4 + 2m_a^2 m_b^2 (2\rho - 1)^2$. With all this, we are ready to write the mass eigenstates of the fermionic fields:

$$\begin{aligned} \nu'_L &= U_{\text{PMNS}}(\nu_L - \frac{m_D}{M_D} N_L) \\ N'_L &= N_L + \frac{m_D^T}{M_D} \nu_L \\ N'_R &= U_R(N_R - \frac{M_\Psi}{M_D} \nu_R) \\ \nu'_R &= \nu_R + \frac{M_\Psi}{M_D} N_R \end{aligned} \quad (4.65)$$

And more important, due to its use in **LanHEP** code, the interaction fields in terms of mass eigenstates

fields:

$$\begin{aligned}
\nu_{Lj} &= (U_{\text{PMNS}}^\dagger)_{jk} \nu'_{Lk} + \frac{(m_D)_{jk}}{M_{Dk} N'_{Lk}} \\
N_{Lj} &= N'_{Lj} - \frac{(m_D^T)_{jk}}{M_{Dk}} \nu'_{Lk} \\
N_{Rj} &= (U_R^\dagger)_{jk} N'_{Rk} + \frac{M_{\Psi k}}{M_{Dk}} \nu'_{Rk} \\
\nu_{Rj} &= \nu'_{Rj} - \frac{M_{\Psi k}}{M_{Dk}} N'_{Rk}
\end{aligned} \tag{4.66}$$

Chapter 5

Conclusions and Future Work

The neutrino portal model is a viable alternative to connect the dark sector and the standard model, as it is able to saturate the observed relic density, as well as to separate the relic density and direct detection phenomenologies avoiding the Higgs-Portal tension. Relic density and unitarity can provide the most stringent constraints to the parameter space since they restrict the masses of fermions to being very close to each other, however, at the moment there are no constraints at the moment for how big these masses can be. Is necessary to do in the future a random parameter scan that provides us the points where this model is allowed, for that purpose the inclusion of CMB-indirect detection would be necessary. The positive aspect of this model is that in the future with more precise sensibility and maybe a better understanding of the background, a signal for direct detection could be observed at relatively small masses or for indirect detection due to the "Dark Higgs" decays can significantly increase the annihilation cross-section.

Since the realization of the two-generation model arrived quite late in the development of this work, all the phenomenology will remain as future work. This is a more accurate model that can incorporate the neutrino oscillation phenomenology in the same framework and for that reason is more complicated, but the development of the minimal model with one generation will make the path easier at the time of studying this model. Also, we can give some information in advance for the two-generation model: due to more families of VL-fermion is possible that it would be more restrictive because it would have more channels for the scattering and annihilation process that can contribute to increasing the detection signal simulated, but at the same time the new channels would decrease the relic density and this could lower the intensity of these signals. Because one of the neutrino mass eigenstates is massless, the loops of this model will be a little more delicate to deal with.

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Appendix A

Triangle Loop

Fortunately, the general problem for a triangle loop diagram in this kind of model has already been solved in the FPDVDM article [21]. Here we show how this calculation is implemented for the case of one generation in NFPVDM. Triangle diagrams for the minimal model effective VVZ vertex are shown in Figure 4.2. Once we implement this vertex, the Dark Sector will be able to connect with the standard model. Following the appendix of [21], and according to previous articles [39, 40], we can write the most general CP conserving effective lagrangian as follows:

$$\begin{aligned}
\mathcal{L}_{eff} = & F_{Z1} i (\partial_\mu V_\nu^+ V^{-\nu} Z^\mu - \partial_\mu V_\nu^- V^{+\nu} Z^\mu) \\
& + \frac{F_{Z2}}{M_V^2} i (\partial_\mu V_\alpha^+ V_\beta^- - \partial_\mu V_\beta^- V_\alpha^+) (i \partial^\alpha i \partial^\beta Z^\mu) \\
& + F_{Z3} i (\partial_\alpha Z_\beta - \partial_\beta Z_\alpha) V^{+\alpha} V^{-\beta} \\
& - F_{Z5} i \epsilon^{\alpha\beta\mu\rho} (i \partial_\rho V_\alpha^+ V_\beta^- - i \partial_\rho V_\beta^- V_\alpha^+) Z_\mu \\
& - \frac{F_{Z8}}{M_V^2} i (\epsilon^{\mu\alpha\rho\sigma} (i \partial_\rho V_\alpha^+ V_\beta^- - i \partial_\rho V_\beta^- V_\alpha^+) i \partial^\beta i \partial_\sigma Z_\mu - \epsilon^{\mu\beta\rho\sigma} (i \partial_\rho V_\alpha^+ V_\beta^- - i \partial_\rho V_\beta^- V_\alpha^+) i \partial^\alpha i \partial_\sigma Z_\mu)
\end{aligned} \tag{A.1}$$

In momentum space, this is:

$$\mathcal{L}_{eff} = -i V^{\alpha\beta\rho}(p_1, p_2, q) V_\alpha^+(p_1) V_\beta^-(p_2) Z_\delta(q). \tag{A.2}$$

This will lead us to the effective vertex with the following Lorentz structure.

$$\begin{aligned}
V^{\alpha\beta\delta} = & F_{1Z} (p_1 - p_2)^\delta g^{\alpha\beta} + \frac{F_{2Z}}{M_V^2} (p_1 - p_2)^\delta q^\alpha q^\beta + F_{3Z} (q^\alpha g^{\delta\beta} - q^\beta g^{\delta\alpha}) \\
& - i F_{5Z} \epsilon^{\alpha\beta\mu\rho} (p_1 - p_2)_\rho - i \frac{F_{8Z}}{M_V^2} (\epsilon^{\delta\alpha\rho\sigma} q^\beta - \epsilon^{\delta\beta\rho\sigma} q^\alpha) q_\sigma (p_1 - p_2)_\rho
\end{aligned} \tag{A.3}$$

Now we need to calculate the F_{iZ} doing the 1-Loop and matching the result with the Lorentz structure of the effective vertex. This was done considering the two generic graphs in Figure A.1

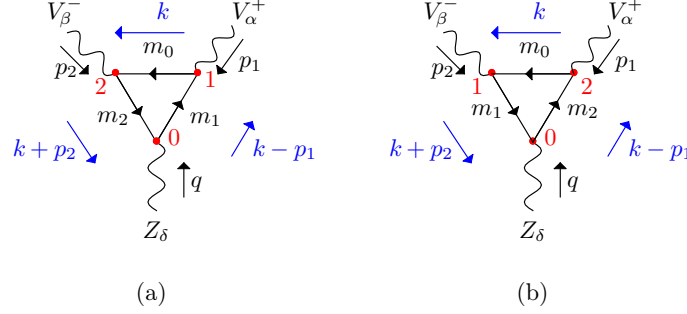


Figure A.1: Generic triangle diagrams for VVZ vertex.

The explicit Loop is:

$$\begin{aligned}
i\mathcal{M}^{\alpha\beta\delta} = & \int \frac{d^4k}{(2\pi)^4} \frac{Tr\{\gamma^\delta(v_0 - a_0\gamma_5)(\not{k} - \not{p}_1 + m_1)\gamma^\alpha(v_2 - a_2\gamma_5)(\not{k} + m_0)\gamma^\beta(v_1 - a_1\gamma_5(\not{k} + \not{p}_2 + m_2))\}}{((k - p_1)^2 - m_1^2)(k^2 - m_0^2)((k + p_2)^2 - m_2^2)} \\
& + \int \frac{d^4k}{(2\pi)^4} \frac{Tr\{\gamma^\delta(v_0 - a_0\gamma_5)(\not{k} - \not{p}_1 + m_2)\gamma^\alpha(v_2 - a_2\gamma_5)(\not{k} + m_0)\gamma^\beta(v_1 - a_1\gamma_5(\not{k} + \not{p}_2 + m_1))\}}{((k - p_1)^2 - m_2^2)(k^2 - m_0^2)((k + p_2)^2 - m_1^2)}
\end{aligned} \tag{A.4}$$

Where v_i and a_i are the vectors and axial constant for the vertices that appear in Fig A.1 (the respective values are shown in Table A.1) and The first and second terms represent Figure A.1 .a and A.1 .b Feynman diagrams, respectively. Since the loop has three Lorentz indices and propagators, the Passariono-Veltman functions to calculate are:

$$C_0 = -i \int \frac{d^4k}{\pi^2} \frac{1}{(k^2 - m_0^2)((k - p_1)^2 - m_1^2)((k + p_2)^2 - m_2^2)} \tag{A.5}$$

$$C^\mu = -i \int \frac{d^4k}{\pi^2} \frac{k^\mu}{(k^2 - m_0^2)((k - p_1)^2 - m_1^2)((k + p_2)^2 - m_2^2)} = -p_1^\mu C_1 + p_2^\mu C_2 \tag{A.6}$$

$$\begin{aligned}
C^{\mu\nu} = & -i \int \frac{d^4k}{\pi^2} \frac{k^\mu k^\nu}{(k^2 - m_0^2)((k - p_1)^2 - m_1^2)((k + p_2)^2 - m_2^2)} \\
= & g^{\mu\nu} C_{00} + p_1^\mu p_1^\mu C_{11} + p_2^\mu p_2^\mu C_{22} - \{p_1 p_2\}^{\mu\nu} C_{12}
\end{aligned} \tag{A.7}$$

$$\begin{aligned}
C^{\mu\nu\rho} &= -i \int \frac{d^4k}{\pi^2} \frac{k^\mu k^\nu k^\rho}{(k^2 - m_0^2)((k - p_1)^2 - m_1^2)((k + p_2)^2 - m_2^2)} \\
&= -\{p_1 g\}^{\mu\nu\rho} C_{001} + \{p_2 g\}^{\mu\nu\rho} C_{002} + \{p_1 p_1 p_2\}^{\mu\nu\rho} C_{112} - \{p_1 p_2 p_2\}^{\mu\nu\rho} C_{122} \\
&\quad - p_1^\mu p_1^\mu p_1^\mu C_{111} + p_2^\mu p_2^\mu p_2^\mu C_{222}
\end{aligned} \tag{A.8}$$

Considering the two generic diagrams, we have that these functions for A.1.a and for A.1.a are the same, but with the change $m_1 \rightarrow m_2$ and $m_2 \rightarrow m_1$.

Now we will show the result for the form factors in terms of the Veltman-Passarino functions:

$$\begin{aligned}
f_1 &= \frac{V_{PPP}}{4} (1 - (m_1^2 - m_2^2) \Delta C_1 - (m_1^2 + m_2^2) \bar{C}_1 + 8 \bar{C}_{001}) - \frac{V_{PMM}}{2} m_2 m_1 \bar{C}_1 + \\
&\quad + \frac{V_{MPM}}{4} m_2 m_0 (\bar{C}_0 + 2 \bar{C}_1) + \frac{V_{MMP}}{4} m_0 m_1 (\bar{C}_0 + 2 \bar{C}_1)
\end{aligned} \tag{A.9}$$

$$f_2 = -V_{PPP} M_V^2 (\bar{C}_{12} + 2 \bar{C}_{112}) \tag{A.10}$$

$$\begin{aligned}
f_3 &= \frac{V_{PPP}}{4} (-2 + (m_1^2 - m_2^2) \Delta C_1 - 2 m_0 \bar{C}_0 + (m_1^2 + m_2^2) \bar{C}_1 - 4 m_0^2 \bar{C}_1 + 8 \bar{C}_{00} + 8 \bar{C}_{001}) \\
&\quad - \frac{V_{MMP}}{4} m_0 m_1 (2 \Delta C_1 - C_0) - \frac{V_{MPM}}{4} m_2 m_0 (2 \Delta C_1 + \bar{C}_0) + \frac{V_{PMM}}{2} m_2 m_1 \bar{C}_1
\end{aligned} \tag{A.11}$$

$$\begin{aligned}
f_5 &= -\frac{A_{PPP}}{4} ((m_1^2 - m_2^2) \Delta C_1 + (m_1^2 + m_2^2) \bar{C}_1) - \frac{A_{MMP}}{4} m_0 m_1 (\bar{C}_0 + 2 \bar{C}_1) \\
&\quad - \frac{A_{MPM}}{4} m_2 m_0 (\bar{C}_0 + 2 \bar{C}_1) - \frac{A_{PMM}}{2} m_2 m_1 \bar{C}_1
\end{aligned} \tag{A.12}$$

$$f_8 = \frac{A_{PPP} M_V^2 \bar{C}_{12}}{2} \tag{A.13}$$

Where the $\bar{C}_i = \frac{C_i^{(a)} + C_i^{(b)}}{2}$ and $\Delta C_i = \frac{C_i^{(a)} - C_i^{(b)}}{2}$, with C_i the scalar components of C -tensors Veltman-Passarino functions, (a) and (b) make reference to the denominator of first and second

term of eq. A.4, respectively. The factors V_i and A_i are:

$$\begin{aligned}
V_{PPP} &= \frac{1}{\pi^2}(v_0 v_2 v_1 + v_0 a_2 a_1 + a_0 v_2 a_1 + a_0 a_2 v_1) & A_{PPP} &= \frac{1}{\pi^2}(a_0 a_2 a_1 + a_0 v_2 v_1 + v_0 a_2 v_1 + v_0 v_2 a_1) \\
V_{PMM} &= \frac{1}{\pi^2}(v_0 v_2 v_1 + v_0 a_2 a_1 - a_0 v_2 a_1 - a_0 a_2 v_1) & A_{PMM} &= \frac{1}{\pi^2}(a_0 a_2 a_1 + a_0 v_2 v_1 - v_0 a_2 v_1 - v_0 v_2 a_1) \\
V_{MPM} &= \frac{1}{\pi^2}(v_0 v_2 v_1 - v_0 a_2 a_1 + a_0 v_2 a_1 - a_0 a_2 v_1) & A_{MPM} &= \frac{1}{\pi^2}(a_0 a_2 a_1 - a_0 v_2 v_1 + v_0 a_2 v_1 - v_0 v_2 a_1) \\
V_{MMP} &= \frac{1}{\pi^2}(v_0 v_2 v_1 - v_0 a_2 a_1 - a_0 v_2 a_1 + a_0 a_2 v_1) & A_{MMP} &= \frac{1}{\pi^2}(a_0 a_2 a_1 - a_0 v_2 v_1 - v_0 a_2 v_1 + v_0 v_2 a_1)
\end{aligned} \tag{A.14}$$

Finally, we have to evaluate the $f_i(v_0, a_0, v_2, a_2, v_1, a_1, m_0, m_2, m_1)$ function ($i = 1, 2, 3, 5, 8$) with the respective coupling values and add by the four diagrams contributions:

$$\begin{aligned}
F_{iZ} &= \frac{1}{2} f_i(v_{Z\nu\nu}, a_{Z\nu\nu}, v_{Vn_D\nu}, a_{Vn_D\nu}, v_{Vn_D\nu}, a_{Vn_D\nu}, m_{n_D}, m_\nu, m_\nu) \\
&+ \frac{1}{2} f_i(v_{ZNN}, a_{ZNN}, v_{Vn_DN}, a_{Vn_DN}, v_{Vn_DN}, a_{Vn_DN}, m_{n_D}, m_N, m_N) \\
&+ f_i(v_{Z\nu N}, a_{Z\nu N}, v_{Vn_D\nu}, a_{Vn_D\nu}, v_{Vn_DN}, a_{Vn_DN}, m_{n_D}, m_\nu, m_N).
\end{aligned} \tag{A.15}$$

Vertex	v_i	a_i
$Z\nu\nu$	$\frac{e}{4s_W c_W} c_L^2$	$\frac{e}{4s_W c_W} c_L^2$
ZNN	$\frac{e}{4s_W c_W} s_L^2$	$\frac{e}{4s_W c_W} s_L^2$
$Z\nu N$	$\frac{e}{4s_W c_W} s_L c_L$	$\frac{e}{4s_W c_W} s_L c_L$
$V^\pm n_D \nu$	$\frac{-\sqrt{2}g_D}{4}(s_L + s_R)$	$\frac{-\sqrt{2}g_D}{4}(s_L - s_R)$
$V^\pm n_D N$	$\frac{\sqrt{2}g_D}{4}(c_L + c_R)$	$\frac{\sqrt{2}g_D}{4}(c_L - c_R)$

Table A.1: Vector and Axial coupling values for the vertex that appear in the triangle loops.

A.1 Functions used

The following are functions used in the code to calculate the PV functions at $q^2 \rightarrow 0$.

$$\begin{aligned}
x_1 &= -\frac{M_V^2 + m_0 - m_1^2}{M_V^2} \\
x_2 &= -\frac{M_V^2 + m_0 - m_2^2}{M_V^2} \\
y &= \frac{m_0^2}{m_V^2}
\end{aligned} \tag{A.16}$$

we use x instead of x_1 or x_2 if $m_1 = m_2$

$$f(x, y) = \int_0^1 \frac{d\rho}{\rho^2 + x\rho + y} = \begin{cases} \frac{2}{\lambda} \arctan\left(\frac{\lambda}{2y+x}\right); & \lambda = \sqrt{4y - x^2} \quad \text{if } y > \frac{x^2}{4} \\ \frac{1}{\sqrt{x^2 - 4y}} \ln\left(\frac{2y+x+\lambda}{2y+x-\lambda}\right); & \lambda = \sqrt{x^2 - 4y} \quad \text{if } y < \frac{x^2}{4} \end{cases} \quad (\text{A.17})$$

$$\begin{aligned} C_{int}(M_V^2, m_0^2, m_1^2, n) &= - \int_0^1 \frac{d\rho M_V^2 \rho^n}{(M_V^2 \rho(1 - \rho) + (m_1^2 - m_0^2)\rho + m_0^2)} \\ &= \begin{cases} f(x, y) & \text{if } n = 0 \\ \frac{1}{2} \ln\left(\frac{m_1^2}{m_0^2}\right) - \frac{1}{2} x f(x, y) & \text{if } n = 1 \\ \frac{1}{n-1} - x C_{int}(n-1) - y C_{int}(n-2) & \text{if } n > 1 \end{cases} \end{aligned} \quad (\text{A.18})$$

$$\begin{aligned} L_{int}(M_V^2, m_0^2, m_2^2, \mu^2, n) &= \int_0^1 \rho^n \ln\left(\frac{-(M_V^2 \rho(1 - \rho) + m_2^2 \rho + m_1^2(1 - \rho))}{\mu^2}\right) \\ &= \frac{1}{n+1} \left[-\ln\left(\frac{m_1^2}{\mu^2}\right) - 2C_{int}(n+2) - x C_{int}(n+1) \right] \end{aligned} \quad (\text{A.19})$$

$$\begin{aligned} \Omega C_{int}(M_V^2, m_0^2, m_1^2, m_2^2, \mu^2 = M_V^2, l, n) &= - \int_0^1 d\omega \int_0^1 d\rho \frac{M_V^2 \rho^l \omega^n}{M_V^2 \rho(1 - \rho) + ((m_2^2(1 - \omega)) + m_1^2 \omega - m_0^2)\rho + m_0^2} \\ &= \begin{cases} \frac{1}{x_1 - x_2} (L_{int}(M_V^2, m_0^2, m_1^2, \mu^2, l-1) - L_{int}(M_V^2, m_0^2, m_2^2, \mu^2, l-1)) & \text{if } n = 0 \\ \frac{1}{x_1 - x_2} \left(\frac{1}{nl} - \Omega C_{int}(l+1, n-1) - x_2 \Omega C_{int}(l, n-1) - y \Omega C_{int}(l-1, n-1) \right) & \text{if } n > 0 \end{cases} \end{aligned} \quad (\text{A.20})$$

$$\begin{aligned} \Omega L_{int} &= \int_0^1 d\omega \int_0^1 d\rho \rho^l \omega^n \ln\left(-\frac{M_V^2 \rho(1 - \rho) + (m_2^2(1 - \omega) + m_1^2 \omega - m_0^2)\rho + m_1^2}{\mu^2}\right) \\ &= \frac{1}{n+1} (L_{int}(M_V^2, m_0^2, m_1^2, \mu^2, l) + (x_2 - x_1) \Omega C_{int}(M_V^2, m_0^2, m_1^2, m_2^2, l+1, n+1)) \end{aligned} \quad (\text{A.21})$$

Then, PV functions are related to this functions in the following way

$$C_0(M_V, m_0, m_1, m_2) = \begin{cases} -\frac{C_{int}(M_V^2, m_0^2, m_1^2, 1)}{M_V^2} & \text{if } m_1^2 = m_2^2 \\ -\frac{\Omega C_{int}(M_V^2, m_0^2, m_1^2, m_2^2, 1, 0)}{M_V^2} & \text{if } m_1^2 \neq m_2^2 \end{cases} \quad (\text{A.22})$$

$$C_1(M_V, m_0, m_1, m_2) = \begin{cases} \frac{C_{int}(M_V^2, m_0^2, m_1^2, 2)}{2M_V^2} & \text{if } m_1^2 = m_2^2 \\ \frac{\Omega C_{int}(M_V^2, m_0^2, m_1^2, m_2^2, 2, 1)}{M_V^2} & \text{if } m_1^2 \neq m_2^2 \end{cases} \quad (\text{A.23})$$

$$C_2 = C_1 \quad (\text{A.24})$$

$$C_{11}(M_V, m_0, m_1, m_2) = \begin{cases} -\frac{C_{int}(M_V^2, m_0^2, m_1^2, 3)}{3M_V^2} & \text{if } m_1^2 = m_2^2 \\ -\frac{\Omega C_{int}(M_V^2, m_0^2, m_1^2, m_2^2, 3, 2)}{M_V^2} & \text{if } m_1^2 \neq m_2^2 \end{cases} \quad (\text{A.25})$$

$$C_{22} = C_{11} \quad (\text{A.26})$$

$$C_{12}(M_V, m_0, m_1, m_2) = \begin{cases} -\frac{C_{int}(M_V^2, m_0^2, m_1^2, 3)}{6M_V^2} & \text{if } m_1^2 = m_2^2 \\ \frac{-\Omega C_{int}(M_V^2, m_0^2, m_1^2, m_2^2, 3, 1) + \Omega C_{int}(M_V^2, m_0^2, m_1^2, m_2^2, 3, 2)}{M_V^2} & \text{if } m_1^2 \neq m_2^2 \end{cases} \quad (\text{A.27})$$

$$C_{00}(M_V, m_0, m_1, m_2, \mu) = \begin{cases} -\frac{L_{int}(M_V^2, m_0^2, m_1^2, \mu^2, 1)}{2} & \text{if } m_1^2 = m_2^2 \\ -\frac{\Omega L_{int}(M_V^2, m_0^2, m_1^2, m_2^2, \mu^2, 1, 0)}{2} & \text{if } m_1^2 \neq m_2^2 \end{cases} \quad (\text{A.28})$$

$$C_{111}(M_V, m_0, m_1, m_2) = \begin{cases} \frac{C_{int}(M_V^2, m_0^2, m_1^2, 4)}{4M_V^2} & \text{if } m_1^2 = m_2^2 \\ -\frac{\Omega C_{int}(M_V^2, m_0^2, m_1^2, m_2^2, 4, 3)}{M_V^2} & \text{if } m_1^2 \neq m_2^2 \end{cases} \quad (\text{A.29})$$

$$C_{222} = C_{111} \quad (\text{A.30})$$

$$C_{112}(M_V, m_0, m_1, m_2) = \begin{cases} \frac{C_{int}(M_V^2, m_0^2, m_1^2, 4)}{12M_V^2} & \text{if } m_1^2 = m_2^2 \\ -\frac{\Omega C_{int}(M_V^2, m_0^2, m_1^2, m_2^2, 4, 2) - \Omega C_{int}(M_V^2, m_0^2, m_1^2, m_2^2, 4, 3)}{M_V^2} & \text{if } m_1^2 \neq m_2^2 \end{cases} \quad (\text{A.31})$$

$$C_{122} = C_{112} \tag{A.32}$$

$$C_{001}(M_V, m_0, m_1, m_2, \mu) = \begin{cases} \frac{L_{int}(M_V^2, m_0^2, m_1^2, \mu^2, 2)}{4} & \text{if } m_1^2 = m_2^2 \\ \frac{\Omega L_{int}(M_V^2, m_0^2, m_1^2, m_2^2, \mu^2, 2, 1)}{2} & \text{if } m_1^2 \neq m_2^2 \end{cases} \tag{A.33}$$

$$C_{002} = C_{001} \tag{A.34}$$

Appendix B

Kinetic Mixing

For kinetic mixing we use the general expression of gauge boson vacuum polarization tensor [41] and the result of [21], then in the $q^2 \rightarrow 0$ limit read

$$\begin{aligned}\Pi_T^{ZV} &= \frac{1}{3-2\epsilon} \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} \right) \Pi_{\mu\nu}^{ZV} \\ \Pi_T^{ZV}(q^2 \rightarrow 0) &\sim \frac{gg_D}{64\pi^2 c_W} [3m_\nu^2 \mathcal{F}_m^{ZV}(r_\nu, r_\psi) + q^2 (\mathcal{F}_{qT}^{ZV}(r_\nu, r_\psi))]\end{aligned}\tag{B.1}$$

$$\begin{aligned}\Pi_L^{ZV} &= \frac{q^\mu q^\nu}{q^2} \Pi_{\mu\nu}^{ZV} \\ Pi_L^{ZV}(q^2 \rightarrow 0) &\sim \frac{gg_D}{64\pi^2 c_W} [3m_\nu^2 \mathcal{F}_m^{ZV}(r_\nu, r_\psi) + q^2 (\mathcal{F}_{qL}^{ZV}(r_\nu, r_\psi))]\end{aligned}\tag{B.2}$$

Where $r_\nu = \frac{m_\nu}{M_\Psi}$ and $r_\psi = \frac{M_\Psi}{m_N}$. Then the total contribution is:

$$Pi^{ZV}(q^2 \rightarrow 0) \sim \frac{gg_D}{64\pi^2 c_W} [6m_\nu^2 \mathcal{F}_m^{ZV}(r_\nu, r_\psi) + q^2 (\mathcal{F}_{qT+qL}^{ZV}(r_\nu, r_\psi))]\tag{B.3}$$

And the interest constants are:

$$\epsilon_{ZV} = \frac{gg_D}{64\pi^2 c_W} \mathcal{F}_{qT+qL}^{ZV}(r_\nu, r_\psi)\tag{B.4}$$

$$\Delta m_{ZV}^2 = \frac{gg_D}{64\pi^2 c_W} 6m_\nu^2 \mathcal{F}_m^{ZV}(r_\nu, r_\psi)\tag{B.5}$$

And the functions used are:

$$\begin{aligned} \mathcal{F}_{qT+qL}^{ZV}(r_\nu, r_\psi) = & \frac{2(r_\nu^2 - 1)(r_\psi - 1)}{3} \left(\frac{3r_\nu^6 r_\psi^6 - 5r_\nu^6 r_\psi^4 - 21r_\nu^4 r_\psi^4 + 22r_\nu^4 r_\psi^2 - 21r_\nu^2 r_\psi^2 - 5r_\nu^2 + 3}{(r_\nu r_\psi^2 - 1)^4} \right. \\ & \left. + 6 \frac{r_\nu^8 r_\psi^8 - 3r_\nu^6 r_\psi^4 + 12r_\nu^4 r_\psi^4 - 3r_\nu^4 r_\psi^2 + r_\nu^2}{(r_\nu r_\psi^2 - 1)^5} \ln(r_\nu r_\psi) \right) \end{aligned} \quad (\text{B.6})$$

$$\mathcal{F}_m^{ZV}(r_\nu, r_\psi) = (r_\nu^2 - 1)(r_\psi^2 - 1) \left(\frac{1 - 4r_\psi^2 + r_\nu^2 r_\psi^2}{r_\psi^2 (r_\nu^2 r_\psi^2 - 1)^2} + \frac{4(r_\nu^2 r_\psi^2 - r_\nu^2 + 1 \ln(r_\nu r_\psi))}{(r_\nu^2 r_\psi^2 - 1)^3} \right) \quad (\text{B.7})$$